An enhanced fletcher-reeves-like conjugate gradient methods for image restoration

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ABSTRACT
Noise is an unavoidable aspect of modern camera technology, causing a decline in the overall visual quality of the images. Efforts are underway to diminish noise without compromising essential image features like edges, corners, and other intricate structures. Numerous techniques have already been suggested by many researchers for noise reduction, each with its unique set of benefits and drawbacks. Denoising images is a basic challenge in image processing. We describe a two-phase approach for removing impulse noise in this study. The adaptive median filter (AMF) for salt-and-pepper noise identifies noise candidates in the first phase. The second step minimizes an edge-preserving regularization function using a novel hybrid conjugate gradient approach. To generate the new improved search direction, the new algorithm takes advantage of two well-known successful conjugate gradient techniques. The descent property and global convergence are proven for the new methods. The obtained numerical results reveal that, when applied to image restoration, the new algorithms are superior to the classical fletcher reeves (FR) method in the same domain in terms of maintaining image quality and efficiency.

Keywords:
Conjugate gradient methods
Global convergence
Image restoration
Impulse noise reduction
Optimization

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1. INTRODUCTION

The subject of image restoration has been widely researched and applied in numerous fields of science and engineering. It requires the reconstruction of an original scene from a deteriorated observation. For example, air turbulence degrades star pictures viewed by ground-based telescopes. Images are often exposed to noise due to environmental factors, transmission channels, and other related elements during acquisition, compression, and transmission. As a result, the image quality is affected, leading to distortion and loss of image information. Noise also impacts later image processing tasks, such as the analysis and tracking of images, as well as video processing. Thus, image denoising is a crucial aspect of modern image processing systems.

Image denoising aims to restore the original image quality by minimizing noise from a noise-rich image. However, since noise, edge, and texture are high-frequency constituents, it is challenging to differentiate between them during denoising. As a result, restored images may lose some important details. Overall, the challenge in image processing systems is to recover relevant information from noisy images during noise removal to produce high-quality images.

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There are certain cases where stellar pictures must be recovered even if they have not been observed within the atmosphere. The fundamental goal of this research is to create a class of iterative optimization algorithms applicable to edge-preserving regularization (EPR) objective functions. To reduce impulse noise, a two-phase technique was recently developed in [1]. For salt-and-pepper noise, the adaptive median filter (AMF) is used, while for random-valued noise, the adaptive center-weighted median filter (ACWMF) is used, which is first improved by applying the variable window technique to increase its detection capabilities in severely damaged pictures [2]. We only utilize the salt-and-pepper noise in this study. Let $X$ represent the true picture and $A = \{1, 2, 3, \ldots, M\} \times \{1, 2, 3, \ldots, N\}$ represent the index set of $X$. Let $N \subset A$ denote the set of noise pixel indices detected during the first phase. Also, let $P_{ij}$ be the set of pixel’s four nearest neighbors at position $(i, j) \in A$, $y_{ij}$ denote the discovered pixel value of the actual picture at position $(i, j)$, and $u_{ij} = [u_{i_j}]_{(i,j) \in N}$ denote a lexicographically ordered column vector of length $c$, where $c$ represents the size of $N$. Then, minimizing the following function will recover the noise pixels.

$$f_u(u) = \sum_{(i,j) \in N} \left[ |u_{ij} - y_{ij}| + \frac{\beta}{2}(2 \times S^1_{ij} + S^2_{ij}) \right]$$

(1)

where $\beta$ is the regularization parameter,

$$S^1_{ij} = 2 \sum_{(m,n) \in P_{ij} \cap \partial X} \phi_a (u_{ij} - y_{m,n})$$

and

$$S^2_{ij} = \sum_{(m,n) \in P_{ij} \cap \partial X} \phi_a (u_{ij} - y_{m,n}).$$

Function (1) is a hypothetical function that preserves the edges $\phi_a = \sqrt{\alpha + x^2}, \alpha > 0$, that can be used to describe impulsive noise in general. Minimizing (1) defines the essence of a slavish AMF introduced in [3]. This is a typical method for locating pixels that may be contaminated. In practice, the non-smooth data-fitting term might be dropped because it is not needed in the second phase, when only poor-quality pixels are recovered after reduction. As a result, a number of optimization strategies for minimizing the following smooth EPR functional may be utilized (such as [4]–[7]).

$$f_u(u) = \sum_{(i,j) \in N} \left[ 2 \times S^1_{ij} + S^2_{ij} \right]$$

(2)

The conjugate gradient (CG) method for image correction is quite effective for solving unconstrained optimization problems of the form (3),

$$Min \ f(x), x \in R^n$$

(3)

due to their low memory requirements and simplicity of coding [4]–[7]. To solve (1), an iterative computation of a new solution vector is done using (4).

$$x_{k+1} = x_k + \alpha_k d_k$$

(4)

The step length $\alpha_k$ is calculated is traditionally obtained through a one-dimensional line search which, in practice, is usually inexact due to cost and impracticality considerations. For quadratic functions, $\alpha_k$ can be computed exactly using [8].

$$\alpha_k = -\frac{g^T_k d_k}{d_k^T g_k}$$

(5)

However, for general functions, $\alpha_k$ is computed to ensure that the obtained search direction is sufficiently downhill through satisfying the strong Wolfe conditions [2].

$$f(x_k + \alpha_k d_k) \leq f(x_k) + \delta \alpha_k g^T_k d_k$$

(6a)

and

$$d^T_k g(x_k + \alpha_k d_k) \geq \sigma d^T_k g_k$$

(6b)

where $0 < \delta < \sigma < 1$. CG methods compute search directions using (7).

$$d_{k+1} = -g_{k+1} + \beta_k s_k$$

(7)
where $\beta_k$ is a normally referred to as a conjugacy parameter and both $d_k$ and $d_{k+1}$ satisfy the conjugacy condition $d_k^T Q d_j = 0, \forall i \neq j$, for a symmetric matrix $Q \in \mathbb{R}^{n \times n}$. A variety of equations have been published to compute the scalar $\beta_k$. Two well-known conjugate gradient approaches are Fletcher [9] and Dai and Yuan [10], with

$$\beta_k^{FR} = \frac{g_{k+1}^T g_{k+1}}{g_k^T g_k}, \quad \text{and} \quad \beta_k^{BV} = \frac{g_{k+1}^T s_{k+1}}{d_k^T y_k} \quad (8)$$

respectively. The two methods have been the focus of many studies, not just because of their historical significance, but also because of their proven global convergence. Many other variants have been examined in an attempt to improve the numerical behavior of the CG methods, given their attractive storage requirements; see, for example, [11]–[14]. CG methods can be utilized in solving problems related to machine learning, fluid mechanics, solution of nonlinear equations and differential equations, deep learning, in addition to other applications. Another possible area of application is human performance technology (HPT). HPT is largely based on computer systems’ numerical performance improvement (PI) characteristics, which rely on logical judgements enabled by specialized algorithms [15]. PI also helps to widen the scope of instructional design by using a systems perspective to address performance opportunities and obstacles. CG methods have proven valuable in solving issues in the adoption of mobile electronic performance support systems (EPSS) improved the job performance and efficiency of mobile users, according to a cross-sectional qualitative research [16].

In order to improve the computational efficiency of the standard CG method, a special type of conjugate gradient methods have recently been extensively investigated [17]–[20]. The approaches in [9], [18] propose a conjugacy condition of the form.

$$\beta_k^{HY} = \frac{\|g_{k+1}\|^2}{2/\alpha_k(f_k-f_{k+1})}, \quad \beta_k^{BG} = \frac{\|g_{k+1}\|^2}{(f_k-f_{k+1})/\alpha_k-g_k^T d_k/2} \quad (9)$$

Unlike the traditional CG algorithms, the aforementioned approach has the unique characteristic of consistently constructing better descent directions while satisfying the conjugacy conditions, as evidenced by the reported results. In the following section, a quadratic model will be exploited to derive new conjugacy parameters $\beta_k$, giving rise to new CG algorithms.

2. NEW CONJUGATE GRADIENT COEFFICIENTS

The formulation of the new CG method presented here exploits a classical quadratic model characterized by its simplicity. Iiduka and Narushima [21] propose the following choice for $\beta_k$.

$$\beta_k = \frac{g_{k+1}^T Q s_k}{d_k^T Q s_k} \quad (10)$$

where $Q$ is the constant Hessian of some quadratic function. The parameter $\beta_k$ satisfies a conjugacy condition of the form:

$$d_{k+1}^T Q d_k = 0 \quad (11)$$

In our derivation we will introduce an appropriate approximation to the quantity $d_k^T Q s_k$, essential to our proposed method. Assume $f$ is a quadratic function of the form:

$$f_{k+1} = f_k + s_k^T g_k + \frac{1}{2} s_k^T Q(x_k) s_k. \quad (12)$$

This quadratic function’s gradient is explicitly given by (13).

$$g_{k+1} = g_k + Q(s_{k+1}) s_k \quad (13)$$

As a result, curvature information may be expressed as (14).

$$s_k^T Q(x_k) s_k = 2 s_k^T y_k + 2(f_{k+1} - f_k) \quad (14)$$

From (14), we obtain:
\[ d_k^T Q(x_k) s_k = 2 \frac{\alpha_k (s_k^T d_k)^2}{(s_k^T y_k + 2(f_k - f_{k+1}))} \]  

(15)

plugging (15) into (11), we get

\[ \beta_k = \frac{g_k^T y_k}{2\alpha_k (s_k^T d_k)^2 / (s_k^T y_k + 2(f_k - f_{k+1}))} \]  

(16)

given that conjugacy holds and exact line search, (16) becomes

\[ \beta_k^{\text{BL1}} = \frac{g_{k+1}^T g_{k+1}}{2\alpha_k (g_k^T d_k)^2 / (s_k^T y_k + 2(f_k - f_{k+1}))} \]  

(17)

or, alternatively,

\[ \beta_k^{\text{BL2}} = \frac{g_{k+1}^T g_{k+1}}{2\alpha_k (g_k^T d_k)^2 / (-s_k^T g_k + 2(f_k - f_{k+1}))} \]  

(18)

and

\[ \beta_k^{\text{BL3}} = \frac{g_{k+1}^T g_{k+1}}{2\alpha_k (g_k^T d_k)^2 / (s_k^T y_k + 2(f_k - f_{k+1}))} \]  

(19)

The new expressions for \( \beta_k \) are collectively referred to here as BL algorithms. The algorithmic framework is given next. BL algorithm:

Stage 1. Given \( x_1 \in \mathbb{R}^n \). Initialize \( k = 1 \) and \( d_1 = -g_1 \). If \( \|g_1\| \leq 10^{-6} \), then stop.

Stage 2. Compute \( \alpha_k > 0 \) satisfying conditions (6).

Stage 3. Compute \( x_{k+1} = x_k + \alpha_k d_k \) and \( g_{k+1} = g(x_{k+1}) \). If \( \|g_{k+1}\| \leq 10^{-6} \), then terminate.

Stage 4. Evaluate \( \beta_k \) using (17 – 19), then construct \( d_{k+1} \) by (7).

Stage 5. Set \( k = k + 1 \) and continue with step 2.

Theorem 1. The quantities \( \{x_k\} \) and \( \{d_k\} \), computed by the new methods satisfy.

\[ d_k^T g_{k+1} < 0 \quad \text{and} \quad d_k^T g_{k+1} = \beta_k d_k^T g_k \]  

(20)

Proof: If \( d_k = -g_k \) then \( d_k^T g_1 < 0 \). Suppose that \( d_k^T g_k < 0 \) for any \( k \). From (8) and (19), it is easy to show that

\[ d_{k+1}^T g_{k+1} = \beta_k d_k^T g_{k+1} + \beta_k d_k^T g_k < 0 \]  

(21)

The following is the outcome of using the (22).

\[ d_{k+1}^T g_{k+1} = \beta_k [d_k^T g_{k+1} - (2\alpha_k (g_k^T d_k)^2 / (s_k^T y_k + 2(f_k - f_{k+1}))))] \]  

(22)

We obtain, using (16) and (21),

\[ d_{k+1}^T g_{k+1} = \beta_k d_k^T g_k \]  

(23)

From the downhill property of the search direction, it is obvious that \( d_k^T g_k < 0 \), thus we get

\[ d_{k+1}^T g_{k+1} < 0 \]  

(24)

The proof is thus complete.

3. CONVERGENCE ANALYSIS

In order to establish the global convergence of the BL algorithms, the following assumptions are needed:

- The level set \( \Omega = \{ x \in \mathbb{R}^n / f(x) \leq f(x_1) \} \) is bounded.
- In some neighborhood \( A \) of \( \Omega \), the gradient \( g \) of the objective function is Lipschitz continuous, namely, there exists some constant \( L > 0 \) such that
\[
\|g(o) - g(r)\| \leq L\|o - r\|, \forall r, o \in \Lambda
\]  
(25)

(see [22] for more details). The theorems in [23] have proven to be useful in proving global convergence. We adopt some of those here and prove them for our methods and some of the results in [24], [25].

Lemma 1. Assume that assumptions 1 and 2 hold. Then for any iteration a method that produces \( a_k \) by doing the Wolfe line search, the (26) holds.

\[
\sum_{k=1}^{\infty} (g_k^T dk_k)^2 \leq \infty
\]  
(26)

Theorem 2. Assume that Assumptions 1 and 2 above hold. If formula \( \beta_k \) satisfies (20), then we have:

\[
limit_{k \to \infty} \|g_k\| = 0
\]  
(27)

Proof: By induction, assume that (27) does not hold. The (8) may be expressed as \( d_{k+1} + \beta_k d_k \). Upon squaring both sides, we get

\[
\|d_{k+1}\|^2 + \|g_{k+1}\|^2 + 2d_{k+1}^T g_{k+1} = (\beta_k)^2 \|d_k\|^2
\]  
(28)

Using (23), the following results hold:

\[
\|d_{k+1}\|^2 = \frac{(d_{k+1}^T g_{k+1})^2}{(d_k^T g_k)^2} \|d_k\|^2 - 2d_{k+1}^T g_{k+1} - \|g_{k+1}\|^2
\]  
(29)

Upon dividing both sides of (29) by \( (d_{k+1}^T g_{k+1})^2 \), we get

\[
\frac{\|d_{k+1}\|^2}{(d_{k+1}^T g_{k+1})^2} = \frac{\|d_k\|^2}{(d_k^T g_k)^2} - \frac{\|g_{k+1}\|^2}{(d_{k+1}^T g_{k+1})^2} - \frac{2}{d_{k+1}^T g_{k+1}}
\]  
(30)

This yield

\[
\frac{\|d_{k+1}\|^2}{(d_{k+1}^T g_{k+1})^2} \leq \frac{\|d_k\|^2}{(d_k^T g_k)^2} - \left( \frac{\|g_{k+1}\|}{(d_{k+1}^T g_{k+1})^2} + \frac{1}{\|g_{k+1}\|^2} \right) + \frac{1}{\|g_{k+1}\|^2} \leq \frac{\|d_k\|^2}{(d_k^T g_k)^2} + \frac{1}{\|g_{k+1}\|^2}
\]  
(31)

Hence,

\[
\frac{\|d_{k+1}\|^2}{(d_{k+1}^T g_{k+1})^2} \leq \sum_{i=1}^{k+1} \frac{1}{\|g_i\|^2}
\]  
(32)

Assume that \( c_1 > 0 \) exists such that \( \|g_k\| > c_1 \) for every \( k \in n \). Then

\[
\frac{\|d_{k+1}\|^2}{(d_{k+1}^T g_{k+1})^2} \leq \frac{k+1}{c_1^2}
\]  
(33)

We can see that, using the assumption and (33) as a guide,

\[
\sum_{k=1}^{\infty} (g_k^T dk_k)^2 = \infty
\]  
(34)

By Lemma 1, we may conclude that \( \lim_{k \to \infty} \|g_k\| = 0 \) holds.

4. NUMERICAL RESULTS

The BL1, BL2, and BL3 algorithms performance is examined in the domain of minimizing salt-and-pepper impulse noise (3). The test images are listed in Table 1. Table 1 also reports the numerical results for comparing the classical fletcher reeves (FR) method to the newly derived ones in terms of the number of iterations, function/gradient evaluation count in addition to peak signal-to-noise ratio (PSNR). All simulations are run using MATLAB 2015a. It is worth emphasizing that the major focus of the study is on how fast the problem of reducing carbon emissions in (3) can be tackled efficiently. The pixel quality of the corrected pictures is assessed using PSNR value given by (35),

\[
\text{PSNR} = 10 \log_{10} \left( \frac{255^2}{MSE} \right)
\]
\[ PSNR = 10 \log_{10} \frac{255^2}{\sum_{i,j}(u_{i,j}^* - u_{i,j})^2} \]  

(35)

where \( u_{i,j}^* \) and \( u_{i,j} \) denote the pixel values of the corrected and the original image, respectively. For both procedures, the following are the termination conditions (36).

\[
\frac{|f(u_k) - f(u_{k-1})|}{f(u_k)} \leq 10^{-4} \quad \text{and} \quad \|f(u_k)\| \leq 10^{-4}(1 + |f(u_k)|)
\]

(36)

Figures 1 to 4, show the obtained results by applying the algorithms to the noisy pictures. Figures 1(a), 2(a), 3(a) and 4(a) are the images corrupted with 70% salt-and-pepper noise; Figures 1(b), 2(b), 3(b) and 4(b) are results of the FR method; Figures 1(c), 2(c), 3(c) and 4(c) are results of the BL1 method; Figures 1(d), 2(d), 3(d) and 4(d) are results of the BL2 method; Figures 1(e), 2(e), 3(e) and 4(e) are results of the BL3 method. These results show that the suggested image correction methods BL1, BL2, and BL3 are both effective and efficient.

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<th>PSNR (dB)</th>
<th>NI</th>
<th>NF</th>
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Table 1. Numerical results of FR and new algorithms

Figures 1(a), 2(a), 3(a) and 4(a) are results of the BL1 method; Figures 1(b), 2(b), 3(b) and 4(b) are results of the BL2 method; Figures 1(c), 2(c), 3(c) and 4(c) are results of the BL3 method. These results show that the suggested image correction methods BL1, BL2, and BL3 are both effective and efficient.

![Figure 1](image1.png)

Figure 1. The noisy image and the corresponding corrected images results for each algorithm with 70% salt-and-pepper noise applied to 256*256 Lena image (a) original image, (b) FR output, (c) BL1 output, (d) BL12 output, and (e) BL3 output

![Figure 2](image2.png)

Figure 2. The noisy image and the corresponding corrected images results for each algorithm with 70% salt-and-pepper noise applied to 256x256 house image; (a) original image, (b) FR output, (c) BL1 output, (d) BL2 output, and (e) BL3 output
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Figure 3. The noisy image and the corresponding corrected images results for each algorithm with 70% salt-and-pepper noise applied to 256×256 Elaine image (a) original image, (b) FR output, (c) BL1 output, (d) BL2 output, and (e) BL3 output

Figure 4. The noisy image and the corresponding corrected images results for each algorithm with 70% salt-and-pepper noise applied to 256×256 cameraman image (a) original image, (b) FR output, (c) BL1 output, (d) BL2 output, and (e) BL3 output

5. CONCLUSION

The focus of this research has been on the creation of novel, modified conjugate gradient formulae that outperform the traditional FR CG approach for picture restoration. The results confirm to the effectiveness of the strategy used in this research to derive variations of the traditional CG technique. The novel techniques have demonstrated global convergence under the rigorous Wolfe line search conditions. The testing simulations have demonstrated that, in the majority of instances, the novel approaches, BL1, BL2, and BL3, significantly reduce iteration counts and function evaluations while maintaining the same picture restoration quality.

REFERENCES


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