

## A Novel Displacement-amplifying Compliant Mechanism Implemented on a Modified Capacitive Accelerometer

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### ABSTRACT

The micro-accelerometers are devices used to measure acceleration. They are implemented in applications such as tilt-control in spacecraft, inertial navigation, oil exploration, etc. These applications require high operating frequency and displacement sensitivity. But getting both high parameter values at the same time is difficult, because there are physical relationships, for each one, where the mass is involved. When the mass is reduced, the operating frequency is high, but the displacement sensitivity decreases and vice versa. The implementation of Displacement-amplifying Compliant Mechanism (DaCM) supports to this dependence decreases. In this paper the displacement sensitivity and operation frequency of a Conventional Capacitive Accelerometer are shown (CCA). A Capacitive Accelerometer with Extended Beams (CAEB) is also presented, which improves displacement sensitivity compared with CCA, and finally the implementation of DACM's in the aforementioned devices was also carried out. All analyzed cases were developed considering the in-plane mode. The Matlab code used to calculate displacement sensitivity and operating frequency relationship is given in Appendix A.

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## 1. INTRODUCTION

MEMS accelerometers are miniaturized device sensors used to measure acceleration/deceleration, velocity and vibrations applied on these devices. MEMS accelerometers are extensively used in applications ranging from motion sensing games, automotive, robotics navigators [1]. To extract the acceleration value, the sensor has a movable proof mass connected to a fixed frame through spring structures. When there is an external acceleration, the seismic mass is displaced from its rest position. The magnitude of this displacement is proportional to the magnitude of the acceleration and inversely proportional to the stiffness of the spring structures [2].

The microaccelerometers are devices that stand out due to their high displacement sensitivity and operation frequency [3].

A mechanical amplifier is a device that has the ability to transform the displacement applied to the input in an amplified version of it, obtained at the output of the system. The use of these mechanisms allows obtaining amplification or gain of mechanical signals due to the assembling of simple parts such as rigid beams connected by assembly bolts [4].

The displacement sensitivity is defined as the displacement of the moving mass per unit of gravitational acceleration  $g$  [5]. For the calculation of such sensitivity, it is necessary to start from Newton's second law that establishes that the acceleration of an object is directly proportional to the net force, that acts on it and inversely proportional to its mass. This law is represented by (1).

$$\mathbf{F} = m\mathbf{a} \quad (1)$$

Where  $\mathbf{F}$  is the net force,  $m$  is the mass of the system and  $\mathbf{a}$  is the acceleration. Suspension beams, connected to the mobile mass, are affected by the inertia of the mass, so their length is changed, as a result of the sense of acceleration. This deformation is proportional to the force that causes it. Thus, the relationship between the movement of beams and the force acting directly or indirectly is according to Hooke's law, which is expressed by (2).

$$\mathbf{F} = k\mathbf{x} \quad (2)$$

Where  $\mathbf{x}$  is the displacement and  $k$  is the stiffness constant or constant of the spring, given by (3):

$$k = Et \left( \frac{w_b}{l_b} \right)^3 \quad (3)$$

where  $E$  is the Young's modulus of the material,  $t$ ,  $w_b$  and  $l_b$  are the thickness, width and length of the suspension beams, respectively.

In (4) it shows the equivalence of (1) and (2).

$$m\mathbf{a} = k\mathbf{x} \quad (4)$$

In order to obtain the displacement sensitivity, we proposed to use  $\mathbf{x}$ , obtained from (4):

$$\mathbf{x} = \frac{m\mathbf{a}}{k}$$

The expression for the displacement sensitivity calculation, here developed, considers the effect of acceleration and allows us to get very close values to those obtained through simulation.

Displacement sensitivity will remain constant within a frequency range before any resonance frequency [6]. This happens since, as it is widely known, before the resonance frequency, the device will operate according to the design requirements, under those it was developed. It is recommended to design for high operation frequencies, to avoid low resonance frequencies.

In this work, an improvement on the displacement sensitivity and operating frequency with the implementation of Displacement-amplifying Compliant Mechanism (DaCM), is presented.

To obtain the operation frequency of an accelerometer, (6) is used [7].

$$f = \frac{1}{2\pi} \sqrt{\frac{NEtw_b^3}{ml_b^3}} \quad (6)$$

where  $N$  is the number of suspension beams.

The design and simulations were realized considering the in-plane mode of operation.

## 2. RESEARCH METHOD

### 2.1. Theoretical Analysis and Simulation of Displacement Sensitivity and Operating Frequency of a Capacitive Conventional Accelerometer (CCA).

The accelerometers are devices used to measure acceleration and vibration. These devices convert the acceleration of the gravity or of the movement into an electrical analogical signal, proportional to the force applied to the system [8]. Figure 1 shows the main elements of a CCA.

The purposes of this work are:

- a. To improve the displacement sensitivity and operating frequency.
- b. To validate the theoretical results of accelerometers (CCA and CAEB) responses with Ansys software, which makes modeling by Finite Element Method.

Figure 2 shows the dimensions of the capacitive accelerometer. Table 1 shows the properties of Silicon.

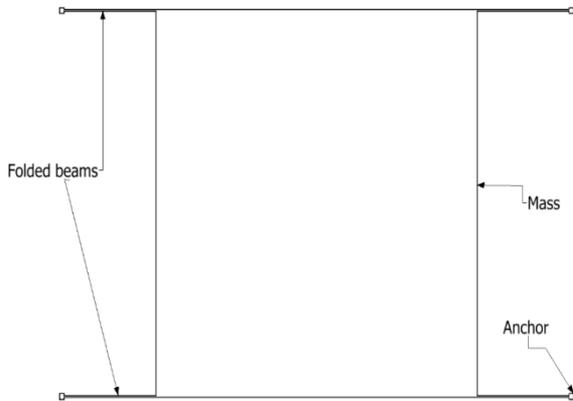


Figure 1. Main elements of the CCA

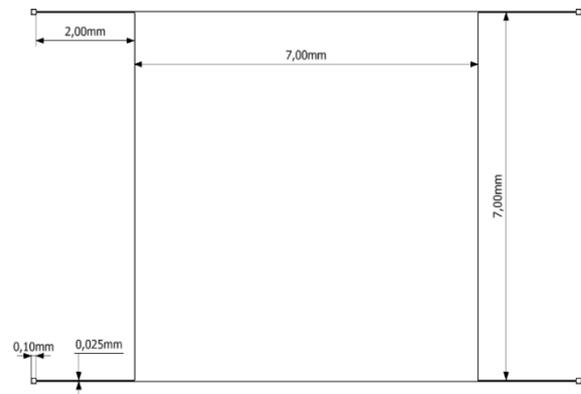


Figure 2. Dimensions of the CCA

Table 1. Properties of Silicon [9]

Property	Value
Density, $\rho$	2330 kg/m <sup>3</sup>
Thermal Expansion Coefficient, $\alpha$	2.6 x 10 <sup>-6</sup> 1/°C
Young's modulus, $E$	131 GPa
Poisson ratio	0.33 Dimensionless

To obtain the displacement sensitivity and operating frequency (5) and (6), were employed respectively. Table 2 shows the calculated values.

Table 2. Values of CCA in the in-plane mode

Parameter	Value
Displacement sensitivity, $x$	1.09 $\mu\text{m/g}$
Operating frequency, $f$	475.35 Hz

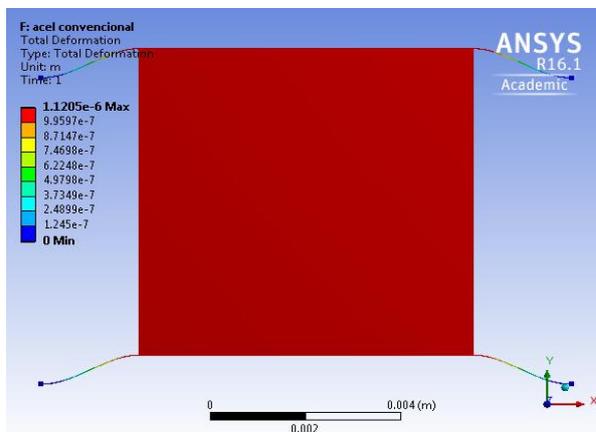


Figure 3. Displacement sensitivity of CCA

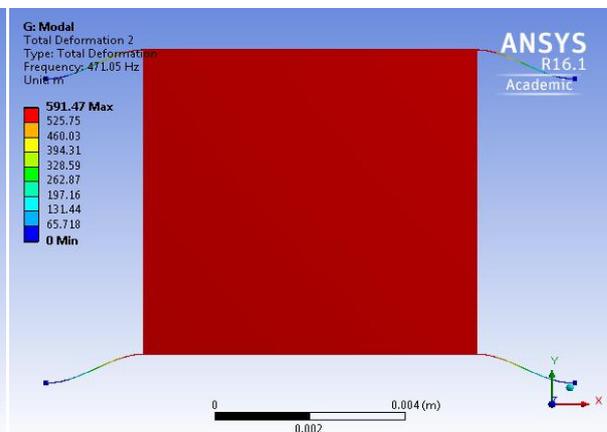


Figure 4. Operating frequency of CCA

In order to validate the calculated values, in simulations, 1 g ( $9.81 \text{ m/s}^2$ ) is applied. Figures 3 and 4 show the displacement sensitivity, as well as the operating frequency that corresponds to the in-plane vibration mode, respectively.

The theoretical result of displacement sensitivity has a value of  $1.09 \text{ }\mu\text{m/g}$ , while the simulation result value is  $1.12 \text{ }\mu\text{m/g}$ . From these results, the displacement sensitivity has an error of 2.7%. For the case of the operating frequency theoretical value is 475.35 Hz and simulated result is 471.05 Hz; the error is 0.9%, with respect to the theoretical results.

**2.2. Theoretical Analysis and Simulation of Displacement Sensitivity and Operating Frequency of a Capacitive Accelerometer with Extended Beams (CAEB).**

Once the calculations have been made for a CCA subjected at 1g, the case of the CAEB will be analysed, with the same acceleration condition. Figure 5 shows the dimensions of this accelerometer, generated from a change in the geometry of the conventional mass, intended to extend the length of the suspension beams, without excessively reduce the value of the mass, as it is required by equation of sensitivity (5). Table 3 shows the theoretical results obtained by (5) and (6).

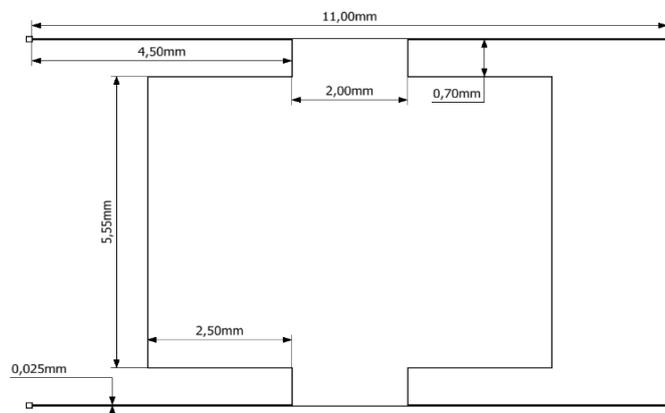


Figure 5. Dimensions of an accelerometer with extended beams

Table 3. Values of CAEB in-plane mode

Parameter	Value
Displacement sensitivity, $x$	$10.74 \text{ }\mu\text{m/g}$
Operating frequency, $f$	152.06 Hz

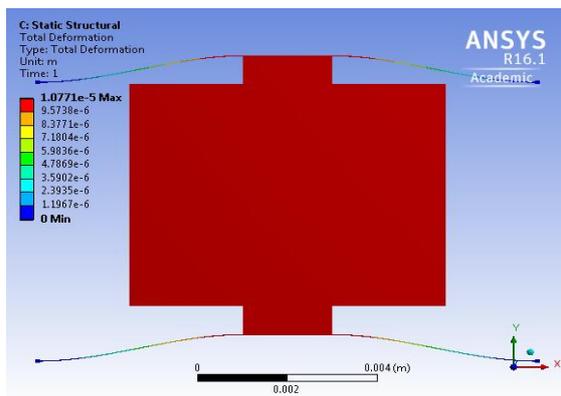


Figure 6. Displacement sensitivity of CAEB

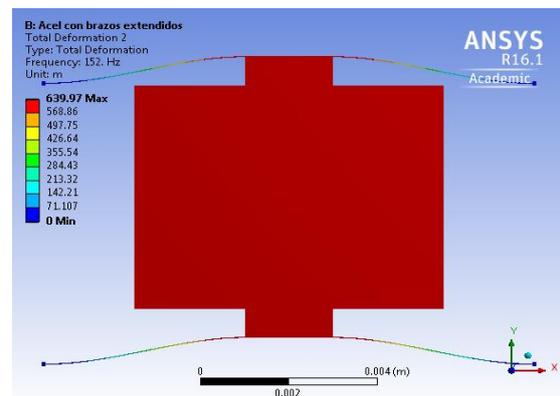


Figure 7. Operating frequency of CAEB

Figures 6 and 7 show the displacement sensitivity and the operating frequency that corresponds to the vibration in-plane mode, respectively. The data were obtained by simulations.

The theoretical result of displacement sensitivity has a value of  $10.74 \mu\text{m/g}$  while the simulation result value is  $10.77 \mu\text{m/g}$ . From these results, the displacement sensitivity has an error of 0.27%. For the case of the operating frequency theoretical value is 475.35 Hz and simulated result is 152 Hz, the error is 0.04%, with respect to the theoretical results.

It can be observed that the displacement sensitivity for the case of CAEB, is approximately 10 times higher than in the case of CCA, but in the case of the operating frequency, the response is 3 times lower. With the implementation of Displacement-amplifying Compliant Mechanism (DaCM), the mass will be reduced to half of the original size. This fact will improve the operating frequency and displacement sensitivity.

### 3. RESULTS AND ANALYSIS

In this section, the simulation results of CCA and CAEB with the implementation of DaCM's are presented.

Figure 8 shows the dimensions of CCA with the DaCM adapted. As it can be observed, the height of this CCA is reduced by a half, compared with the previous CCA. It must be mentioned that the novel DaCM geometry is generated by relevant structural modifications to the one reported in [6]. The simulations were carried out as it was described in sections 2.1 and 2.2, for the structures without DaCM. Figures 9 and 10 show the simulation results of displacement sensitivity and operating frequency for the in-plane mode, respectively.

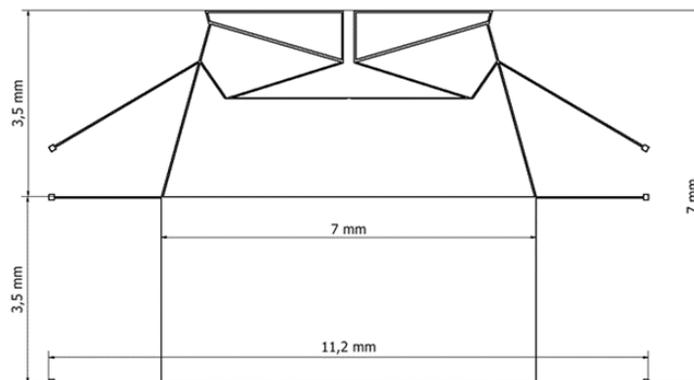


Figure 8. Dimensions of CCA with DaCM

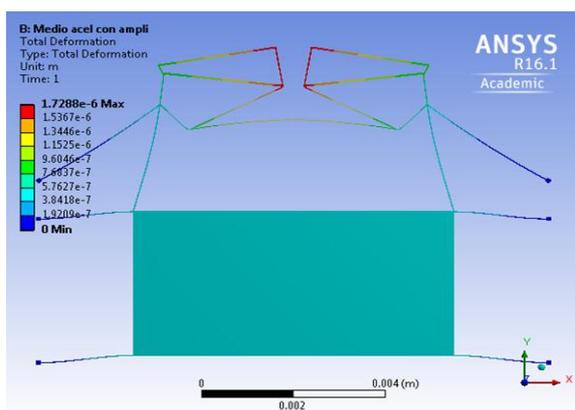


Figure 9. Displacement sensitivity of CCA with DaCM

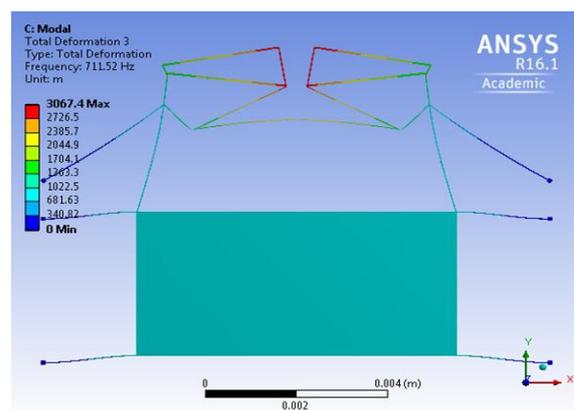


Figure 10. Operating frequency of CCA with DaCM

As it can be noted, the DaCM implementation improved 53% and 51% with respect to the displacement sensitivity and operating frequency, respectively, compared with CCA.

In [6] a similar accelerometer to CCA with diferent DaCM is presented, showing an improvement of 61% in displacement sensitivity and 35% in operating frequency compared to a system provided in [10], similar to the CCA here analyzed. This fact indicates that our improvements in displacement sensitivity are very close to [6]. On the other hand, our percentage of the improvement in the operating frequency is higher. It must be mention that comparing the total size of our system with the used in [6], our proposed device is 36% smaller.

Table 4 shows a comparison of the simulation results between CCA and CCA with DaCM.

**Table 4. Comparison of the simulation results for the in-plane mode**

Displacement Sensitivity of CCA	Displacement sensitivity of CCA with DaCM	Operating frequency of CCA	Operating frequency of CCA with DaCM
1.12 $\mu\text{m/g}$	1.72 $\mu\text{m/g}$	471.11 Hz	711.52 Hz

Figure 11 shows the dimensions of CAEB with DaCM.

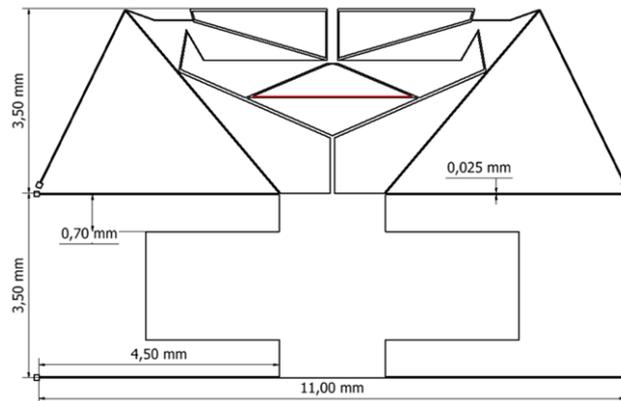


Figure 11. Dimensions of CAEB with DaCM

The displacement sensitivity and operating frequency are shown in Figures 12 and 13, respectively.

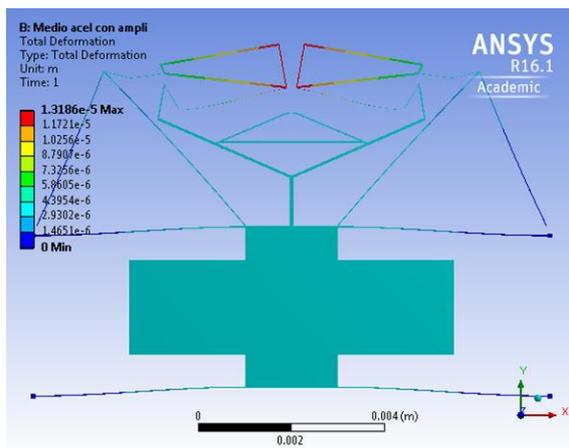


Figure 12. Displacement sensitivity of CAEB with DaCM

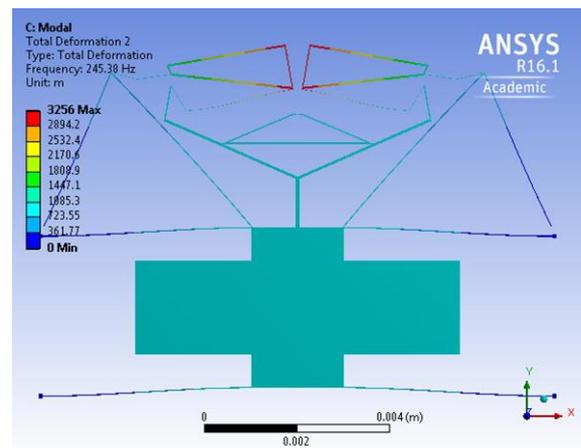


Figure 13. Operating frequency of CAEB with DaCM

A resume of the simulation results between CAEB and CAEB with DaCM is shown in Table 5.

Table 5. Comparison of the simulation results for the in-plane mode.

Displacement sensitivity of CAEB	Displacement sensitivity of CAEB with DaCM	Operating frequency of CAEB	Operating frequency of CAEB with DaCM
10.77 $\mu\text{m/g}$	13.18 $\mu\text{m/g}$	152 Hz	245.38 Hz

As it can be observed, the implementation of DaCM to the CAEB, improve the displacement sensitivity by 22.4%, and the operating frequency by 61.4%, compared with alone CAEB the response. The area of the complete system is of 7 mm x 7 mm.

#### 4. CONCLUSIONS

The error between simulated and calculated results of displacement sensitivity, for the in-plane mode of vibration is very small, 2.7% for CCA and 0.27% for CAEB. For the operating frequency the error is 0.9% and 0.04% for CCA and CAEB, respectively. Therefore, it can be concluded that the equations used to calculate these parameters are appropriated for the geometries of these systems.

The extension on the length of the CAEB's beams improves the displacement sensitivity, 10 times higher than in the case of CCA. But, in the case of the operating frequency, the answer is reduced by 3 times.

The implementation of DaCM to the CCA, improves the displacement sensitivity by 53% and the operating frequency by 51%, compared with CCA response.

The implementation of DaCM to the CAEB, improves the displacement sensitivity by 22.4%, and the operating frequency by 61.4%, compared with CAEB response.

In both cases, the implementation of DaCM, required modification on mass of the accelerometers. The total size of the systems are similar, but the performance is better for the case of the accelerometer with extended beams, especially for the improvement of the operation frequency.

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#### APPENDIX A

Code generated en MATLAB for calculation of displacement sensitivity and operation frequency.

```
clear all
```

```

close all
clc

wb=25*10^-6
E=131*10^9;
h=25*10^-6;
g=9.81
pi=3.141592;

lb=input('Provide the arm's length ');
k1=(wb/lb)^3;
k=E*h*k1;
m=input('Provide the mass of the system');
x=(m*g)/(4*k)
f=(1/(2*pi))*((4*k)/m)^0.5

```

## BIOGRAPHIES OF AUTHORS



**Margarita Tecpoyotl Torres** received the Mathematician degree from the University of Puebla, Mexico, in 1991. From this University, she was also graduated as Electronic Engineer in 1993. She received the M.Sc. and Ph.D. degrees in Electronics from National Institute of Astrophysics, Optics and Electronics, INAOE, México, in 1997 and 1999, respectively. Dr. Tecpoyotl works, since 1999, at CIICAp of Autonomous University of Morelos, Mexico, where she is currently titular professor. She has been visiting research scientist in University of Bristol (2001), UK. She led the Winner team of Boot Camp, UAEM Potential, obtaining a scholarship from TECHBA for the Full Immersion program in Silicon Valley (2014). In 2015 she won the third place in the Royal Academy of Engineering in the Leaders in Innovation Fellowships final pitch session, in UK. She has currently four patents titles, two copyrights, and one trademark. She is co-founder of INNTECVER. She holds the status of National Researcher (SNI), in Mexico since 1999. Her main research interest includes MEMS, Antenna design, Microwave devices, entrepreneurship and innovation; and also, development of educational programs.



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