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Modified Projective Synchronization of Chaotic Systems with Noise Disturbance, an Active Nonlinear Control Method

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ABSTRACT

The synchronization problem of chaotic systems using active modified projective nonlinear control method is rarely addressed. Thus the concentration of this study is to derive a modified projective controller to synchronize the two chaotic systems. Since, the parameter of the master and follower systems are considered known, so active methods are employed instead of adaptive methods. The validity of the proposed controller is studied by means of the Lyapunov stability theorem. Furthermore, some numerical simulations are shown to verify the validity of the theoretical discussions. The results demonstrate the effectiveness of the proposed method in both speed and accuracy points of views.

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1. INTRODUCTION

Master-slave synchronization of chaotic systems is strikely nonlinear, since the aperiodic and nonregular behavior of chaotic systems and their sensitivity to the initial conditions. Chaotic behavior may appear in many physical systems. So, chaos synchronization subject has received a great deal of attention in the last to decades, due to its potential applications in physics, chemistry, electrical engineering, secure communication and so on[1]. Up to now, many types of controling methods are revealed and investigated for control and synchronization of chaotic systems. Active method[2, 3, 4, 5, 6], adaptive method [7, 8, 9], linear feedback method [10, 11], nonlinear feedback method [12, 14, 15], sliding mode method [16, 17, 18], impulsive method [19], phase method [20], generalized method [21], robust synchronization [13] and projective method [22, 23, 24] are some of the introduced methods by the researchers. Among these methods, synchronization with some types of projective methods are extensively investigated in the last decades, since the faster synchronization due to its synchronization scaling factors, which master and slave chaotic systems would be synchronized up to a proportional rate. Projective lag method [25], modified projective synchronization (MPS) [26, 27, 28], function projective synchronization (FPS)[29], modified function projective synchronization [30, 28], generalized function projective synchronization [31, 32] and modified projective lag synchronization[33, 34] are some generalized schemes of projective method, which utilize some type of scaling factors.

When the parameters of a chaotic system are known beforehand, active related methods are preferably chosen than adaptive methods. Active synchronization problem of two chaotic systems with known parameters are vastly investigated by the researchers. For example, in [5, 3, 35], the active controlling method is studied for synchronization of two typical chaotic systems. And also, in [2], an active method for controling the behavior of a unified chaotic system is presented. Chaos synchronization of complex Chen and Lu chaotic systems are addressed in citeMahmoud, with designing an active control method. Furthermore, in [36] active synchronization of two different fractional order chaotic system is studied.

Consequently, the modified projective synchronization of two chaotic systems with known system parameters by acitve control method are rarely investigated by the researchers. Therefore, in the present study, the modified projective synchronization problem is achieved by means of active nonlinear control method. An appropriate feedback controller is designed to control the behavior the state variables of the follower system to track the trajectories of the leader system state variables. In Section 2, the problem of chaos synchronization is discussed. In addition, the validity of the proposed synchronization method is verified by means of Lyapunov stability theorem. Then, in Section 3, some experiments are derived to show the effectiveness of the proposed method. Moreover, some simulations are carried out. Finally, some concluding remarks are given in Section 4.

2. SYNCHRONIZATION

A wide variety of chaotic systems can be represented as follows:

$$\dot{\boldsymbol{x}} = f(\boldsymbol{x})\boldsymbol{\Phi} + F(\boldsymbol{x}) + \boldsymbol{\eta} \tag{1}$$

Where $\mathbf{x} = (x_1, x_2, \cdots, x_n)^T$ is the state variables vector of the system (1). $\boldsymbol{\Phi} = (\phi_1, \phi_2, \cdots, \phi_n)^T \in \mathbf{R}^{n\times 1}$ and $\eta = (\eta_1, \eta_2, \cdots, \eta_n)^T \in \mathbf{R}^{n\times 1}$ are two vectors denoting the unknown parameter vector of the system and the external distributive noise of the system, respectively. $f(\boldsymbol{x}) \in \mathbf{R}^{n\times n}$ and $F(\boldsymbol{x}) \in \mathbf{R}^{n\times 1}$ stand for the linear and nonlinear matrix of functions, respectively. Let the dynamical system (1) as the leader system. Then the follower system can be given by another chaotic function as follows:

$$\dot{\boldsymbol{y}} = g(\boldsymbol{y})\hat{\boldsymbol{\Phi}} + G(\boldsymbol{y}) + \boldsymbol{u} \tag{2}$$

Where $\boldsymbol{y} = (y_1, y_2, \cdots, y_n)^T$ presents the state variables vector of the follower system (2). $\hat{\boldsymbol{\Phi}} = (\hat{\phi}_1, \hat{\phi}_2, \cdots, \hat{\phi}_n) \in \mathbf{R}^{n\times 1}$ denotes the estimation of leader system parameters vector $\boldsymbol{\Phi}$. Moreover, $g(\boldsymbol{y}) \in \mathbf{R}^{n\times n}$ and $G(\boldsymbol{y}) \in \mathbf{R}^{n\times 1}$ are the linear and nonlinear matrix of functions, respectively. In the proposed active nonlinear control method, an appropriate controller $\boldsymbol{u}$ is designed which the states of leader system (1) are synchronized with their corresponding states at the follower chaotic sytem (2), base on the modified projective synchronization error that is defined as follows:

$$\boldsymbol{e} = \boldsymbol{y} - \boldsymbol{\Lambda}\boldsymbol{x} \tag{3}$$

Where $\boldsymbol{\Lambda} = diag\{\lambda_1, \lambda_2, \cdots, \lambda_n\}$ represents the modified scaling factors and $\boldsymbol{e} = (e_1, e_2, \cdots, e_n)^T \in \mathbf{R}^{n\times 1}$ stands for synchronization error vector. Then the dynamical synchronizaton error can be obtatined as follows:

$$\begin{aligned}\dot{\boldsymbol{e}} &= \dot{\boldsymbol{y}} - \boldsymbol{\Lambda}\dot{\boldsymbol{x}} \\ &= g(\boldsymbol{y})\boldsymbol{\Phi} + G(\boldsymbol{y}) + \boldsymbol{u} - f(\boldsymbol{x})\boldsymbol{\Phi} - \boldsymbol{\Lambda}F(\boldsymbol{x}) - \boldsymbol{\Lambda}\boldsymbol{\eta}\end{aligned} \tag{4}$$

Where $\bar{\boldsymbol{\eta}}$ denotes the estimation of noise distrubance $\boldsymbol{\eta}$.

Definition 1. For the leader system (1) and the follower system (2), the chaos synchronization would be achieved if an appropriate control is designed to force the state variables of the follower system to track the trajectories of the leader one, meanly, the synchronization error vector (3) converges to zero, as time goes to infinity,i. e:

$$\lim_{t\to\infty} \|e(t)\| = 0$$

which $\|.\|$ denotes 2-norm. Chaos synchronization can be achieved by deriving an appropriate feedback controller, which is the subject of the following theorem.

Theorem 1. The leader system (1) with the state variables vector $\boldsymbol{x}$ and the follower system (2) with the state variables vector $\boldsymbol{y}$, the parameters vector $\boldsymbol{\Phi}$ and any noise disturbance vector $\boldsymbol{\eta}$, would be synchronized for any initial state variables $\boldsymbol{x}(0)$ and $\boldsymbol{y}(0)$, if the active feedback control law is defined as follows:

$$\boldsymbol{u} = -\big[g(\boldsymbol{y}) - f(\boldsymbol{x})\big]\boldsymbol{\Phi} - \big[G(\boldsymbol{y}) - \boldsymbol{\Lambda}F(\boldsymbol{x})\big] + \boldsymbol{\Lambda}\bar{\boldsymbol{\eta}} - \boldsymbol{K}\boldsymbol{e} \tag{5}$$

Where $\bar{\boldsymbol{\eta}}$ can be estimated dynamically as follows:

$$\dot{\bar{\boldsymbol{\eta}}} = -\boldsymbol{\Lambda}\boldsymbol{e} - \boldsymbol{\Psi}(\bar{\boldsymbol{\eta}} - \boldsymbol{\eta}) \tag{6}$$

Where $\boldsymbol{K} = diag\{k_1, k_2, \cdots, k_n\}$ and $\boldsymbol{\Psi} = diag\{\psi_1, \psi_2, \cdots, \psi_n\}$ are two diagonal matrix with positive values for their main diagonal elements.

Proof. Let the Lyapunov stability function as follows:

$$V = \frac{1}{2}\boldsymbol{e}\boldsymbol{e}^T + \frac{1}{2}(\bar{\boldsymbol{\eta}} - \boldsymbol{\eta})(\bar{\boldsymbol{\eta}} - \boldsymbol{\eta})^T \tag{7}$$

It is obvious that the Lyapunov function defined in (7) is positive definite. With calculating its time derivative, we have:

$$\dot{V} = \dot{\boldsymbol{e}}\boldsymbol{e}^T + \dot{\bar{\boldsymbol{\eta}}}(\bar{\boldsymbol{\eta}} - \boldsymbol{\eta})^T \tag{8}$$

Then, substituting the dynamical representation of synchronization error vector (4) and consequently considering the proposed feedback controller (5) and the noise estimation (6), one can get:

$$\dot{V} = -\boldsymbol{K}\boldsymbol{e}\boldsymbol{e}^T - \boldsymbol{\Psi}(\hat{\boldsymbol{\eta}} - \boldsymbol{\eta})(\hat{\boldsymbol{\eta}} - \boldsymbol{\eta})^T \tag{9}$$

Therefore, derivative of V is negative definite, when $\boldsymbol{K}$ and $\boldsymbol{\Psi}$ are diagonal matrix with positive elements on their primary diagonal elements. In the following section, some numerical results are given to show the effectiveness of the proposed synchronization method.

3. NUMERICAL SIMULATIONS

This section is devoted to the synchronization of two different chaotic or hyperchaotic systems. In the following subsection, chaos synchronization between two chaotic systems, Zhang chaotic system and Lorenz chaotic system is addressed. Then, the synchronizaton problem between two hyperchaotic system as Chen hyperchaotic system and Lorenz hyperchaotic system is studied in the last subsection

3.1. chaotic systems

Chaos synchronization between Zhang chaotic system [14] and the Lü chaotic system [37] is addressed in this subsection. The Zhang chaotic system is given by a three simple integer-based and nonlinear differential equations that depends on the three positive real parameters as follows

$$\begin{aligned} \dot{x_1} &= a(x_2 - x_1) - x_2 x_3 \\ \dot{x_2} &= b x_1 - x_1^2 \\ \dot{x_3} &= -c x_3 + x_2^2 \end{aligned} \tag{10}$$

Where x_1, x_2 and x_3 are the state variables of the system and a, b, and c are the three constant parameters of the system. When $a = 10, b = 30$ and $c = 6$, the behaviour of the system is chaotic. The phase portraits of the system is shown in Fig. 1, with initial state variables $x_1(0) = 5, x_2(0) = 2$ and $x_3(0) = 30$.

In addition, the Lü chaotic system can be described as follows:

$$\begin{aligned} \dot{y_1} &= \alpha_1(y_2 - y_1) \\ \dot{y_2} &= \alpha_2 y_2 - y_1 y_3 \\ \dot{y_3} &= y_1 y_2 - \alpha_3 y_3 \end{aligned} \tag{11}$$

Where y_1, y_2 and y_3 are the state variables of the system and α_1, α_2 and α_3 are the parameter of the system. The chaotic behavior of the Lü system is shown in Fig. 2, with system parameters as: $\alpha_1 = 2.1, \alpha_2 = 30$ and $\alpha_3 = 0.6$, and state variables initial values as: $x_1(0) = 4.3, x_2(0) = 7.2$ and $x_3(0) = 5.8$.

The Zhang chaotic system (10) can be rewritten based on the leader system (1) as follows:

$$\begin{aligned} \dot{x_1} &= a(x_2 - x_1) - x_2 x_3 + \eta_1 \\ \dot{x_2} &= b x_1 - x_1^2 + \eta_2 \\ \dot{x_3} &= -c x_3 + x_2^2 + \eta_3 \end{aligned} \tag{12}$$

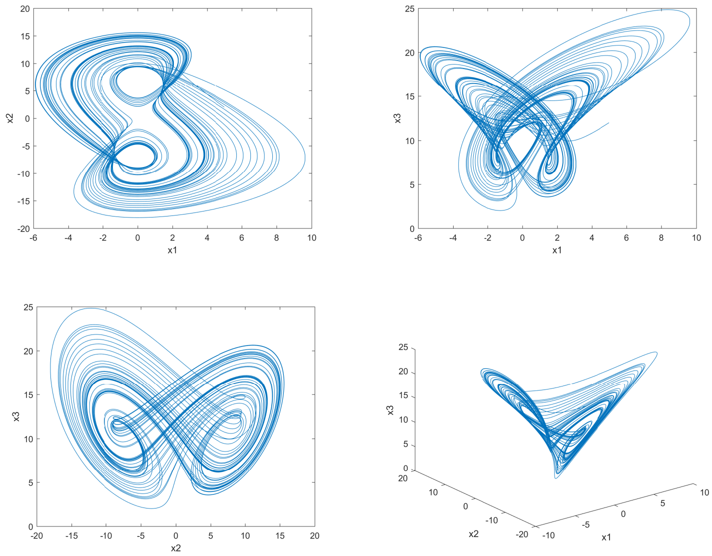

Figure 1. Phase portraits of hte Zhang chaotic system

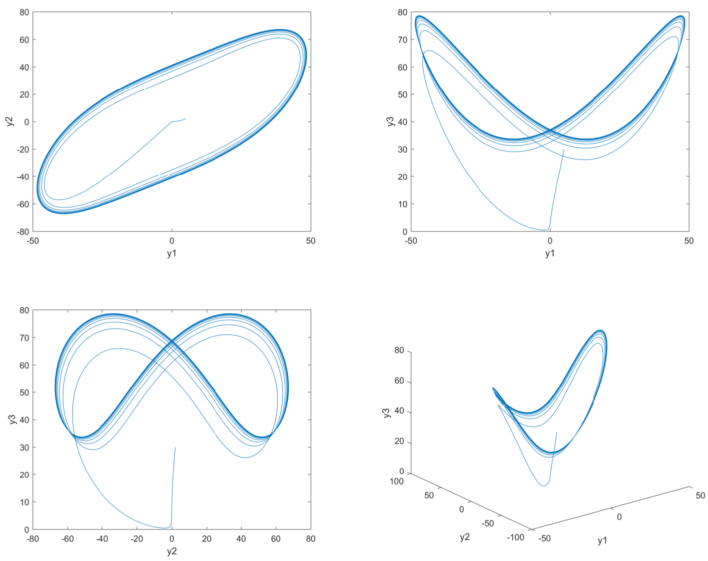

Figure 2. Phase portraits of hte Lu chaotic system

Where η_1, η_2 and η_3 are the three noise disturbance corresponding to the state variables x_1, x_2 and x_3, respectively. Then, the Lü chaotic system (11) can be represented as the follower system as follows:

$$\begin{aligned} \dot{y_1} &= a(y_2 - y_1) + u_1 \\ \dot{y_2} &= by_2 - y_1 y_3 + u_2 \\ \dot{y_3} &= y_1 y_2 - c y_3 + u_3 \end{aligned} \tag{13}$$

According to the proposed control law (5) and noise disturbance estimation (6), we define the following feedback controller as:

$$\begin{aligned} u_1 &= -ay_2 + \lambda_1 a x_1 + a e_1 - \lambda_1 x_2 x_3 + \lambda_1 \bar{\eta}_1 - k_1 e_1 \\ u_2 &= -by_2 + y_1 y_3 + \lambda_2 (b x_1 - x_1^2) + \lambda_2 \bar{\eta}_2 - k_2 e_2 \\ u_3 &= -y_1 y_2 + c e_3 + \lambda_3 x_2^2 + \lambda_3 \bar{\eta}_3 - k_3 e_3, \end{aligned} \tag{14}$$

and the noise disturbance estimation as:

$$\begin{aligned} \dot{\bar{\eta}}_1 &= -\lambda_1 e_1 - \psi_1 (\bar{\eta}_1 - \eta_1) \\ \dot{\bar{\eta}}_2 &= -\lambda_2 e_2 - \psi_2 (\bar{\eta}_2 - \eta_2) \\ \dot{\bar{\eta}}_3 &= -\lambda_3 e_3 - \psi_3 (\bar{\eta}_3 - \eta_3) \end{aligned} \tag{15}$$

Assume the parameter of the Zhang chaotic system as $a = 10, b = 30$ and $c = 6$ and the initial values for the drive chaotic system (12) are taken as, $x_1(0) = 12, x_2(0) = 5$, and, $x_3(0) = 6.5$. In additiion, the initial values of the response L system (3) are selected as: $y_1(0) = 2, y_2(0) = 15$ and $y_3(0) = 0$. Consider the nosie disturbance values as $\eta_1 = 0.8, \eta_2 = 0.6$ and $\eta_3 = 0.3$ and also their corresponding estimation ititial values as $\bar{\eta}_1 = 0.15, \bar{\eta}_2 = 0.2$ and $\bar{\eta}_3 = 0.1$. Let the gain constants as $k_1 = 2, k_2 = 2, k_3 = 2, \phi_1 = 1.5, \phi_2 = 1.5$ and $\phi_3 = 1.5$.

The validify of the proposed synchronization method for contorling the behavior of the Lu chaotic system (13) to track the motion trajectories of the Zhang chaotic system (12) and the noise disturbance estimation are shown in Figure 3 and 4, respectively. Figure 3 shows that the state variables of the system (13) track effectively the motion trajectories of the leader chaotic system. In addition, in Figure 4 exhibit that the distance between noise disturbance and its estimation values converge to zero.

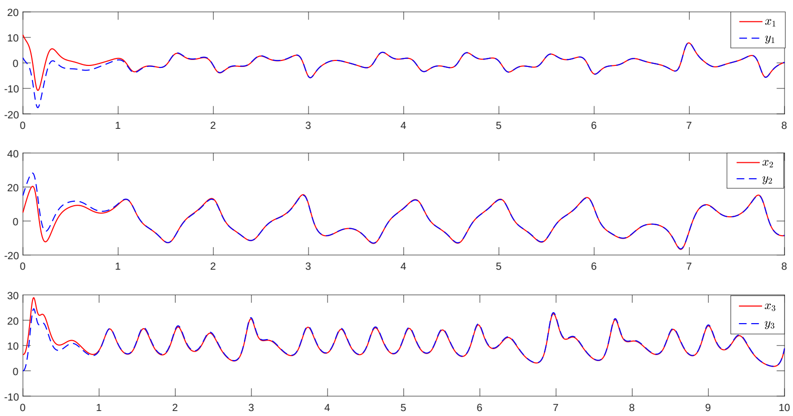

Figure 3. Time responce of the drive Zhang chaotic system and the response Lorenz chaotic system

3.2. Hyperchaotic systems

In this subsection, the synchronization between two hyperchaotic systems as Chen hyperchaotc system and Lorenz hyperchaotic system is investivated via the proposed control method. The Chen hyperchaotic

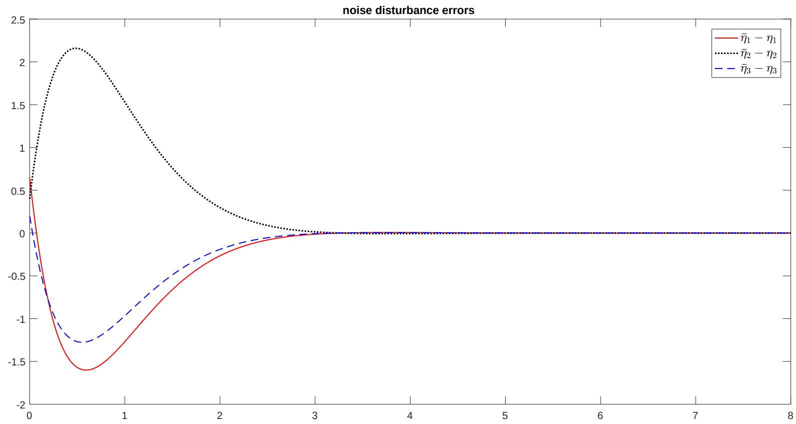

Figure 4. Time responce of the noise disturbance estimation

system is introduced in [38], as an extention of a three-dimensional Chen chaotic system as follows:

$$\begin{aligned} x_1 &= a(x_2 - x_1) + x_4 \\ x_2 &= dx_1 + cx_2 - x_1x_3 \\ x_3 &= x_1x_2 - bx_3 \\ x_4 &= x_1x_2 + rx_4 \end{aligned} \tag{16}$$

Where x_1, x_2, x_3 and x_4 are the state variables and a, b, c and d are the parameter of the system. The phase prortait of the system (16) is shown in Fig. 5, with state variables $x_1(0) =, x_2(0) =, x_3(0) =$ and $x_4(0) =$ and the parameters as $a = 35, b = 3, c = 12, d = 7$ and r=0.5 . As it can be seen the behavior of the system (16) is hyperchaotic. The Lorenz hyperchaotic system, which was introduced in [39], can be described as follows:

$$\begin{aligned} y_1 &= \alpha_1(y_2 - y_1) + y_4 \\ y_2 &= -y_1y_3 + \alpha_3y_1 - y_2 \\ y_3 &= y_1y_2 - \alpha_2y_3 \\ y_4 &= -y_1y_3 + \alpha_4y_4 \end{aligned} \tag{17}$$

Where y_1, y_2, y_3 and y_4 are the state variabels, a, b, c and d are parameter of the system. The chaotic behavior of the Lorenz hyperchaotic system is shown in Fig. 6, with initial values for the system state variables as $x_1(0) =, x_2(0) =, x_3(0) =$ and $x_4(0) =$ and the system parameters as $\alpha_1 = 36, \alpha_2 = 3, \alpha_3 = 20$ and $\alpha_4 = 1.3$

The leader system can be defined based on the Chen hyperchaotic system (16) as follows:

$$\begin{aligned} x_1 &= a(x_2 - x_1) + x_4 + \eta_1 \\ x_2 &= dx_1 + cx_2 - x_1x_3 + \eta_2 \\ x_3 &= x_1x_2 - bx_3 + \eta_3 \\ x_4 &= x_1x_2 + rx_4 + \eta_4 \end{aligned} \tag{18}$$

Where η_1, η_2, η_3 and η_4 are the noise disturbances of the system. Then, consider the Lorenz hyperchaotic system (17), as the follower system as follows:

$$\begin{aligned} y_1 &= a(y_2 - y_1) + y_4 + u_1 \\ y_2 &= -y_1y_3 + dy_1 - y_2 + u_2 \\ y_3 &= y_1y_2 - by_3 + u_3 \\ y_4 &= -y_1y_3 + cy_4 + u_4 \end{aligned} \tag{19}$$

Where u_1, u_2, u_3 and u_4 are the feedback controller of the system.

The proposed chaos synchronization between the leader Chen hyperchaotic System (18) and the follower Lorenz hyperchaotic system (19) can be achived by designing an appropriate control law and noise estimation law as follows:

$$\begin{aligned} u_1 &= -ay_2 + \lambda_1 a x_1 + ae_1 - e_4 + \lambda_1 \bar{\eta}_1 - k_1 e_1 \\ u_2 &= +y_1 y_3 - \lambda_2 x_1 x_3 - dy_1 + \lambda_2 d x_1 + y_2 + c\lambda_2 x_2 + \lambda_2 \bar{\eta}_2 - k_2 e_2 \\ u_3 &= -y_1 y_2 + \lambda_3 x_1 x_2 + be_3 + \lambda_3 \bar{\eta}_3 - k_3 e_3 \\ u_4 &= y_1 y_3 + \lambda_4 x_1 x_2 - cy_4 + \lambda_4 r x_4 + \lambda_4 \bar{\eta}_4 - k_4 e_4, \end{aligned} \tag{20}$$

and,

$$\begin{aligned} \dot{\bar{\eta}}_1 &= -\lambda_1 e_1 - \psi_1(\bar{\eta}_1 - \eta_1) \\ \dot{\bar{\eta}}_2 &= -\lambda_2 e_2 - \psi_2(\bar{\eta}_2 - \eta_2) \\ \dot{\bar{\eta}}_3 &= -\lambda_3 e_3 - \psi_3(\bar{\eta}_3 - \eta_3) \\ \dot{\bar{\eta}}_4 &= -\lambda_4 e_4 - \psi_4(\bar{\eta}_4 - \eta_4) \end{aligned} \tag{21}$$

Now, some numerical results related to the proposed synchronization of two hyperchaotic systems are given. Consider the parameter of the leaer Chen hyperchaotic system (18) as $a = 35, b = 3, c = 12, d = 7$ and $r = 0.5$ and its initial values are taken as, $x_1(0) = 11, x_2(0) = 5, x_3(0) = 9$, and, $x_4(0) = 13$. In additiion, the initial values of the response Lorenz hyperchaotic system (19) are selected as: $y_1(0) = 1, y_2(0) = 11, y_3(0) = 2$ and $y_4(0) = 3$. Consider the nosie disturbance values as $\eta_1 = 0.8, \eta_2 = 0.6, \eta_3 = 0.3$ and $\eta_4 = 0.5$. Let the gain constants as $k_1 = 2, k_2 = 2, k_3 = 2, k_4 = 2, \phi_1 = 1.5, \phi_2 = 1.5, \phi_3 = 1.5$ and $\phi_4 = 1.5$.

The effectiveness of the synchronization method for the contorling behavior of the Lorenz hyperchaotic system (19) to track the motion trajectories of the Chen hyperchaotic system (18) and the noise disturbance estimation are illustrated in Figure 3 and 4, respectively. Figure 3 shows that the state variables of the system (19) track effectively the motion trajectories of the leader chaotic system(18). In addition, in Figure 4 exhibit that the distance between noise disturbance and its estimation values converge to zero.

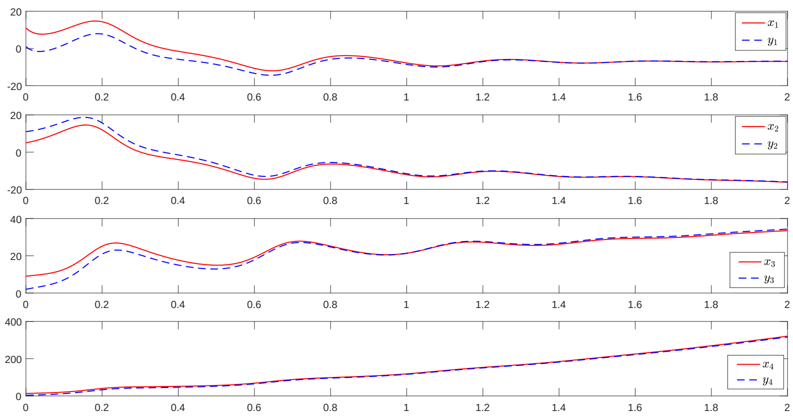

Figure 5. Time responce of the drive Zhang chaotic system and the response Lorenz chaotic system

4. CONCLUSION

In this research, some results related to the modified projective synchronization of known chaotic/hyperchaotic systems with noise disturbances are derived. Since the paramers of the leader system is considered knonwn, an appropriated active nonlinear feedback control law with designed via modified projective synchronization error. The validity of the proposed method is proved by means of Lyapunov stability theorem. Furtheremore,

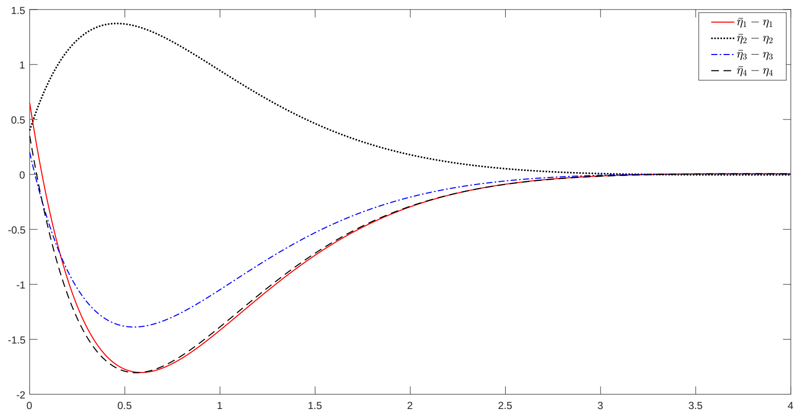

Figure 6. Time responce of the noise disturbance estimation

its effectiveness is verified by some numerical simulations of the chaotic and hyperchaotic systems. Finally, some figures are shown to verify the accuracy of the theorical discussions. As it can be seen from these results, the motion trajectories of the leader chaotic systems containing noise disturbances can effectively track by the state variables of the follower chaotic systems state variabels, which affected by proposed control method.

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