

# Packets Wavelets and Stockwell Transform Analysis of Femoral Doppler Ultrasound Signals

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## ABSTRACT

Ultrasonic Doppler signals are widely used in the detection of cardiovascular pathologies or the evaluation of the degree of stenosis in the femoral arteries. The presence of stenosis can be indicated by disturbing the blood flow in the femoral arteries, causing spectral broadening of the Doppler signal. To analyze these types of signals and determine stenosis index, a number of time-frequency methods have been developed, such as the short-time Fourier transform, the continuous wavelets transform, the wavelet packet transform, and the S-transform

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## 1. INTRODUCTION

Fourier analysis is a basic tool in signal processing. It is indispensable in many areas of research; unfortunately it has limitations when implemented beyond the strict framework of its definition: the area of stationary finite energy signals. In Fourier analysis, all the temporal aspects become illegible in the spectrum. The study of non-stationary signals therefore requires either an extension of the Fourier Transform (or stationary methods), introducing a temporal aspect, or the development of specific methods.

A first solution, implemented intuitively in the mid-century, corresponds to Fourier analysis sliding window or short time Fourier transform (*STFT*), which was introduced in 1945 by *D. Gabor* with the idea of a time-frequency plan where time becomes an additional parameter of frequency [1]. This method shows that a joint exact location in time and frequency is impossible, and introduces the idea of a discrete basis, minimum, resulting in a few coefficients of the signal energy distribution in time-frequency plan.

Other methods are used in this work, namely the continuous wavelet transform and wavelet packets. these two variances of the wavelet transform have existed in a latent state in both mathematics and signal processing, but the real expansion began in the early 1980s. The last method used in this work (based on the wavelet transform) is the *S*-transform proposed by *Stokwellet al.*; it is similar to *STFT* with an exception that the amplitude and width of the analysis window are a function of frequency, as in the wavelet transform [2].

## 2. METHODS OF ANALYSIS

### 2.1. Short-Time Fourier Transform

This method is based on the decomposition of the signal into small segments in which the Fourier transform is applied; there by generating a localized spectrum analytically *STFT* is given by the following relationship:

$$X_{\tau}(f) = \int_{-T/2}^{T/2} X(t)w(t-\tau)e^{-j2\pi ft} dt \quad (1)$$

Where  $w(t-\tau)$  is a selected window function. The action of this window is to locate in time, the resulting local spectrum. This localization window is then shifted in time to produce the local spectrum for the duration of the existence of  $x(t)$ . The resulting spectral power is called spectrogram [3]-[5].

### 2.2. Continuous Wavelet Transform

The continuous wavelet transform (*CWT*) is defined by:

$$CWT(a,b) = \int_{-\infty}^{+\infty} x(t)\Psi_{a,b}^*(t)dt \quad (2)$$

Where  $x(t)$  represents the analyzed signal,  $a$  and  $b$  represent respectively the scaling factor (dilatation/compression coefficient) and the time (shifting coefficient), and the superscript asterisk (\*) denotes the complex conjugation.  $\Psi_{a,b}(t)$  is obtained by scaling the wavelet at time  $b$  and scale  $a$ :

$$\Psi_{a,b}(t) = \frac{1}{\sqrt{|a|}} \Psi\left(\frac{t-b}{a}\right) \quad (3)$$

Where  $\psi(t)$  represents the wavelet time function [6].

### 2.3. The Wavelet Packet Transform

Wavelet packets used to decompose the signal to a large number of bases and selected with a certain criterion; the one that best represents the signal. During decomposition, the low-pass and high-pass filtered versions of the signal are decomposed. The approximation of the details and the details of the details are therefore added to the approximation of the signal. The coefficients from this decomposition are characterized by three parameters: the level of decomposition, frequency index, and time index. Each wavelet packet is carrying triple information  $\{f, s, p\}$ , frequency, scale, and position where the wavelet has only two parameters: scale and position [7]-[8].

In wavelet packet analysis, the signal is decompose into approximations and details. The approximation is then itself decomposed into approximation and detail in second level, and the process is repeated. For a decomposition of  $n$  level, there are  $(n+1)$  possibilities to decompose or to code the signal. The Figure 1 shows the decomposition of a digital signal in wavelet transform at three levels.

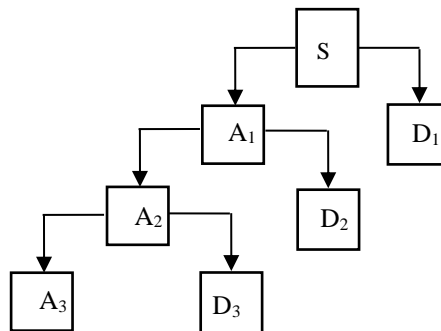


Figure 1. Wavelet transform decomposition scheme

Where  $S = A_1 + D_1$ ,  $S = A_2 + D_2 + D_1$ ,  $S = A_3 + D_3 + D_2 + D_1$ . In wavelet packets analysis, decomposition into approximation and detail is made only on approximations but also on details. In other words, when analyzing in wavelet packets, it is no longer only the filtered low-pass versions of the signal that are decomposed, but also the filtered high-pass versions. In another way, the high frequencies are also cut into sub-bands and the decomposition tree deviate symmetrically. The wavelet packets decomposition leads to a decomposition into frequency sub-bands of the signal [9]-[11].

#### 2.4. The S-Transform

The  $S$ -transform provides a time-frequency representation of a signal. It only combines a dependent frequency resolution with simultaneous location of the real and imaginary part of the spectrum. It was published for the first time in 1996 by Stokwell. The basic idea of this time-frequency distribution is similar to the Fourier transform sliding window, except that the amplitude and width of the analysis window are variable depending on the frequency as is the case in wavelet analysis [11]-[12]. The  $S$ -transform of a function  $x(t)$  can be defined as a transform into wavelets with a quite specific mother wavelet multiplied by a phase factor:

$$S(\tau, f) = e^{i2\pi f\tau} W(\tau, d) \quad (4)$$

Where  $W(\tau, d)$  is the continuous wavelet transform of the signal  $x(t)$  defined by:

$$W(\tau, d) = \int_{-\infty}^{+\infty} x(t) w(t - \tau, d) dt \quad (5)$$

Where the mother wavelet is defined by:

$$w(t, f) = \frac{|f|}{\sqrt{2\pi}} e^{-\frac{t^2 f^2}{2}} e^{-i2\pi f t} \quad (6)$$

Let us note that the dilatation factor  $d$  is the reverse of the frequency  $f$  [13]-[14].

### 3. RESULTS AND ANALYSIS

The Doppler signals studied in this work are from the S' Marie hospital in Leicester (England); the signal files are in .wav form with a sampling frequency of 44 KHz and a duration of 4.34s, corresponding to 191 390 samples. The Figure 2 shows a temporal representation and spectral analysis of a signal database. From the results of Keeton and Sadik given in [15]-[16], a Doppler signal can be regarded as a Gaussian signal in a segment of 10 ms to 12 ms. Although it has not always guaranteed that the Doppler signals are Gaussian at 10 ms, this remains true for segments below 10 ms.

The principal objective of this research is to compare STFT, CWT, PWT and S-transform methods in the case of the resolution of time-frequency of ultrasonic Doppler signals. The purpose of the time-frequency analysis is to provide a more informative description of the signal revealing the temporal variation of its frequency content. A solution, which is considered as the more intuitive, consists in associating a non-stationary signal a sequence of Fourier transforms short-term to adapt the successive observation windows to structural variations of the signal in such a way that the stationarity assumptions are locally satisfied [16]. The disadvantage of the Fourier transform is the stationary of signals, and therefore does not allow obtaining time information. STFT implicitly is considered for a non-stationary signal as a series of quasi-stationary situations across the analysis window.

The temporal resolution of such an analysis is determined by the width of the window, the frequency resolution is determined by the width of its Fourier transform. For a highly non-stationary signal as an ultrasonic Doppler signal, good temporal resolution is required, which requires working with a short window. The major disadvantage of this transform is the limitation of the frequency resolution. The problem of the STFT is that it uses a fixed size of window covering the time-frequency domain. Another disadvantage of this transform is the fixed temporal and frequency resolution [17]. Processing in general, or by using wavelet S-transform, offer the possibility to have a window that adapts according to the irregularities of the

signal. Wavelets are a family of functions localized in time and frequency and form an orthonormal basis. They are generated one from the other by translation and dilatation. Each wavelet is used to decompose the signal and is used as each exponential function in the Fourier transform. The difference is that the wavelet functions are well localized in time unlike exponentials [18].

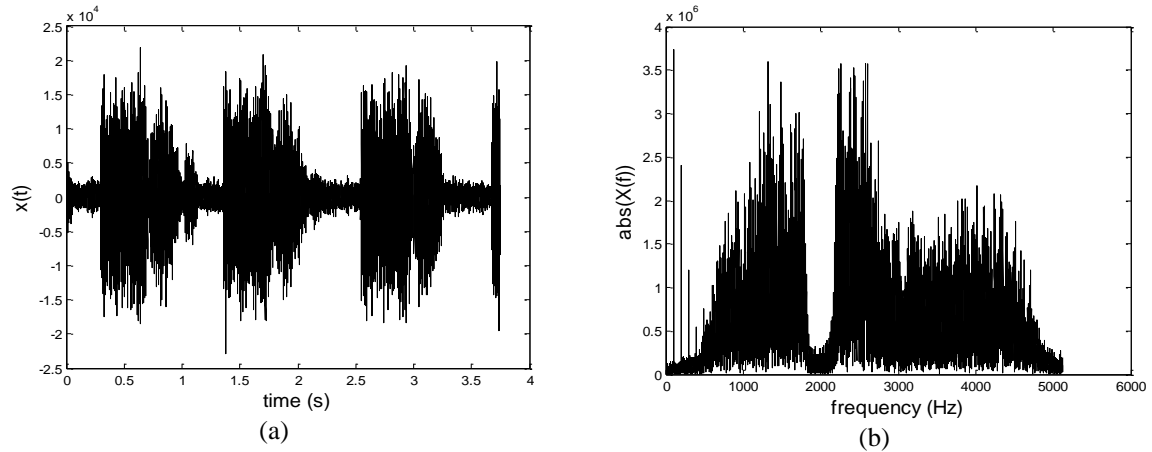


Figure 2. (a) Temporal representation. (b) Spectrum of Doppler Ultrasound Signal

The wavelet packets transform is a generalization of the DWT; it allows decomposing the details approximation. This transform allows decomposing the Doppler signal into sub-frequency bands by means of a filter bank. The coefficients of this decomposition give a time-frequency representation that can monitor the velocity of blood in the arteries. The wavelet-based transform like CWT, PWT or S-transform are designed to give good time resolution with a poor frequency resolution at high frequencies and a good frequency resolution with a poor temporal resolution at low frequencies. The major drawback of these transforms is the choice of the mother wavelet. Using the STFT or wavelet-based transform requires a compromise between time and frequency resolutions. For the STFT, narrower analysis window will provide better temporal resolution, but the concentration around the origin of the Fourier transform will necessarily be less, which implies a poorer frequency resolution. For further transforms, the compromise is similar, and depends on the scan frequency: increasing the analysis frequency implies improving the time resolution, but decreasing the frequency resolution [15].

The wavelet-based transforms have been designed for non-stationary signals since they incorporate the concept of scale to transformation, which gives better time-frequency resolution: a compressed wavelet to analyze the high frequency detail and a dilated wavelet to detect underlying trends of low frequency. In sonograms obtained by STFT, CWT, PWT and S-transform are given in Figure 3. The horizontal axis ( $t$ ) shows the time and the frequency ( $f$ ) is shown on the vertical axis. The gray level intensity represents the power level corresponding to a frequency for each point in the time axis [19].

It is clear that the CWT, PWT and S-transforms could help to improve the quality of sonograms of Doppler. Ultrasonic Doppler signals sampled contain a wealth of information on blood flow. The most comprehensive way to show this information is to perform a time-frequency analysis and present the results as sonograms. For the S-transform, a linear time-frequency representation is presented. This method surpasses the problem of the sliding window Fourier transform of fixed length, and addresses the notion of phase in the wavelet transform for non-stationary signals analysis. This transform provides a very suitable space for feature extraction and location in time and frequency discriminating information in the ultrasonic Doppler signal [14].

In Figure 3, one can observe an improvement in quality of sonograms obtained by the wavelet transform over those obtained by STFT. The sonograms obtained by STFT give a low-quality spectral interpretation in terms of location of minimum and maximum frequencies. The advantage of the S-transform is optimizing the time-frequency resolution.

It is clear from Figure 3 that there is a certain qualitative improvement in sonograms obtained by the S-transform compared to those obtained by STFT. Sonograms obtained by STFT give false frequencies, and spectral analysis by the STFT produces unclear sonograms because of the distortion in spectral estimation caused by sliding window. The advantage of the S-transform compared to STFT is the optimization of time-frequency resolution and the dynamic localization of the spectrum in the time-frequency plan. The second

advantage of the  $S$ -transform is a better location of systolic peaks used to determine the spectral broadening index ( $SBI$ ).

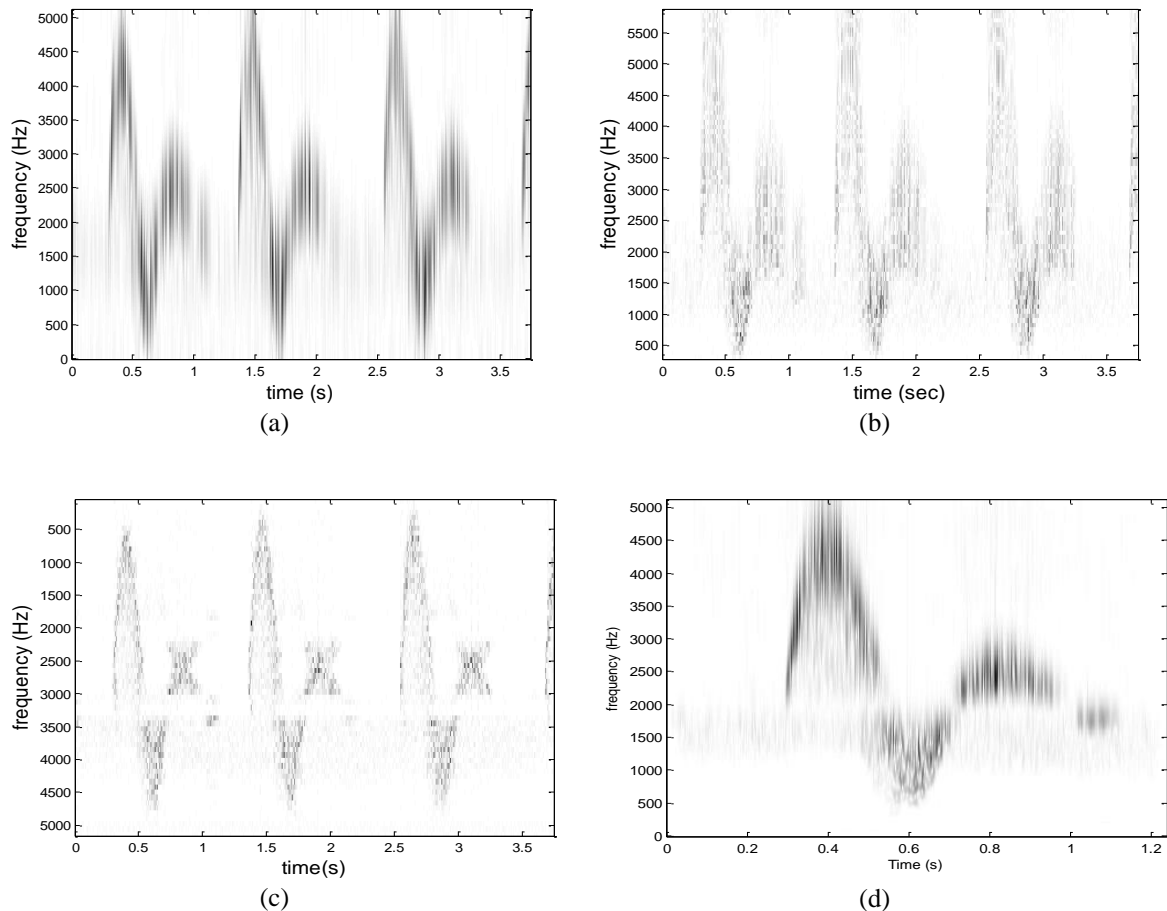


Figure 3. Femoral arterial Doppler sonograms: (a) using  $STFT$ , (b) using  $CWT$ , (c) using  $PWT$ , (d) using  $S$ -transform

Particularly because of the intrinsic limitations of the spectrogram and particularly time-frequency resolution of problems, other types of frequency-time representation are preferred. The time-frequency and time-scale analysis's have been developed to meet a need for demonstrating phenomena that are very localized in time and frequency. Unlike the spectrogram, the resolution of the time-frequency representation obtained by the  $S$ -transform or wavelet-transform is dependent on frequency and time. Both well localized in time and frequency, the  $S$ -transform or wavelet transform has properties of "zoom" making it an ideal tool for detecting phenomena of high frequency and short duration. It is worth reminding that wavelet transforms possess a good temporal resolution at high frequencies and the vice-versa [19]. The wavelet transforms utilize a set of analytical functions built by expansion / compression and translation of a function called mother wavelet. In our study, we chose the Morlet wavelet that has a similar form to Doppler ultrasound signals [20].

#### 4. CALCULATING SBI

Speeds of the red blood cells in a vessel are determined from the Ultrasound Doppler signals. The temporal evolution of these echoes is sonograms. A variation of these speeds translates directly a variation frequency, or temporal level sonogram. In fact, such situations exist where structures are found in the arteries (carotid or femoral). These directly affect the blood flow that becomes non-uniform in their neighborhood. This causes enlargement of the Doppler signal spectrum near the systolic peak quantified by what we call the

spectral broadening index (*SBI*). However, this index is highly correlated with the nature of the envelope frequency ( $f_{max}$  and  $f_{mean}$ ). In fact, the frequency generated from sonograms is embedded in noise emanating from different sources reflecting (the wall of the artery, skin, etc.) that can see in Figure 4.

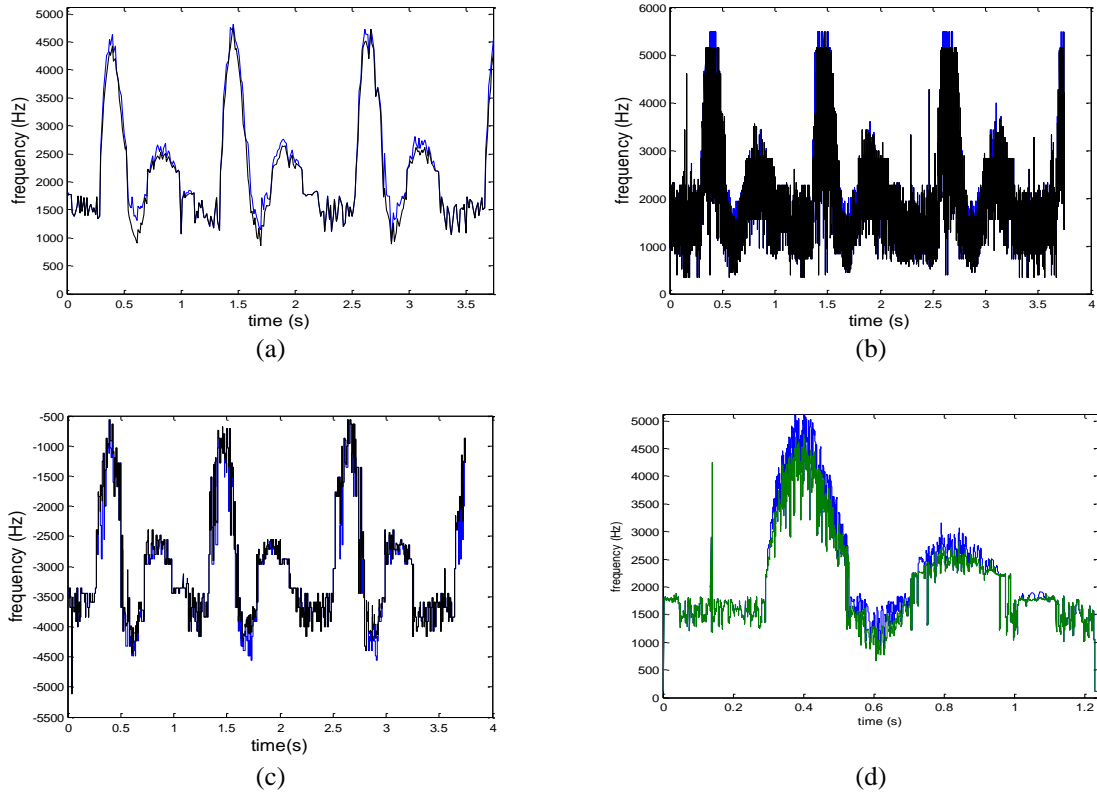


Figure 4. Maximum and mean frequency envelopes of the femoral arterial extracted from (a) STFT, (b) CWT, (c) PWT, (d) S-transform sonograms

A filter applied to the spectral envelopes is required to determine the systolic peak and estimate the *SBI* index. The Figure 5 illustrate the systolic peaks that are clearly defined and thus allowing better temporal location of systolic evolution. The severity of the stenosis (given in percentage %) is expressed by the ratio of the diameter reduced by the stenosis and the actual diameter of the artery.

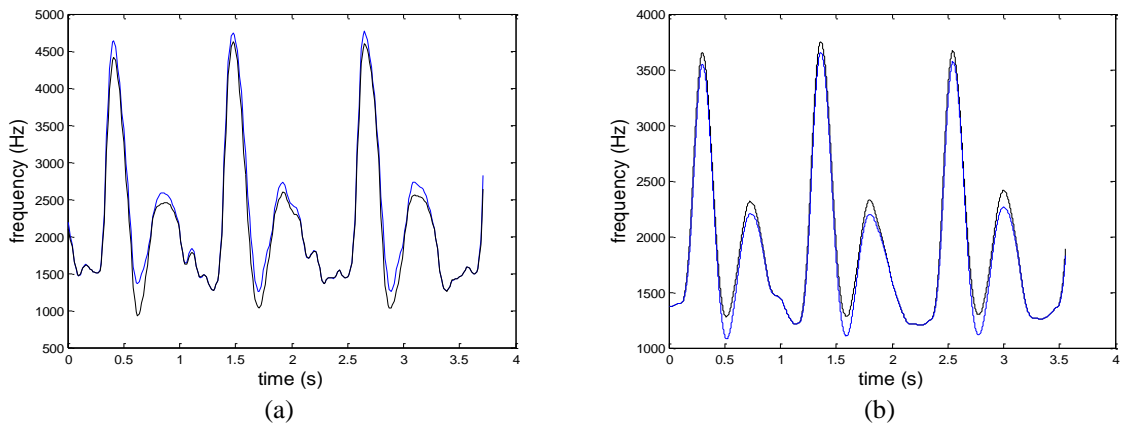


Figure 5. Filtered envelopes  $f_{max}$  and  $f_{mean}$  of the artery femoral snograms: (a) using STFT, (b) using CWT

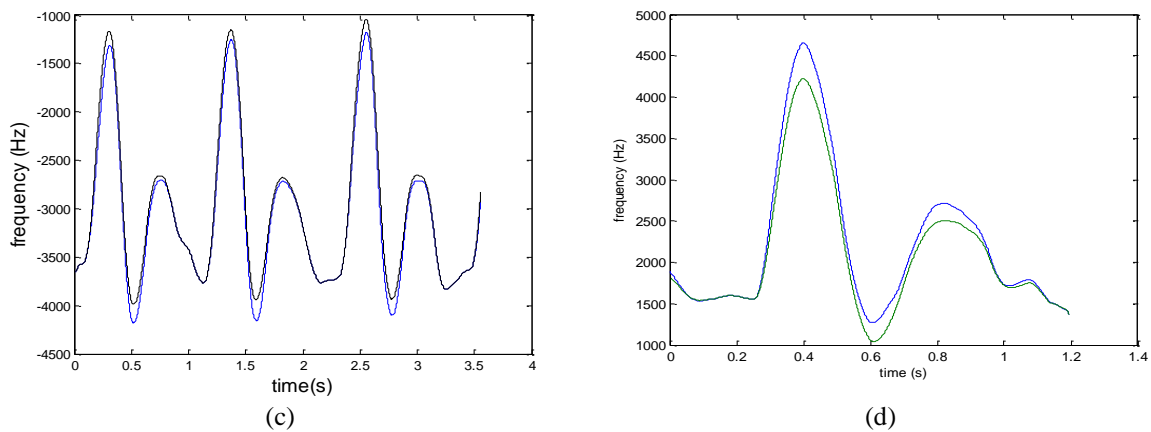


Figure 5. Filtered envelopes  $f_{max}$  and  $f_{mean}$  of the artery femoral snograms: (c) PWT, (d) using S-transform

In Figure 6, the upstream flow of the stenosis is laminar and red blood cells flow with a speed called average speed  $V_{mean}$ . At the stenosis, the red blood cell velocity increases because of arterial constriction, and this maintains a constant flow. In this case, the speed is maximum and is called  $V_{max}$ . Immediately downstream of the stenosis, when the diameter increases, suddenly appears vortexed completely disrupting the flow, this allows the red blood cells to take multiple speeds and in all directions. The average value of these speeds gives the average flow velocity  $V_{mean}$  [15]. Since the speed  $V_{max}$  and  $V_{mean}$  respectively represent the frequencies  $f_{max}$  and  $f_{mean}$  Doppler spectrum, one can express the degree of stenosis according to  $f_{max}$  and  $f_{mean}$  :

$$SBI = \frac{f_{max} - f_{mean}}{f_{max}} \tag{7}$$

According to this equation, the  $SBI$  is then used in our study to calculate the degree of stenosis. Since the  $SBI$  is calculated by the ratio of  $(f_{max} - f_{mean})$  and  $f_{max}$ , it is necessary to use larger values of  $f_{max}$  and  $f_{mean}$  by the fact that using small values may introduce a important errors in the  $SBI$  calculation. For this reason, the  $SBI$  is calculated at the systolic peak, where the flow rates (frequencies of the Doppler spectrum) are maximal. The degree of severity of the stenosis, (given in percentage %) is expressed by the diameter reduced by the stenosis and the real artery diameter [15].

$$Degree\ of\ stenosis = \frac{(A - B)}{A} \times 100\% = \frac{V_{max} - V_{mean}}{V_{max}} \times 100\% \tag{8}$$

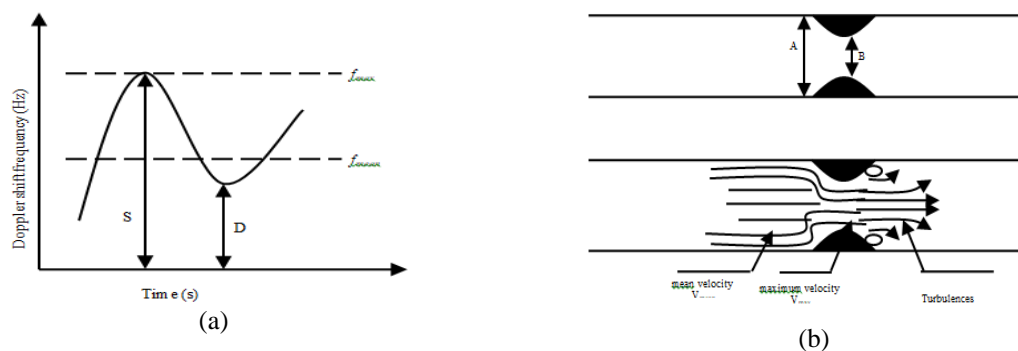


Figure 6. (a) Diagram illustrating the variable involved in the definition of  $SBI$ .  $f_{max}$  is the maximum frequency at peak systole,  $f_{mean}$  is the mean frequency, S is systolic peak, D is end diastolic height. (b) the effect of stenosis on the flow of blood in the arteries

There are two methods of calculation: The first method is to calculate the average parameters ( $f_{max}$  and  $f_{mean}$ ) of each systolic peak, and then to deduce the *SBI*. The second method is to average the *SBI* found from the parameters ( $f_{max}$  and  $f_{mean}$ ) of each systolic peak. The *SBI* different values calculated by various methods applied on *STFT*, *S*-transform, *CWT* and *PWT* modeling transform techniques are illustrated on the Figure 7.

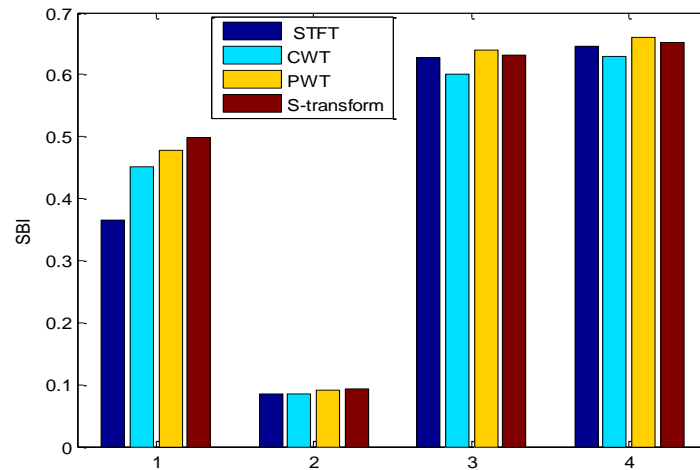


Figure 7. *SBI* magnitude of different Femoral Doppler signals using: *STFT*, *CWT*, *PWT* and *S*-transform methods

These results show that the measurement of spectral broadening quantified by broadening spectral index may be an indication of stenosis severity at the femoral arteries. The *SBI* was calculated from the *STFT*, *S*-transform, *CWT* and *PWT* sonograms. One observed a strong correlation between the value of the *SBI* obtained by *STFT* and that obtained by *S*-transform, *CWT* and *PWT*. The results of this study proved that in spite of the qualitative improvement of the different sonograms; it has no quantitative advantage in employing the *S*-transform, *CWT* and *PWT* compared to the *STFT* for the determination of *SBI* due to its weak variance and with the additional numerical requirements.

## 5. CONCLUSION

The time-frequency analysis methods used in this work aim to show the sonograms of ultrasonic Doppler signals, wavelet-based methods such as *CWT*, *PWT* and *S*-transform and the classical *STFT* method have been compared in terms of their frequency-resolving power and their effects in determining the spectral broadening index in the presence of the stenosis in the ultrasound Doppler signals of the femoral arteries.

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