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Numerical simulation of electromagnetic radiation using high-order discontinuous galerkin time domain method

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ABSTRACT

In this paper, we propose the simulation of 2-dimensional electromagnetic wave radiation using high-order discontinuous Galerkin time domain method to solve Maxwell's equations. The domains are discretized into unstructured straight-sided triangle elements that allow enhanced flexibility when dealing with complex geometries. The electric and magnetic fields are expanded into a high-order polynomial spectral approximation over each triangle element. The field conservation between the elements is enforced using central difference flux calculation at element interfaces. Perfectly matched layer (PML) boundary condition is used to absorb the waves that leave the domain. The comparison of numerical calculations is performed by the graphical displays and numerical data of radiation phenomenon and presented particularly with the results of the FDTD method. Finally, our simulations show that the proposed method can handle simulation of electromagnetic radiation with complex geometries easily.

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1. INTRODUCTION

To date, electromagnetic (EM) phenomena play a crucial role in any aspect of human life. The modern lifestyle has become a source of omnipresent electromagnetic since the used devices generate electromagnetic fields and produce electromagnetic radiation. Television and mobile phone are the good example devices used daily. Furthermore, there are many instances in the real word, which reflect the electromagnetic (EM) phenomena, such as the radiation of microwave [1], laser [2], lightning [3], etc. Shortly, we cannot leave the EM from our life; therefore, EM simulation has been developed by many scientists to figure out any real-world phenomena.

Currently, many scholars have developed research on the numerical simulation of EM, due to the performance of the digital computer is increased significantly but the price is decreased. Thus, the numerical simulation of EM will be more attractive than both experimental and analytical methods since the cost is reduced. Furthermore, the research is aimed to improve the performance of the method concerning both efficiency problems, primarily when the method should deal with the complex problems [4]. It should also be noted that the numerical method is aimed to solve the problem of EM by using Maxwell's equations as the governing equations.

In the beginning, the numerical simulation in EM is performed in the frequency domain [5], [6]. The equation, which is established in the frequency domain, is resulted from the transformation of the time domain equation. As a result, the method is simple however the solution is limited on calculation one frequency at a time. So, it can not be used for broadband frequency analysis. Regarding the limitation, Yee [7] proposed finite difference time domain method (FDTD) to solve Maxwell's equations in the time domain. Furthermore, the numerical simulation for the Maxwell equation is developed, and the numerical methods are usually based on the finite difference, finite volume, and finite element methods.

Finite difference (FD) methods [7], [8] are the most popular methods for numerical simulation of wave propagation. While the methods have gained the both of advantages, i.e., it is simple and robust, they have some disadvantages. For example, they are not well suited to problems with complicated problems, and the handling of the boundary condition is not an easy task. Finite volume (FV) [9], [10] and finite element (FE) methods can handle complicated spatial domains easily [11]-[13]. Unfortunately, FV methods have only second-order accuracy, and FE method based on Bubnov-Galerkin projection suffers from spurious Gibbs oscillation as well as the overshoot or undershoot at sharp gradient region. Numerous efforts have been conducted to improve the performance of the FE method.

In this paper, we described a high order discontinuous Galerkin (DG) method for simulating two-dimensional electromagnetic wave radiation. Discontinuous Galerkin (DG) method is one of the advanced, improved FE methods. The DG method combines the flexibility of finite element methods with the accuracy of spectral methods. The DG method allows unstructured mesh configuration, and inter-element continuity is not required. The basis function is discontinuous across mesh boundaries. With a proper choice of numerical flux at the element boundaries, the spurious Gibbs oscillation can be suppressed, and the DG method only requires communication between mesh that has common faces [14]-[17].

2. GOVERNING EQUATIONS AND NUMERICAL SCHEME

We use the two-dimensional transverse electric (TE) Maxwell's equations as the governing equations [7]. We assumed that there is no field variation in the z-direction, ***E*** field is lying in the (*x*,*y*) plane and ***H*** field is parallel to the z-direction.

$$
\begin{aligned}
\varepsilon_0 \frac{\partial E_x}{\partial t} + \sigma E_x &= \frac{\partial H_z}{\partial y} \\
\varepsilon_0 \frac{\partial E_y}{\partial t} + \sigma E_y &= -\frac{\partial H_z}{\partial x} \\
\mu_0 \frac{\partial H_z}{\partial t} + \sigma^* H_z &= \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}
\end{aligned}
\tag{1}
$$

where ε_0 is the dielectric conductivity, μ_0 is the magnetic susceptibility, σ is the electric conductivity and σ^* is the magnetic resistivity.

To simulate the infinite spatial domain we truncated the domain by adding Berenger's Perfectly Matched Layer (PML) boundary conditions in an outer truncated region [18], [19]. The critical part of Berenger's PML definition for the 2D TE case is that the magnetic field H_z must be split into two components which are denoted as H_{zx} and H_{zy}, the 2D TE Maxwell's equations can be written as follows:

$$
\begin{aligned}
\varepsilon_0 \frac{\partial E_x}{\partial t} + \sigma_y E_x &= \frac{\partial \left(H_{zx} + H_{zy}\right)}{\partial y} \\
\varepsilon_0 \frac{\partial E_y}{\partial t} + \sigma_x E_y &= -\frac{\partial \left(H_{zx} + H_{zy}\right)}{\partial x} \\
\mu_0 \frac{\partial H_{zx}}{\partial t} + \sigma_x^* H_{zx} &= -\frac{\partial E_y}{\partial x} \\
\mu_0 \frac{\partial H_{zy}}{\partial t} + \sigma_x^* H_{zy} &= \frac{\partial E_x}{\partial y}
\end{aligned}
\tag{2}
$$

where the parameters σ_x and σ_y represent an anisotropic electric conductivity in *x* and *y* directions respectively and σ_x^* σ_y^* represent an anisotropic electric conductivity

To simplify matters, let us express Maxwell's equations in conservation form:

$$\frac{\partial \mathbf{q}}{\partial t} + \nabla \bullet \mathbf{F}(\mathbf{q}) = \mathbf{S} \tag{3}$$

where $\mathbf{q} = [\varepsilon_0 E_x \quad \varepsilon_0 E_y \quad \mu_0 H_{zx} \quad \mu_0 H_{zy}]^{\mathrm{T}}$ is the state vector, $\mathbf{S} = -[\sigma_y E_x \quad \sigma_x E_y \quad \sigma_x^* H_{zx} \quad \sigma_y^* H_{zy}]^{\mathrm{T}}$ and $\mathbf{F}(\mathbf{q}) = (\mathbf{F}_x \quad \mathbf{F}_y)^{\mathrm{T}}$, where

$$\mathbf{F}_x = [0 \quad (H_{zx} + H_{zy}) \quad E_y \quad 0]^{\mathrm{T}}$$

$$\mathbf{F}_Y = [-(H_{zx} + H_{zy}) \quad 0 \quad 0 \quad -E_x]^{\mathrm{T}}$$

The boundary conditions on metalic surfaces are taken as perfect electrical conductor (PEC), so the electrical fields are set to be zero. By applying the Bubnov-Galerkin procedure, i.e., integrating the (2) partially twice and still retained the flux terms in each element D^k, we obtain the weak form.

$$\begin{aligned} &\int_{\mathrm{D}^k} \left(\frac{\partial \mathbf{q}_h^k}{\partial t} + \nabla \bullet \mathbf{F}_h^k(\mathbf{q}) \right) l_i^k(\mathbf{x}) d\mathbf{x} = \\ &\oint_{\delta \mathrm{D}^k} \mathbf{n} \bullet \left[\mathbf{F}_h^k - \mathbf{F}^* \right] l_i^k(\mathbf{x}) d\mathbf{x} \end{aligned} \tag{4}$$

where $l_i^k(\mathbf{x})(\mathbf{x})$ is the Lagrange basis function, $\mathbf{F}^*$ is the numerical flux, and $\delta\mathrm{D}^k$ are the faces of an element. In this paper, we used the central difference as the numerical flux for simplicity.

The local solution is approximated by

$$\begin{aligned} \mathbf{q}_h^k(\mathbf{x}, t) &= \sum_{i=1}^{N_p} \mathbf{q}_h^k(\mathbf{x}_i^k, t) l_i^k(\mathbf{x}) \\ &= \sum_{n=1}^{N_p} \hat{\mathbf{q}}_n(\mathbf{x}_i^k, t) \psi_n(\mathbf{x}) \end{aligned} \tag{5}$$

N_{p} is the number of grid points in each element, $\mathbf{q}_h^k$ and $\hat{\mathbf{q}}_n$ are nodal and modal expansion coefficients respectively. $\mathbf{q}_h^k$ and $\hat{\mathbf{q}}_n$ are related by using Vandermonde matrix $\mathbf{V}$

$$\mathbf{V}\hat{\mathbf{q}} = \mathbf{q}\,; \; V_{ij} = \psi_j(\mathbf{r}_i)$$

$\psi_j(\mathbf{r})$ is modal basis function defined on tetrahedron element and obtained by combined Jacobi polynomials.

$$\begin{aligned} &\psi_m(\mathbf{r}) = \sqrt{2} P_i(a) P_j^{(2i+1,0)}(b)(1-b)^i, \\ &(i,j) \geq 0; \quad i + j \leq N, m = j + (N+1)i + 1 - \frac{i}{2}(i-1) \end{aligned} \tag{6}$$

$$\text{with } a = 2\frac{1+r}{1-s} - 1, \quad \mathrm{b} = \mathrm{s} \tag{7}$$

$P_n^{(\alpha,\beta)}(x)$ is the n^{th}-order polynomial Jacobi.

As the FE method procedure, the physical triangular is mapped as shown in Figure 1 to standard triangular by a relation:

$$\Psi(\mathbf{r}) = \mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix} = -\frac{r+s}{2}\begin{pmatrix} x^1 \\ y^1 \end{pmatrix} + \frac{r+1}{2}\begin{pmatrix} x^2 \\ y^2 \end{pmatrix} + \frac{s+1}{2}\begin{pmatrix} x^3 \\ y^3 \end{pmatrix} \tag{8}$$

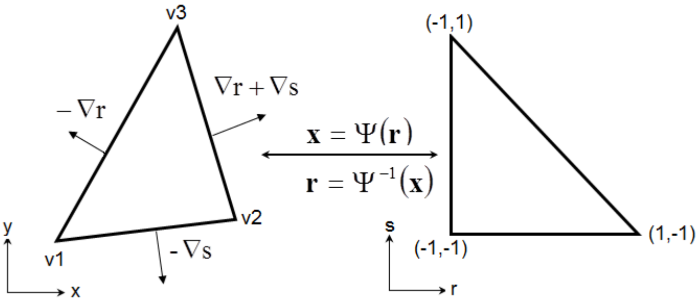


Figure 1. Triangular mapping, adapted from [20]

The detail of 2-D derivation of high order Discontinuous Galerkin method is described in [20]. The semi-algebraic (2) is integrated into time marching by using five stages of fourth order low storage Runge-Kutta scheme as developed by Carpenter & Kennedy [21].

3. RESULTS AND DISCUSSION

In this section, we present 2 (two) numerical examples to demonstrate the performance of the DG method. The first example illustrated the modeling of 6 cm diameter metal cylindrical scatterer in free space. The spatial domain is divided into 1753 triangular elements, and the thickness of PML is 0.024 m as shown in Figure 2. The hard source excitation is the combination of an exponential and sinusoidal pulse with the carrier frequency of 5 MHz.

$$f(t) = \sqrt{\frac{\mu_o}{\varepsilon_o}}\left(1 - e^{-\left(\frac{t}{20}\right)^2}\right)\sin(2\pi f t) \tag{9}$$

We took the time stepping dt=2.43e-12 and polynomial order N=3.

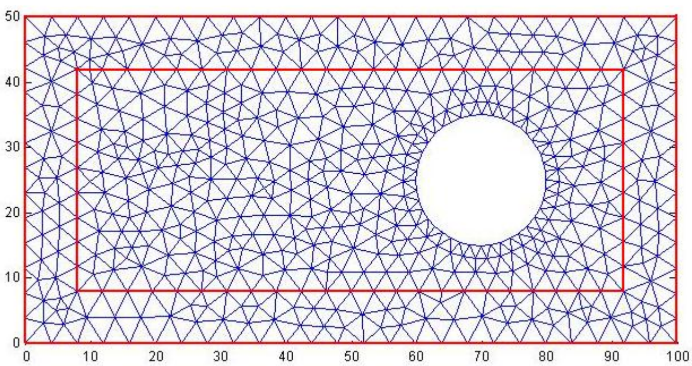


Figure 2. The domain of the first example

Figures 3(a)-3(d) show the snapshots of the Hz fields. Those figures show that the waves will be reflected when the waves hit the metal cylinder, and the wave will be absorbed well when entering the perfectly matched layer.

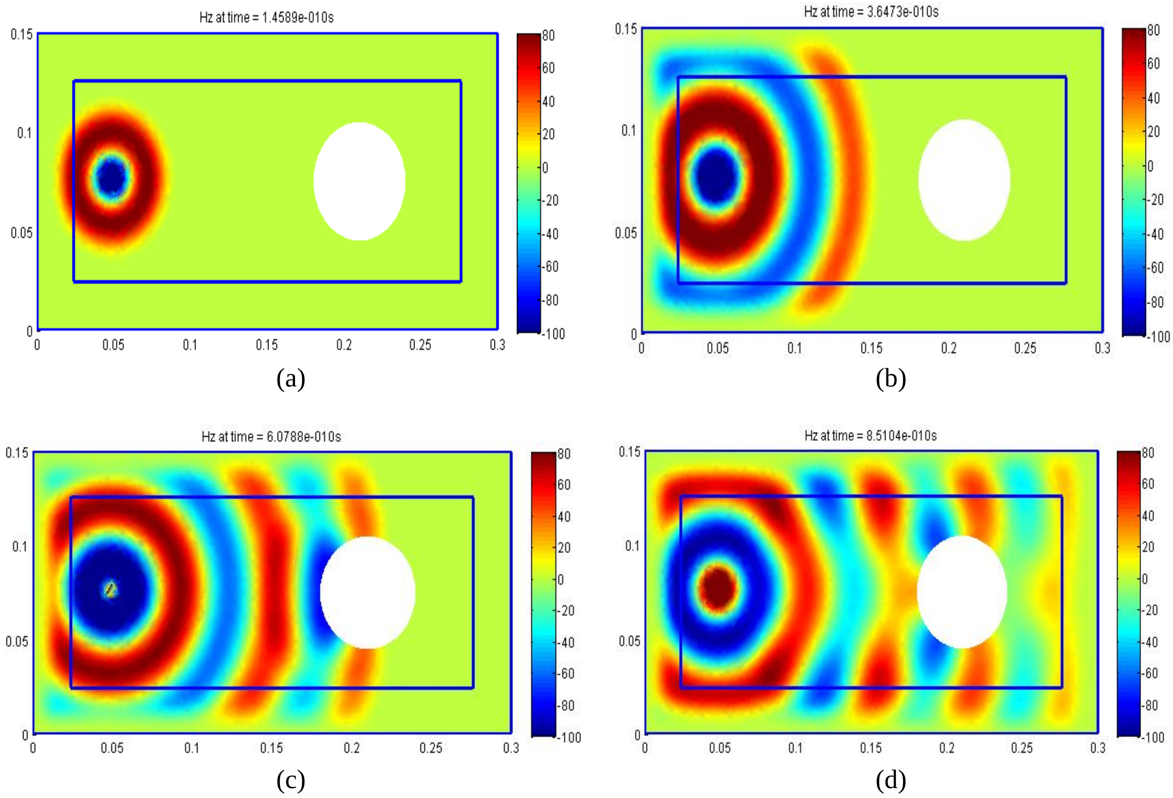


Figure 3. (a) HzPulse propagation att=1.46e-10 s of DG method, (b) HzPulse propagation att=3.65e-10 s of DG method, (c) HzPulse propagation att=6.08e-10 s of DG method, (d) HzPulse propagation att=8.51e-10 s of DG method

The numerical calculations are compared with the results of the FDTD method. The FDTD code is provided by Susan Hagness (https://github.com/cvarin/FDTD/blob/master/Taflove/fdtd2D.m). Figure 3(d) and Figure 4 show the spatial distribution of the magnetic field and Figure 5 shows the comparison of magnetic pulse at position (0.0465, 0.675). The comparisons show excellent agreement.

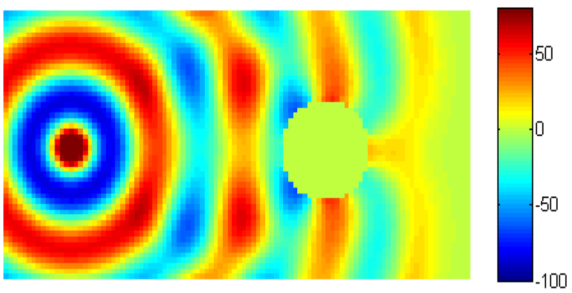


Figure 4. Hz Pulse propagation att =8.51e-10 s of FDTD method

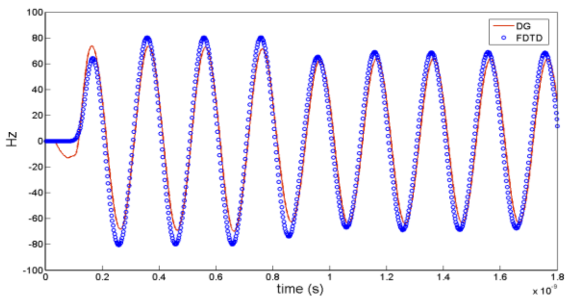
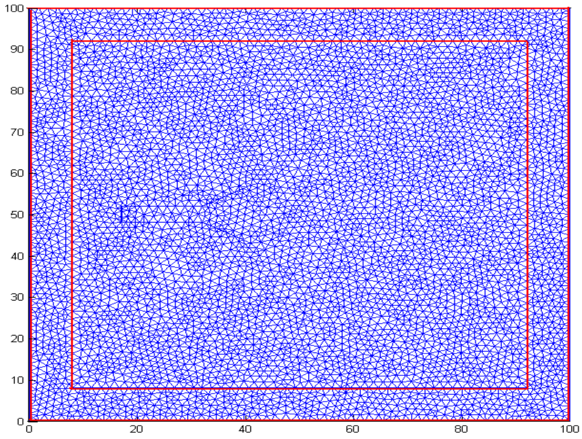
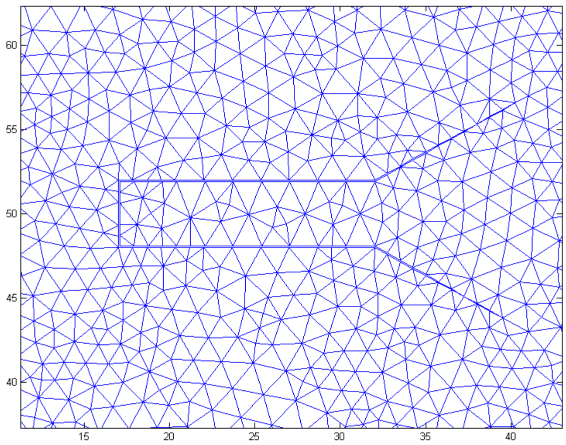


Figure 5. The domain of the first example

The spatial domain of the second example is a two-dimensional horn antenna as shown in Figures 6(a)-6(b) [23], the domain is divided into 11232 triangular elements. Metallic walls are taken as a boundary condition. Similar to the first example, following the hard source excitation is with the carrier frequency of 10 GHz is taken. We took the time stepping Δt=3.12e-013 and polynomial order N=7.



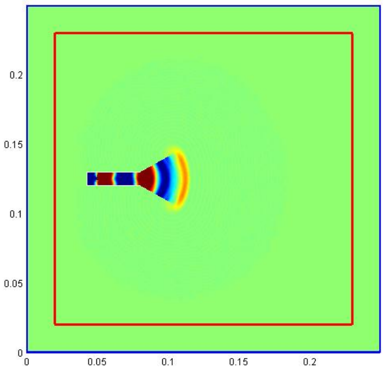
(a)



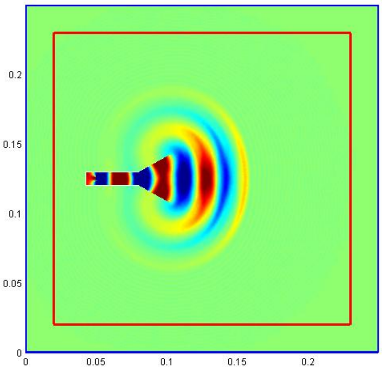
(b)

Figure 6. (a) Horn antenna, (b) Detail of the horn antenna

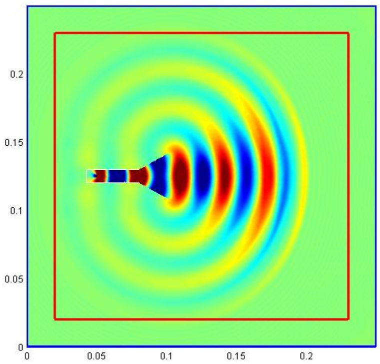
Figures 7(a)-7(e) show the wave radiation of the horn antenna. The metallic wall of the horn antenna acted as a waveguide. The waves initially propagated to the left and right directions. When the wave hit the left end of the metallic wall, it will be reflected right direction, and the wave, which is propagated to the right, will be spread following the divergent metallic wall. Those images demonstrate that the PML technique is adequately useful for absorbing the narrowband signal.



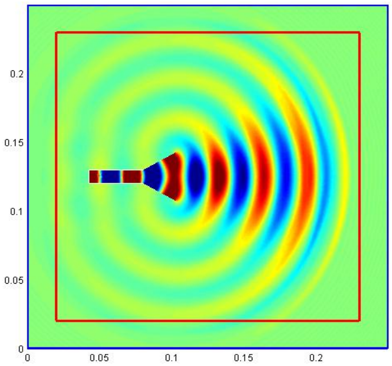
(a)



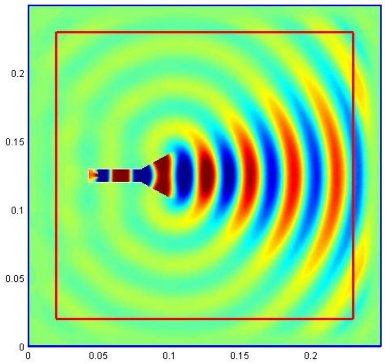
(b)



(c)



(d)



(e)

Figure 7. (a) Hz pulse propagation att=1.87e-10 s, (b) Hz pulse propagation att=3.12e-10 s, (c) Hz pulse propagation att=4.37e-10 s, (d) Hz pulse propagation att=4.99e-10 s, (e) Hz pulse propagation att=6.55e-10 s

4. CONCLUSION

In this paper, we have proposed the simulations of 2-D electromagnetic wave radiation in the time domain using a high order discontinuous Galerkin method and PML boundary condition. The use of unstructured triangular elements makes the methods very attractive and complex geometries can be handled easily. The numerical examples and the comparison with the FDTD method indicate the capability of the proposed approach for electromagnetic wave simulation.

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REFERENCES

[1] R. K. Adair, "Biophysical limits on athermal effects of RF and microwave radiation", *Bioelectromagnetics*, vol.24 (1), 2003, pp. 39–48.
[2] P. V. Shpak et al., "Generation of multi-frequency radiation in pulsed microchip laser with Raman conversion", *Laser Physics Letters*, vol.7 (8), 2010, pp. 555–559.
[3] V. Cooray, "Numerical Solution of Initial Boundary Value Problems Involving Maxwell's Equations in Isotropic Media", *IEEE Transaction on Antennas and Propagation*, vol.14 (3), 2007, pp. 302–307.
[4] L. Sevgi, "Complex Electromagnetic Problems and Numerical Simulation Approaches," *Wiley-IEEE Press*, 2003.
[5] N. A. Elias et al., "SAR Levels for Irradiation by a Crumpled 900 MHz Flexible Diamond Dipole," *International Journal of Electrical and Computer Engineering (IJECE)*, vol. 7 (3), June 2016, pp. 1546-1553.
[6] C.Guesmi, A.Ferchichi, and A.Gharsallah, "A Modified Fractal Bow Tie Antenna for an RFID Reader," *International Journal of Electrical and Computer Engineering (IJECE)*, vol. 4 (3), June 2014, pp.441-446.
[7] K. S. Yee, "Numerical Solution of Initial Boundary Value Problems Involving Maxwell's Equations in Isotropic Media", *IEEE Transaction on Antennas and Propagation*, vol.14 (3), 1966, pp. 302–307.
[8] A. Taflove, "Computational Electrodynamics: The finite-difference-time-domain method", Artech House, Boston, 1995.
[9] D.Baumann, C.Fumeaux, P.Leuchtmann, R. Vahldieck, A, "Finite-volume time-domain (FVTD) modeling of a broadband double-ridged horn antenna," *Journal of Numerical Modelling: Electronic Networks, Devices and Fields*, vol. 17, 2004, pp. 285-298.
[10] C. Fumeaux, D. Baumann, R.Vahldieck, "Finite-volume time-domain (FVTD) modeling of a broadband double-ridged horn antenna," *Journal of IEEE Transactions on Antennas and Propagation*, vol. 54, no. 3, 2006, pp.844-851.
[11] J. F. Lee, R. Lee, and A. Cangellaris, "Time-domain finite-element methods", *IEEE Transactions on Antennas and Propagation*, vol. 45 (3), 1997, pp. 430-442.
[12] G. Rodrigue and D. White, A Vector Finite Element Time-Domain Method for Solving Maxwell's Equations on Unstructured Hexahedral Grids, *SIAM Journal on Scientific Computing*, vol.23 (3),pp. 683–706.
[13] J. Ali, R. Yahya, N. Abdullah, and S.Z. Sapuan," FDTD analysis of the incident field coupling to Printed Circuit Board conductors connected with diode and MESFETs: GTEM validation", *International Journal of Electrical and Computer Engineering (IJECE)*, Vol. 7, No. 6, December 2017, pp. 2936~2941.
[14] J. S. Hesthaven and T. Warburton, "High-order nodal methods on unstructured grids, I. Time Domain Solution of Maxwell's Equations," *J. Computational Physics*, vol. 181, 2002, pp. 1-34.
[15] J. S. Hesthaven and T. Warburton, "High-order nodal discontinuous Galerkin methods for Maxwell eigen value problem," Phil. Trans. Roy. Soc. London, Series A, *Mathematical and Physical Sciences*, 362 (1816) ,2004, pp. 493-524.
[16] G. Cohen, X. Ferrieres, and S. Pernet, "A spatial high-order hexahedral discontinuous Galerkin method to solve Maxwell's equations in time domain," *J. Computational Physics*, vol. 217, 2006, pp. 340-363.
[17] Pranowo, "A novel space–time Discontinuous Galerkin method for solving of one-dimensional Electromagnetic wave propagations," *TELKOMNIKA Telecommunication Computing Electronics and Control*, Vol.15, No.3, September, pp. 1310~1316. 2017.
[18] J. P. Berenger, "A perfectly matched layer for the absorption of electromagnetic waves," *J. Computational Physics*, vol. 114, 1994, pp. 185-200.
[19] Pranowo,C. "Pemodelan Awal Ground Penetrating Radar dengan Metode Discontinuous Galerkin dan PML Berenger,"*JNTETI*, Vol. 5, No. 2, Mei 2016, pp. 115-121.
[20] J. S. Hesthaven and T. Warburton, Nodal discontinuous Galerkin methods: algorithms, analysis, and applications, Springer, New York, 2008.
[21] M. H.Carpenter and C. A. Kennedy., 1994, Fourth-order 2N-Storage Runge-Kutta Schemes, NASA Technical Memorandum 109112, NASA Langley Research.

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