

## LMS Adaptive Filters for Noise Cancellation: A Review

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### ABSTRACT

This paper reviews the past and the recent research on Adaptive Filter algorithms based on adaptive noise cancellation systems. In many applications of noise cancellation, the change in signal characteristics could be quite fast which requires the utilization of adaptive algorithms that converge rapidly. Algorithms such as LMS and RLS proves to be vital in the noise cancellation are reviewed including principle and recent modifications to increase the convergence rate and reduce the computational complexity for future implementation. The purpose of this paper is not only to discuss various noise cancellation LMS algorithms but also to provide the reader with an overview of the research conducted.

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## 1. INTRODUCTION

The concept of noise cancellation has recently gained much attention and has been identified as a vital method to eliminate noise contained in useful signals [1-2]. The application of this technique can be found in various industrial and communication appliances, such as machineries, hands-free phones and transformers [3,4]. Additionally, noise cancellation has also been implemented in the field of image processing, biomedical signal, speech enhancement and echo cancellation [5-7]. As the noise from the surrounding environment severely reduces the quality of speech and audio signals it is quite necessary to suppress noise and enhance speech and audio signal quality, hence the acoustics applications of noise cancellation has become the thrust area of research. The basic concept of Adaptive Noise Canceller (ANC) which removes or suppresses noise from a signal using adaptive filters was first introduced by Widrow [8]. Due to long impulse responses, the computational requirements of adaptive filters are very high especially during implementation on digital signal processors. Where as in case of non-stationary environments and colored background noise convergence becomes very slow if the adaptive filter receives a signal with high spectral dynamic range [9]. To overcome this problem numerous approaches have been proposed in the last few decades. For example, the Kalman filter and the Wiener filter, Recursive-Least-Square (RLS) algorithm, were proposed to achieve the optimum performance of adaptive filters [10-12]. Amongst these the Least Mean Square (LMS) algorithm is most frequently used because of its simplicity and robustness. Though, the LMS lacks from substantial performance degradation with colored interference signals [13]. Other algorithms, such as the Affine Projection algorithm (APA), became alternative approaches but its computational complexity increases with the projection order, restricting its use in acoustical environments [14]. Noise from the surroundings automatically gets added to the signal in the process of transmission of information from the source to receiver side. The usage of adaptive filters is one of the most popular proposed solutions to reduce the signal corruption caused by predictable and unpredictable noise. Adaptive filters have been used in a broad range of application for nearly five decades. It includes adaptive noise

cancellation, adaptive system identification, linear prediction, adaptive equalization, inverse modeling, etc. Noise is assumed to be a random process and adaptive filters have the capability to adjust their impulse response to filter out the correlated signal in the input. They require modest or no a priori knowledge of the signal and noise characteristics. In addition adaptive filters have the potential of adaptively tracking the signal under non-stationary conditions. It has the unique characteristic of self-modifying [14] its frequency response to change the behavior in time and allowing the filter to adapt the response to the input signal characteristics change. The basic principle of an adaptive filter is shown in Figure 1.

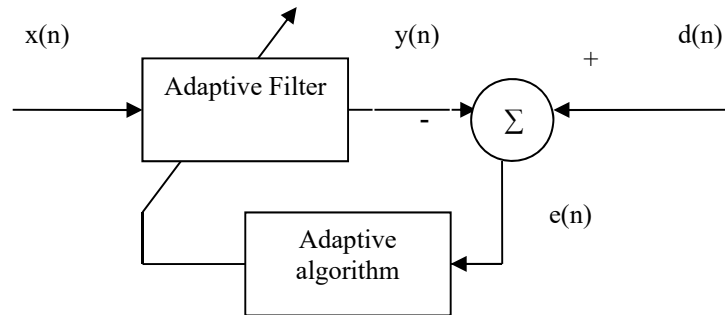


Figure 1. Adaptive Filter

The objective is to filter the input signal,  $x(n)$ , with an adaptive filter in such a manner that it matches the desired signal,  $d(n)$ . In order to generate the error signal the desired signal,  $d(n)$ , is subtracted from the filtered signal,  $y(n)$ . An adaptive algorithm is driven by the error signal which generates the filter coefficients in a manner that minimizes the error signal. Unlike from the fixed filter design, here the filter coefficients are tunable, are adjusted in dependency of the environment that the filter is operated in, and can therefore track any potential changes in this environment. Using this concept, adaptive filters can be tailored to the environment set by these signals. However, if the environment changes filter through a new set of factors, adjusts for new features [15]. The adaptive filter constitutes a vital part of the statistical signal processing. The application of an adaptive filter offers a smart solution to the problem wherever there is a need to process signals that result from operation in an environment of unknown statistics, as it typically provides a significant enhancement in performance over the use of a fixed filter designed by conventional methods [17-18]. The aim of this paper is to review the existing noise cancellation techniques for enhancing speech and audio signal quality and to provide the understanding of suitability of various developed models. Prior to this, a brief review of the adaptive noise cancellation methods and its application is presented in the next section. Finally, a perception on upcoming research is suggested for further consideration.

## 2. ADAPTIVE NOISE CANCELLATION

Acoustic noise cancellation is indispensable from the health point of view as extensive exposures to high level of noise may cause serious health hazards to human being. The conventional noise cancellation method [19] uses a reference input signal (correlated noise signal) which is passed through the adaptive filter to make it equal to the noise that is added to original information bearing signal. Subsequently this filtered signal is subtracted from noise corrupted information signal. This makes the corrupted signal a noise free signal. The fundamental concept of noise cancellation [19] is to produce a signal that is equal to a disturbance signal in amplitude and frequency but has opposite phase. These two signals results in the cancellation of noise signal. The original Adaptive noise cancellation (ANC) [20] uses two sensors to receive the noise signal and target signal separately. The relationship between the noise reference  $x(n)$  and the component of this noise that is contained in the measured signal  $d(n)$  may be determined by Adaptive noise cancellation shown in Figure 2

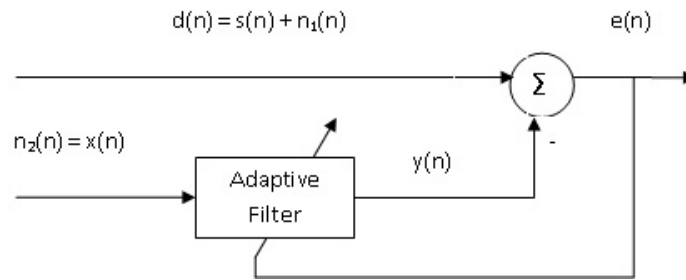


Figure 2. Adaptive noise cancelling

If several unrelated noises corrupt the measurement of interest then several adaptive filters may be deployed in parallel as long as suitable noise reference signals are available within the system. In noise cancelling systems the objective is to produce a system output  $e(n) = [s(n) + n_1] - y(n)$  which is a best fit in the least squares sense to the signal  $s(n)$ . This objective is achieved by adjusting the filter through an adaptive algorithm and feeding the system output back to the adaptive filter and to minimize total system output power [20]. In an adaptive noise cancelling system, the system output serves as an error signal for the adaptive process.

**2.1. Digital Filters**

The purpose of digital filters is to separate signals that have been combined and to restore signals that have been distorted in some way [22]. Signal separation is required when a signal has been contaminated with interference, noise, or other signals whereas restoration is used when a signal has been distorted in some way. Broadly the digital filters are classified as Weiner and Kalman filters [23].

**2.1.1. Wiener filter**

A Wiener filter [24] is a digital filter, which is designed to reduce the mean square difference between some desired signal and the filtered output. It is occasionally called a minimum mean square error filter. A Wiener filter [25] can be finite-duration impulse response (FIR) filter or an infinite-duration impulse response (IIR) filter or a [26]. Generally the formulation of an FIR Wiener filter results in a set of linear equations and has a closed-form solution whereas the formulation of an IIR Wiener filter [27] results in a set of non-linear equations. The Wiener filter represented by the coefficient vector  $w$  is depicted in Figure 3. The filter accepts the input signal  $y(m)$ , and generates an output signal  $\hat{x}(m)$ , where  $\hat{x}(m)$  is the least mean square error estimate of a desired or target signal  $x(m)$ . The filter input–output relation is shown in Equation 1.

$$\begin{aligned}
 x(m) &= \sum_{k=0}^{p-1} w_k y(m-k) \\
 &= w^T y
 \end{aligned}
 \tag{1}$$

where  $m$  is the discrete-time index,  $y^T = [y(m), y(m-1), \dots, y(m-P+1)]$  is the filter input signal, and the parameter vector  $w^T = [w^0, w^1, \dots, w^{P-1}]$  is the Wiener filter coefficient vector.

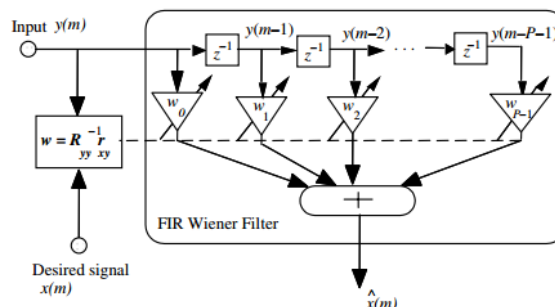


Figure 3. Illustration of a Wiener Filter Structure

### 2.1.2. Kalman Filter

The Kalman filter is a mathematical power tool which plays an important role in computer graphics as we include sensing of the real world in our systems. The Kalman filter can also be termed as a set of mathematical equations that implement a predictor-corrector type estimator which is optimal in the sense that it minimizes the estimated error covariance—when some presumed conditions are met. For the past decade the Kalman filter has been the active area of research and application, particularly in the area of autonomous or assisted navigation. The Kalman filter [28] (and its variants such as the extended Kalman filter [29] and unscented Kalman filter [30] is one of the most popular data fusion algorithms in the field of information processing. [31-36].

### 2.2. Adaptive Filters

An adaptive filter [37] is a system with a linear filter which consists of transfer function restrained by variable parameters and a means to adjust those parameters according to an optimization algorithm. Adaptive linear filters [38] are linear dynamical system with variable or adaptive structure and parameters and have the property to modify the values of their parameters, i.e. their transfer function, during the processing of the input signal, in order to generate signal at the output which is without undesired components, noise, and degradation and also interference signals.

Figure.4 shows the basic concept of an adaptive filter [39] whose primary objective is to filter the input signal,  $x(n)$ , with an adaptive filter in such a manner that it matches the desired signal,  $d(n)$ . The desired signal,  $d(n)$ , is subtracted from the filtered signal,  $y(n)$ , to produce an error signal which in turn drives an adaptive algorithm that generates the filter coefficients in a manner that minimizes the error signal. The adaptation adjusts the characteristics of the filter through an interaction with the environment in order to reach the desired values. Contrary to the conventional filter design techniques, adaptive filters do not have constant filter coefficients and no priori information is known, such a filters with adjustable parameters are called an adaptive filter. Adaptive filter adjust their coefficients to minimize an error signal and may be termed as finite impulse response (FIR) [40], infinite impulse response (IIR) [41], lattice and transform domain filter. Generally adaptive digital filters consist of two separate units: the digital filter, with a structure determined to achieve desired processing (which is known with an accuracy to the unknown parameter vector) and the adaptive algorithm for the update of filter parameters, with an aim to guarantee fastest possible convergence to the optimum parameters from the point of view of the adopted criterion. Majority of adaptive algorithms signify modifications of the standard iterative procedures for the solution of the problem of minimization of criterion function in real time. The most common form of adaptive filters are the transversal filter using least mean square (LMS) algorithm [42] and recursive least square (RLS) algorithm [43].

### 2.3. Adaptive Algorithms

Adaptive algorithms [44] have been extensively studied in the past few decades and the most popular adaptive algorithms are the least mean square (LMS) algorithm and the recursive least square (RLS) algorithm. Attaining the best performance of an adaptive filter requires usage of the best adaptive algorithm with low computational complexity and a fast convergence rate.

#### 2.3.1. Least-Mean-Square Algorithm (LMS)

A very straightforward approach in noise cancelling is the use of LMS algorithm which was developed by Windrow and Hoff [45]. This algorithm uses a gradient descent to estimate a time varying signal. The gradient descent method finds a minimum, if it exists, by taking steps in the direction negative of the gradient and it does so by adjusting the filter coefficients in order to minimize the error. The gradient is the del-operator and is applied to find the divergence of a function, which is the error with respect to the nth coefficient in this case. The LMS algorithm has been accepted by several researchers for hardware implementation because of its simple structure. In order to implement it, modifications have to be made to the original LMS algorithm because the recursive loop in its filter update formula prevents it from being pipelined.

The following equation shows the detail of LMS algorithm,  
Weights evaluation –

$$w_i(n+1) = w_i(n) + \mu * e(n) * x(n-i) \quad (2)$$

Filtering output –

$$y(n) = \sum_{i=0}^{M-1} w_i(n) * x(n-i) \quad (3)$$

Error estimation (where error is the desired output)–

$$e(n) = d(n) - y(n) \quad (4)$$

where the output of an adaptive filter  $y(n)$  and the error signal  $e(n)$  are given by (3) and (4), respectively. In these equations,  $x(n)$  is the input signal vector, and  $w(n)$  is the tap weight vector of the adaptive filter. The equations employ the current estimate of the weight vector. From these equations it is clear that at each iteration, the information of most recent values ( $d(n)$ ,  $x(n)$ ,  $w(n)$  and  $e(n)$ ) are required and the iterative procedure is started with an initial guess  $w(0)$ .  $\mu$  is the step size that depends on the power spectral density of the reference input  $x(n)$  and filter length  $M-1$  and control the stability and convergence speed of the LMS algorithm.

In the recent times, a new version of the LMS algorithm with time varying convergence parameter has been proposed **Error! Reference source not found..** The time-varying LMS (TV-LMS) [47] algorithm has shown better performance than the conventional LMS algorithm in terms of less mean square error MSE and faster convergence. The TV-LMS algorithm is based on utilizing a time-varying convergence parameter  $\mu$  with a general power decaying law for the LMS algorithm. The basic concept of TV-LMS algorithm is to exploit the fact that the LMS algorithms need a larger convergence parameter value to speed up the convergence of the filter coefficients to their optimal values. After the coefficients converge to their optimal values, the convergence parameter ought to be small for better estimation accuracy. In other words, we set the convergence parameter to a large value in the initial state in order to speed up the algorithm convergence.

### 2.3.2. NLMS Algorithm

The main weakness of the conventional type LMS lies in its complexity in selecting a suitable value for the step size parameter that guarantees stability. In order to overcome, NLMS has been proposed in controlling the convergence factor of LMS through modification into a time-varying step size parameter. As NLMS employs a variable step size parameter intended at minimizing the instantaneous output error hence converges faster than the conventional LMS [48-49]. The conventional LMS algorithm experiences a gradient noise amplification problem as the convergence factor  $\mu$  is large. The correction applied to the weight vector  $w(n)$  at iteration  $n+1$  is "normalized" with respect to the squared Euclidian norm of the input vector  $x(n)$  at iteration  $n$ . We may express the NLMS algorithm as a time-varying step-size algorithm, calculating the convergence factor  $\mu$  as in Equation 5.

$$\mu(n) = \frac{\alpha}{c + \|x(n)\|^2} \quad (5)$$

where:  $\alpha$  is the NLMS adaption constant, which optimize the convergence rate of the algorithm and should satisfy the condition  $0 < \alpha < 2$ , and  $c$  is the constant term for normalization and is always less than 1. The Filter weights are updated by the Equation 6.

$$w(n+1) = w(n) + \frac{\alpha}{c + \|x(n)\|^2} e(n) x(n) \quad (6)$$

In comparison to LMS, the NLMS has varying step size that makes the NLMS to converge more quickly. In order to best serve various applications several variants of LMS have been developed. Some of the popular variants are Modified Normalized LMS (MN-LMS) algorithm, Leaky LMS, Block LMS, Sign Error LMS, Sign-Data LMS (SD-LMS), Sign-Data Normalized LMS (SDN-LMS), Sign-Sign LMS (SS-LMS) algorithm, Sign-Sign LMS algorithm with leakage term (SS-LMS-LT), Variable step-size LMS (VS-LMS) algorithm, Filtered X-LMS (Fx-LMS) algorithm, Frequency response shaped LMS (FRS-LMS) algorithm, Hybrid LMS (H-LMS) algorithm are summarized in Table 1.

### 2.3.3. Recursive least square (RLS) Algorithm

RLS algorithm is another potential alternative to overcome slow convergence in colored environments [43], which uses the least squares method to develop a recursive algorithm for the adaptive transversal filter. The RLS [82] recursively finds the filter coefficients that minimize a weighted linear least squares cost function relating to the input signals. RLS tracks the time variation of the process to the optimal filter coefficient with relatively very fast convergence speed; though it has increased computational complexity and stability problems as compared to LMS-based algorithms [83]. The RLS algorithm [84-85] has established itself as the "ultimate" adaptive filtering algorithm in the sense that it is the adaptive filter exhibiting the best convergence behavior. Unfortunately, practical applications of the algorithms are often associated with high computational complexity and poor numerical properties. Several different standard

RLS algorithms with varying degrees of computational complexity and stability exist. Amongst all the conventional recursive least squares (CRLS) algorithm is considered to be the most stable, but requires  $O(N^2)$  (on the order of  $N^2$ ) operations per iteration, where  $N$  is the filter length [86].

Fast Transversal RLS Algorithm

Fast transversal filter (FTF) algorithm [87-88] involves the combined use of four transversal filters for forward and backward predictions, joint process and gain vector computation estimation. The merit of FTF algorithm lies in its reduced computational complexity as compared to other available solutions.

Table 1. Variation of LMS algorithm

S. No	Algorithm type	Recursion (Weighted)	Reference
1.	Conventional LMS	$w_i(n+1) = w_i(n) + \mu^* e(n) * x(n-i)$	[45], [48]
2.	NLMS	$w(n+1) = w(n) + \frac{\mu}{c + \ x(n)\ ^2} e(n) x(n)$	[48-49]
3.	(MN-LMS)	$W(n+1) = W(n) + \beta \frac{X(n)}{e + \ X(n)\ ^2} \mu e(n)$ Where, $0 < \beta < 2$	[50-51]
4.	Leaky LMS	$W(n+1) = (1 - \mu\gamma)W(n) + X(n)\mu e(n)$ Where, leaky coefficient $\gamma$ , $0 < \gamma < 1$ $0 < \mu < (\gamma + \lambda_{\max})$	[52-54]
5.	(B-LMS)	$W((k+1)L) = W(kL) + \mu \frac{1}{L} \sum_{l=0}^{L-1} e(kL+l) X(kL+l)$ Where, $l = 0, 1, 2, \dots, L-1$	[55-57]
6.	(SE-LMS)	$W(n+1) = W(n) + X(n)\mu \text{sgn}[e(n)]$ Where, $\text{sgn}(\cdot) = \text{signum function } \text{sgn}[e(n)] = \begin{cases} 1 & \text{for } e(n) > 0 \\ 0 & \text{for } e(n) = 0 \\ -1 & \text{for } e(n) < 0 \end{cases}$	[58-59]
7.	(SD-LMS)	$W(n+1) = W(n) + \text{sgn}[X(n)]\mu e(n)$ Where, $\text{sgn}(\cdot) = \text{signum function}$	[60-62]
8.	(SDN-LMS)	$w_k(n+1) = w_k(n) + \frac{\mu}{ x(n-k) } e(n)x(n-k)$ Where, $\text{sgn}(\cdot) = \text{signum function}$	[63]
9.	(SS-LMS)	$W(n+1) = W(n) + \text{sgn}[X(n)]\mu \text{sgn}[e(n)]$ Where, $\text{sgn}(\cdot) = \text{signum function}$	[64-65]
10.	(VS-LMS)	$w_k(n+1) = w_k(n) + \mu_k e(n)x(n-k)$ Where, $\mu_{\min} < \mu < \mu_{\max}$	[66-69]
11.	(SS-LMS-LT)	$W(n+1) = (1 - \mu\gamma)W(n) + \text{sgn}[X(n)]\mu \text{sgn}[e(n)]$	[89-90]
12.	(FX-LMS)	$W(n+1) = W(n) + X^{(n)}\mu e(n)$ Where, $X^{(n)} = \hat{S}(n)X(n)$	[70-74]
13.	(FRS-LMS)	$W(n+1) = [I - \mu F]W(n) + X(n)\mu e(n)$ Where, $F = \square F_0$ and $\square$ is constant.	[75-77]
14.	(H-LMS)	$W(n+1) = W(n) + X(n)\mu e(n)$ for, $0 \leq n \leq p$ $W(n+1) = W(n) + \frac{X(n)\mu(n)e(n)}{E[X(n)X^T(n)]}$ for, $n \geq p+1$	[78-81]

### 3. CONCLUSION

A comprehensive review has been carried out to identify the existing literature related to adaptive filtering in noise reduction using LMS adaptive algorithms in particular. LMS is preferred over RLS algorithms for various noise cancellation purposes as RLS has increased computational complexity and stability problems as compared to LMS-based algorithms which are robust and reliable. Various LMS adaptive algorithms viz. N-LMS, MN-LMS, Leaky LMS, Block LMS, SE-LMS, SD-LMS, SDN-LMS, SS-LMS, SS-LMS-LT, VS-LMS, FX-LMS, FRS-LMS, H-LMS are dealt in this paper for the purpose of comparison in terms of simplicity and application. The LMS algorithm is relatively simple to implement and is powerful enough to evaluate the practical benefits that may result from the application of adaptivity to the problem at hand. Moreover, it provides a practical frame of reference for assessing any further improvement that may be attained through the use of more sophisticated adaptive filtering algorithms.

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