The cross-association relation based on intervals ratio in fuzzy time series

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ABSTRACT

The fuzzy time series (FTS) is a forecasting model based on linguistic values. This forecasting method was developed in recent years after the existing ones were insufficiently accurate. Furthermore, this research modified the accuracy of existing methods for determining the partitioning universe of discourse, fuzzy logic relationship (FLR), and variation historical data using intervals ratio, cross association relationship, and rubber production Indonesia data, respectively. The modified steps start with the intervals ratio to partition the determined universe discourse. Then the triangular fuzzy sets were built, allowing fuzzification. After this, the FLR are built based on the cross-association relationship, leading to defuzzification. The average forecasting error rate (AFER) was used to compare the modified results and the existing methods. Additionally, the simulations were conducted using rubber production Indonesia data from 2000-2020. With an AFER result of 4.77%<10%, the modification accuracy has a smaller error than previous methods, indicating very good forecasting criteria. In addition, the coefficient values of \( D_1 \) and \( D_2 \) were automatically obtained from the intervals ratio algorithm. The future works modified the partitioning of the universe of discourse using frequency density to eliminate unused partition intervals.

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1. INTRODUCTION

Zadeh [1] introduced the basic concept of fuzzy set theory in 1965 for continuous feasible sets. According to Zadeh, a fuzzy set is expressed by a membership function, where each domain corresponds to one number in range from zero to one [1]. The fuzzy set theory, the concept of linguistic variables, and the fuzzy applications to approximating reasoning were developed by Zadeh [1], [2] which have successfully penetrated into the realm of forecasting using time series data. Time series is the observation of historical data, while forecasting is the future prediction in daily life, such as economics, employment sector, tourism, agriculture, climatology, stock, and others, hence losses can be avoided using a well-timed forecast. The fuzzy time series (FTS) was introduced by Song and Chissom [3], [4] in 1993 to forecast the enrollment data.
The cross-association relation based on intervals ratio in fuzzy time series (Etna Vianita)
2.3. Li’s method

Li’s method described multi-factors high-order of FTS forecasting model and used the triangular fuzzy sets to modify Lee et al. [48] method in grades of membership 1, 0.5, and 0. This method modified the FLR using a cross-association relationship. The FLR is divided into four, hence Li’s method algorithm is:

i) determine the universe of discourse with $D_1$ and $D_2$, which is an arbitrary positive number that elected by the researcher;
ii) partitioning the universe of discourse into several intervals for main factor and influence factors with equal length; iii) define a group of fuzzy sets; iv) fuzzification; v) build FLR using cross-association relationship; and vi) defuzzification.

2.4. Method testing

This research used AFER obtained from [47] to determine the error value between the actual data and the forecasting. The formula of AFER is:

$$AFER = \frac{\sum_{i=1}^{n} |F_i - A_i|}{n} \times 100\%$$

(1)

where $A_i$ is the actual data result and $F_i$ is the forecasting result.

3. RESEARCH METHOD

This research modified Li et al. method [50] of determining $D_1$ and $D_2$ for the build of the universe of discourse. Moreover, it is also partitioning the universe of discourse using intervals ratio [46] with 50th percentile. The following is the proposed method algorithm:

a) Determine the universe of discourse with $D_1$ and $D_2$, which is automatically obtained from the intervals ratio algorithm.

b) Partitioning the universe of discourse into several intervals with different length using intervals ratio.

c) Define a group of triangular fuzzy sets.

Build fuzzy sets of main factor $\{A_k|k = 1, 2, ..., l + 1\}$ on $\{u_i|j = 1, 2, ..., l\}$:

$$A_1 = [Q(1), Q(1), Q(1) + q_k];$$

$$A_k = [Q(k) - q_{k-1}, Q(k), Q(k) + q_k], k = 2, 3, ..., l;$$

$$A_{l+1} = [Q(l), Q(l) + q_l, Q(l) + q_l].$$

where $Q(k)$ is the left endpoint, $q_k$ is the length of $u_{k+1} (k \in \{1, 2, ..., l\})$. Fuzzy sets of influence factors $\{B_k|k = 1, 2, ..., l + 1, i = 1, 2, ..., n\}$ are built on $\{u_i|j = 1, 2, ..., l, i = 2, ..., n\}$ analog to constructing $\{A_k|k = 1, 2, ..., l + 1\}$.

Definition 3.1. A fuzzy number with the main factor $A = (a, b, c)$ is said to be triangular if its membership function is given:

$$\mu_A(x) = \begin{cases} \frac{(x-a)}{(b-a)} & \text{if } a \leq x \leq b, \\ 1 & \text{if } x = b, \\ \frac{(c-x)}{(c-b)} & \text{if } b \leq x \leq c, \\ 0 & \text{otherwise.} \end{cases}$$

Also, the influence factor $B_k^i = (a, b, c)$ is said to be triangular fuzzy number if its membership function is given:

$$\mu_B(x) = \begin{cases} \frac{(x-a)}{(b-a)} & \text{if } a \leq x \leq b, \\ 1 & \text{if } x = b, \\ \frac{(c-x)}{(c-b)} & \text{if } b \leq x \leq c, \\ 0 & \text{otherwise.} \end{cases}$$

d) Fuzzification.

Each data point in the main factor $X$ is fuzzified to a set, where data point data $x_i$ is fuzzified to $A_{(k+1)}$. If $\mu_{A_k}(x_i) \leq \mu_{A_{(k+1)}}(x_i)$, otherwise to $A_k(k \in \{1, 2, ..., l\})$. Similarly, data point $y_i$ in influence
factors \( Y^i \) is fuzzified to \( B_j^i \), where \( y_j^i \) is the \( j^{th} \) investigation of the \((i+1)^{th}\) influence factors \((j \in \{1,2, \ldots, l+1\}, s \in \{1,2, \ldots, n\}, i = 1,2, \ldots, n)\). Then, some fuzzy time series is obtained from that given. Suppose \( F(t), G(t) \) is to be obtained from fuzzy time series of \( X \) and \( Y^i \) \((i = 1,2, \ldots, n)\) respectively.

e) Build FLR using a cross-association relationship.

d) Defuzzification.

The prediction of \( x_t \) at moment \( t \). Four kinds of FLRs can be used in forecasting, four predictions \( \{x_{tS}, x_{tC1}, x_{tSC}, x_{tLC} \mid i = 1,2, \ldots, n\} \) with final prediction \( x_t^* \) at moment \( t \).

\[
x_t^* = \frac{\sum_{i=1}^{n} x_{tS}^i + x_{tC1}^i + x_{tSC}^i + x_{tLC}^i}{\sum_{i=1}^{n} W_1^i + W_2^i + W_3^i + W_4^i}, \text{if } x_{tS}^i \text{ does not exist}
\]

\[
x_t^* = \frac{\sum_{i=1}^{n} x_{tS}^i + x_{tC1}^i + x_{tSC}^i + x_{tLC}^i}{\sum_{i=1}^{n} W_1^i + W_2^i + W_3^i + W_4^i}, \text{if } x_{tS}^i \text{ exists}
\]

where \( W_1^i = \{i\} \text{ no available HSCAFRL exists in } S_{C1}, 1 \leq i \leq n\), \( W_2 = \{i\} \text{ no available high-order long-cross association fuzzy logical relationship (HLCAFLR) exists in } L_{C1}, 1 \leq i \leq n\).

4. RESULTS AND DISCUSSION

In this research, the simulation data is retrieved from the Badan Pusat Statistika (BPS) website [54]. The data of Indonesia’s rubber production and the land area from 2000-2020, which are the main and influence factor respectively, were used. Figure 1 shows that there were 21 data which were used for the forecasting with the help of Microsoft Excel.

4.1. Proposed method

The first step is to determine the maximum and minimum values from the data. According to the data, the main factor has a minimum value \( D_{min} \) of 1125.2 and a maximum value \( D_{max} \) of 3111.3, while the influence factor has minimum and maximum value of 2747.9 and 3305.4, respectively. Following this, the universe of discourse \( U_i = [D_{min} - D_{A1}; D_{max} + D_{A2}] \), with \( D_{A1} \) and \( D_{A2} \) \((i=1,2)\) is determined using intervals ratio algorithm. The intervals ratio algorithm is:

a) The absolute difference between two successive data was taken. For example, the rubber production in 2000 and 2001 were 1125.2 and 1723.3, respectively, hence the absolute difference is 598.1. Similarly, the differences for the next year and the data observations for the influence factor can be calculated.

b) From each absolute difference, the relative difference is calculated. For example, the relative difference between 2001 and 2000 is \( r_{X1} = \frac{|1723.3 - 1125.2|}{1125.2} = 53.2\% \). Table 1 shows the relative differences for year \( n(t) \).
The intervals are calculated as. First, 9.57% and 1.12% for the main and influence factor, respectively. Furthermore, an increase from the horizontal axis of the influence factor increases to 0.03% and then 0.04% with a base of 0.01%, results in the same cumulative sum. Figure 2 illustrated the distribution of cumulative ratios for main factors, where the horizontal axis was plotted from 7.57% to 11.57% basis, on the basis table of 1% and mapping the influence factor 0.02% to the base table shown in Table, basis=1% and mapping the influence factor 0.02% to the base table shown in Table 2, basis=0.01%.

c) Mapping to the base table for each factors by mapping $MIN(r_{Xt1_→r_{Xtn-2}})=0.57\%$ for main factor and $MIN(r_{Y_1t1_→r_{Ytn-1}})=0.02\%$ for influence factor. Mapping the main factor 0.57% to the base table shown in Table, basis=1% and mapping the influence factor 0.02% to the base table shown in Table 2, basis=0.01%.

d) Plot the cumulative distribution of $r_{Xt1}$ on the basis table of 1% and $r_{Y_1t1}$ on the basis table of 0.01%. First, $MIN(r_{Xt1_→r_{Xtn-2}})=0.57\%$ and $MIN(r_{Y_1t1_→r_{Ytn-1}})=0.02\%$; gives a cumulative sum of 1. Furthermore, an increase from the horizontal axis on the principal factor to 1.57% and then 2.57% on a 1% basis, results in the same cumulative sum. Figure 2 illustrated the distribution of cumulative ratios for main factors, where the horizontal axis was plotted from 7.57% to 11.57% for simplicity. Conversely, when the horizontal axis of the influence factor increases to 0.03% and then 0.04% with a base of 0.01%, the cumulative number is 1. Subsequently, the cumulative distributives should be plotted. Figure 3 plotted the horizontal axis from 1.10% to 1.14% for simplicity.

e) Setting $\alpha$ as the 50th percentile. The total number from 2000 to 2020 is 21. Therefore, it is calculated as 50% of the total amount, which is 10. From Figure 2, 9.57% and 10.57% are both greater than 10 and the smaller is chosen as the ratio of the main factor, namely the ratio of the main factors of 9.57%. While from Figure 3, 1.12%, 1.13%, 1.14% all three are greater than 10 and the smaller one is chosen as the ratio of the influence factor, namely the ratio of the influence factor is 1.12%. So, results in a ratio of 9.57% and 1.12% for the main and influence factor, respectively.

f) The intervals are calculated as. First,

Figure 1. Data on Indonesia’s rubber production and land area [54]

<table>
<thead>
<tr>
<th>Years</th>
<th>Actual rubber production data</th>
<th>$r_{Xt}$ of rubber production data</th>
<th>Actual land area data</th>
<th>$r_{Yt}$ of land area data</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>1125.2</td>
<td>-</td>
<td>3046</td>
<td>-</td>
</tr>
<tr>
<td>2001</td>
<td>1723.3</td>
<td>53.2%</td>
<td>2838.4</td>
<td>6.8%</td>
</tr>
<tr>
<td>2002</td>
<td>1226.6</td>
<td>28.8%</td>
<td>2825.5</td>
<td>0.5%</td>
</tr>
<tr>
<td>2003</td>
<td>1396.2</td>
<td>13.8%</td>
<td>2772.5</td>
<td>1.9%</td>
</tr>
<tr>
<td>2004</td>
<td>1662.0</td>
<td>19.0%</td>
<td>2747.9</td>
<td>0.9%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>I</td>
<td>I</td>
<td>I</td>
</tr>
<tr>
<td>2020</td>
<td>2533.5</td>
<td>13.4%</td>
<td>3305.4</td>
<td>1.1%</td>
</tr>
</tbody>
</table>

Table 2. Base table

<table>
<thead>
<tr>
<th>$MIN(r_{Xt1_→r_{Xtn-2}})$</th>
<th>Base</th>
<th>$MIN(r_{Y_1t1_→r_{Ytn-1}})$</th>
<th>Base</th>
</tr>
</thead>
<tbody>
<tr>
<td>$MIN(r_{Xt1_→r_{Xtn-2}})\leq 0.05%$</td>
<td>0.01%</td>
<td>$MIN(r_{Y_1t1_→r_{Ytn-1}})\leq 0.05%$</td>
<td>0.01%</td>
</tr>
<tr>
<td>0.05% &lt; $MIN(r_{Xt1_→r_{Xtn-2}})\leq 0.5%$</td>
<td>0.1%</td>
<td>0.05% &lt; $MIN(r_{Y_1t1_→r_{Ytn-1}})\leq 0.5%$</td>
<td>0.1%</td>
</tr>
<tr>
<td>0.5% &lt; $MIN(r_{Xt1_→r_{Xtn-2}})\leq 5.0%$</td>
<td>1%</td>
<td>0.5% &lt; $MIN(r_{Y_1t1_→r_{Ytn-1}})\leq 5.0%$</td>
<td>1%</td>
</tr>
<tr>
<td>5% &lt; $MIN(r_{Xt1_→r_{Xtn-2}})\leq 50%$</td>
<td>10%</td>
<td>5% &lt; $MIN(r_{Y_1t1_→r_{Ytn-1}})\leq 50%$</td>
<td>10%</td>
</tr>
</tbody>
</table>
The cross-association relation based on intervals ratio in fuzzy time series (Etna Vianita)

\[ \text{truncated}(\text{MIN}(x)) = m.n \times 10^2 \]

\[ \text{truncated}(1125.2) = 1.1 \times 10^3 \]

Second, \( n \) is reduced by 1 to obtain \( n' = 1 - 1 = 0 \). Then, the initial value is given as \( \text{initial} = m.n' \times 10^2 = 1.0 \times 10^3 \). For \( j \geq 1 \), \( \text{lower}_j = \text{upper}_{j-1} \) and \( \text{upper}_j = (1 + \text{ratio})^j \times \text{upper}_0 \), hence intervals \( \text{interval}_j = [\text{lower}_j, \text{upper}_j] \). Furthermore, \( \text{upper}_1 = (1 + 9.57\%)^1 \times 1000 = 1095.7 \), results in an interval \([1000; 1095.7]\). Tables 3 and 4 shows the result of the intervals.

![Figure 2. Distribution of cumulative ratios for main factor](image1)

![Figure 3. Distribution of cumulative ratios for influencing factor](image2)

From the interval ratio algorithm \( D_{11}, D_{21} \) and partition of \( U_i \) is obtained, where the main factor \( D_{12} = 125.2 \) and \( D_{22} = 169.589 \), while for influence factor \( D_{12} = 147.9 \) and \( D_{22} = 16.516 \). Therefore, the universe of discourse for the main factor is:

\[
U_1 = \left[ D_{\text{min}} - D_{11}; D_{\text{max}} + D_{21} \right] \\
U_1 = [1000; 3280.889]
\]

Furthermore, the universe of discourse for the influence factor is:

\[
U_2 = \left[ D_{\text{min}} - D_{12}; D_{\text{max}} + D_{22} \right] \\
U_2 = [2600; 3321.916]
\]
Partition of $U_1$ from the intervals ratio algorithm results in different lengths of intervals for the main factor as shown in Table 3, and partition of $U_2$ from intervals ratio algorithm gets different lengths of intervals for the influence factor as shown in Table 4.

Table 3. Partition of the universe of discourse $U_1$

<table>
<thead>
<tr>
<th>Index $u_i$</th>
<th>Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_{i1}$</td>
<td>[1000; 1095.7]</td>
</tr>
<tr>
<td>$u_{i2}$</td>
<td>[1095.7; 1200.558]</td>
</tr>
<tr>
<td>$u_{i3}$</td>
<td>[1200.558; 1315.452]</td>
</tr>
<tr>
<td>$u_{i4}$</td>
<td>[2994.332; 3280.889]</td>
</tr>
</tbody>
</table>

Table 4. Partition of the universe of discourse $U_2$

<table>
<thead>
<tr>
<th>Index $u_j$</th>
<th>Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_{j1}$</td>
<td>[2600; 2629.12]</td>
</tr>
<tr>
<td>$u_{j2}$</td>
<td>[2629.12; 2658.566]</td>
</tr>
<tr>
<td>$u_{j3}$</td>
<td>[2658.566; 2688.342]</td>
</tr>
<tr>
<td>$u_{j4}$</td>
<td>[3285.123; 3321.916]</td>
</tr>
</tbody>
</table>

After the partitioning of the universe of discourse, the triangular fuzzy sets $A_k (k = 1, 2, 3, ... , 14)$ and $B_k (k = 1, 2, 3, ..., 23)$ is built for the main and influence factors, respectively, corresponding to the linguistic intervals of Tables 3 and 4:

$$A_1 = (1000, 1000, 1095.7), \quad A_2 = (990.842, 1095.7, 1200.558), \quad A_3 = (1085.664, 1200.558, 1315.452),$$
$$A_4 = (2994.332, 3280.889)$$. 

and

$$B_1 = (2600, 2600, 2629.12), \quad B_2 = (2599.674, 2629.12, 2658.566), \quad B_3 = (2628.79, 2658.566, 2688.342),$$
$$B_{23} = (3285.123, 3321.916, 3321.916)$$.

Definition 3.1 was used to describe the grades of membership for each datum corresponding to the triangular fuzzy set $[49, 55] A_k (k = 1, 2, 3, ... , 14)$ and $B_k (k = 1, 2, 3, ..., 23)$ as:

$$A_1 = \emptyset, \quad A_2 = \{0.72, 0.28\}, \quad A_3 = \{0.55, 0.29, 0.77\}, \quad A_4 = \{0.95, 0.12, 0.97\},$$
$$A_5 = \{0.35, 0.36, 0.65\}, \quad A_6 = \{0.88, 0.97, 0.99\}, \quad A_7 = \{1215.2, 1226.6, 1295.2\},$$
$$A_8 = \{1662, 1723.3, 1838.7\}, \quad A_9 = \{0.36, 0.36, 0.36\}, \quad A_{10} = \{0.62, 0.62, 0.62\},$$
$$A_{11} = \{1918, 2148.7, 2176.7\}, \quad A_{12} = \{0.37, 0.37, 0.37\},$$
$$A_{13} = \{0.31, 0.31, 0.31\}, \quad A_{14} = \{2429.5, 2533.5, 2533.5\},$$
$$A_{15} = \{2429.5, 2533.5, 2533.5\}, \quad A_{16} = \{3285.123, 3321.916, 3321.916\}.$$

Example. The actual rubber production data in 2001 is 1723.3. Subsequently, the grades of membership were obtained for data point 1723.3 as 0.05 and 0.95, respectively, corresponding to the triangular fuzzy set $A_7$ and $A_9$. This data has reached maximum membership in the triangular fuzzy set $A_7$. Therefore, the rubber production datum in 2001 is fuzzified by a triangular fuzzy set $A_7$. Table 5 showed the fuzzification of the time series data of rubber production and land area by triangular fuzzy sets.

FLR was developed in cross-relation. Table 6 showed the four FLRs namely HSAFLR, HLAFLR, HSCAFLR, and HLCAFLR. Furthermore, the result of the forecast is displayed in Table 7. Four FLRs are constructed for forecasting, hence four predictions $\{X_{2003}^{HSA}, X_{2003}^{HLA}, X_{2003}^{HSC}, X_{2003}^{HLC}\}$ are calculated, and the
final prediction $x^*_{2003}$ can be obtained. The detailed process of forecasting is described in the following steps:

Step 1: calculate $x^*_{2003_S}$ from the available HSAFLRs in $S$. The fuzzified values of $x_{2000}, x_{2001}, x_{2002}$ of the main factor time series are $A_2, A_7, A_3$ respectively. The available HSAFLRs found in $S$, whose premises are $A_2, A_7, A_3$:

$$A_2, A_7, A_3 \rightarrow A_5(1)$$

By using these FLRs, the prediction $x^*_{2003_S}$ is given as

$$x^*_{2003_S} = \frac{1 \times \bar{x}(5)}{1} = 1381.41$$

where

$$\bar{x}(5) = \frac{0.5m(5-1)+Q(5)+0.5m(5+1)}{2}$$

$$= \frac{0.5(1315.452)+1315.452+0.5(1579.277)}{2} = 1381.41$$

Step 2: $x^*_{2003_L}$ is calculated from the available HLAFLRs in $L$. The fuzzified values of $x_{1999}, x_{2000}, x_{2001}$ of the main factor time series are $NA, A_2, A_7$ respectively. The available HLAFLRs found in $L$, whose premises are $NA, A_2, A_7$:

$$NA, A_2, A_7 \rightarrow A_5(1)$$

By using these FLRs, the prediction $x^*_{2003_L}$ is given as

$$x^*_{2003_L} = \frac{1 \times \bar{x}(5)}{1} = 1381.41$$

where

$$\bar{x}(5) = \frac{0.5m(5-1)+Q(5)+0.5m(5+1)}{2}$$

$$= \frac{0.5(1315.452)+1315.452+0.5(1579.277)}{2} = 1381.41$$

Step 3: $x^*_{2003_SC}$ is calculated from the available HSCAFLRs in $SC$. The fuzzified values of $x_{2000}, x_{2001}, x_{2002}$ of the influence factor time series are $B_{15}, B_9, B_3$ respectively. The available HSCAFLRs found in $SC$, whose premises are $B_{15}, B_9, B_3$:

$$B_{15}, B_9, B_3 \rightarrow A_5(1)$$

By using these FLRs, the prediction $x^*_{2003_SC}$ is given as

$$x^*_{2003_SC} = \frac{1 \times \bar{x}(5)}{1} = 1381.41$$

where

$$\bar{x}(5) = \frac{0.5m(5-1)+Q(5)+0.5m(5+1)}{2}$$

$$= \frac{0.5(1315.452)+1315.452+0.5(1579.277)}{2} = 1381.41$$

Step 4: calculate $x^*_{2003_LC}$ from the available HLCAFLRs in $LC$. The fuzzified values of $x_{1999}, x_{2000}, x_{2001}$ of the influence factor time series are $NA, B_{15}, B_9$ respectively. The available HLCAFLRs found in $LC$, whose premises are $NA, B_{15}, B_9$:

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NA, B15, B9 → A3(1)

The use of these FLRs gives a prediction $x_{2003}^\prime$ as

$$x_{2003}^\prime = \frac{1 \times \bar{x}(5)}{5} = 1381.41$$

where

$$\bar{x}(5) = \frac{0.5m(5-1)+Q(5)+0.5m(5+1)}{2}$$
$$= \frac{0.5m(4)+Q(5)+0.5m(6)}{2}$$
$$= \frac{0.5(1315.452)+1315.452+0.5(1579.277)}{2} = 1381.41$$

The final prediction for $x_{2003}^\prime$ is

$$\frac{1381.41+1381.41+1381.41+1381.41}{4} = 1381.41.$$  

Analog for next observation.

### Table 5. Fuzzification

<table>
<thead>
<tr>
<th>Year</th>
<th>Actual rubber production data</th>
<th>Fuzzified rubber production</th>
<th>Actual land area data</th>
<th>Fuzzified land area</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>1125.2</td>
<td>$A_2$</td>
<td>3046</td>
<td>$B_{15}$</td>
</tr>
<tr>
<td>2001</td>
<td>1723.3</td>
<td>$A_7$</td>
<td>2838.4</td>
<td>$B_9$</td>
</tr>
<tr>
<td>2002</td>
<td>1226.6</td>
<td>$A_3$</td>
<td>2825.5</td>
<td>$B_3$</td>
</tr>
<tr>
<td>2003</td>
<td>1396.2</td>
<td>$A_6$</td>
<td>2772.5</td>
<td>$B_6$</td>
</tr>
<tr>
<td>2004</td>
<td>1662.0</td>
<td>$A_7$</td>
<td>2747.9</td>
<td>$B_7$</td>
</tr>
<tr>
<td>2005</td>
<td>1879.2</td>
<td>$A_{10}$</td>
<td>3305.4</td>
<td>$B_{23}$</td>
</tr>
</tbody>
</table>

### Table 6. Third-order cross-relation FLR

<table>
<thead>
<tr>
<th>Year</th>
<th>HSAFLR</th>
<th>HLAFLR</th>
<th>HSCAFLR</th>
<th>HLCAFLR</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>2001</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>2002</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>2003</td>
<td>$A_2$, $A_7$, $A_3$ → $A_5$</td>
<td>$NA, A_2, A_7$ → $A_5$</td>
<td>$B_{15}, B_9, B_3$ → $A_5$</td>
<td>$NA, B_{15}, B_9$ → $A_3$</td>
</tr>
<tr>
<td>2004</td>
<td>$A_2$, $A_3$, $A_5$ → $A_3$</td>
<td>$A_2$, $A_7$, $A_3$ → $A_7$</td>
<td>$B_9, B_3, B_7$ → $A_7$</td>
<td>$B_{15}, B_9, B_3$ → $A_7$</td>
</tr>
<tr>
<td>2005</td>
<td>$A_3$, $A_3$, $A_3$ → $A_3$</td>
<td>$A_2$, $A_3$, $A_5$ → $A_3$</td>
<td>$B_9, B_3, B_7$ → $A_3$</td>
<td>$B_{15}, B_9, B_3$ → $A_3$</td>
</tr>
</tbody>
</table>

### Table 7. Defuzzification

<table>
<thead>
<tr>
<th>Year</th>
<th>HSAFLR ($\bar{x}_t$)</th>
<th>HLAFLR ($\bar{x}_t$)</th>
<th>HSCAFLR ($\bar{x}_t^\prime$)</th>
<th>HLCAFLR ($\bar{x}_t^\prime$)</th>
<th>Forecasting</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>2001</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>2002</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>2003</td>
<td>1381.41</td>
<td>1381.41</td>
<td>1381.41</td>
<td>1381.41</td>
<td>1381.41</td>
</tr>
<tr>
<td>2004</td>
<td>1658.46</td>
<td>1658.46</td>
<td>1658.46</td>
<td>1658.46</td>
<td>1658.46</td>
</tr>
<tr>
<td>2005</td>
<td>1817.176</td>
<td>1817.176</td>
<td>1817.176</td>
<td>1817.176</td>
<td>1817.176</td>
</tr>
</tbody>
</table>

After this, the forecast value is determined using AFER. The last step involves comparing this method with existing ones as shown in Table 8. According to Table 8, the proposed method had a smaller error than existing ones, with an AFER value of 4.77%.

### Table 8. AFER of the proposed method with existing ones

<table>
<thead>
<tr>
<th>Evaluated criteria</th>
<th>Huang [46]</th>
<th>Lee [48] with triangular fuzzy set</th>
<th>Li [50]</th>
<th>Proposed method of third-order</th>
</tr>
</thead>
<tbody>
<tr>
<td>AFER</td>
<td>10.99%</td>
<td>5.10%</td>
<td>5.06%</td>
<td>4.77%</td>
</tr>
</tbody>
</table>

5. CONCLUSION

This research modified Li’s method in the partition of the universe of discourse and determining the two arbitrary positive numbers using intervals ratio. The proposed method was applied in third-order \((h=3)\) with long relation \(h+1\). Furthermore, this modification has a smaller error than previous methods with an AFER value of \(4.77\% < 10\%\), hence good forecasting criteria. Furthermore, the coefficient values of \(D_1\) and \(D_2\) were automatically obtained from the intervals ratio algorithm. The future works used the frequency density to modify partitioning the universe of discourse to eliminate unused partition intervals.

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REFERENCES


The cross-association relation based on intervals ratio in fuzzy time series (Etna Vianita)


The cross-association relation based on intervals ratio in fuzzy time series (Etna Vianita)

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