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Localization of Distributed Wireless Sensor Networks using Two Sage SDP Optimization

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ABSTRACT

In many applications of wireless sensor network (WSN), the location of sensors is a necessity to evaluate the sensed data and it is not energy and cost efficient to equip all sensors with global positioning systems. In WSN localization, some sensors (called anchors) are aware of their location. Then, the distance measurements between sensors and anchors are used to localize the whole network. WSN localization is a non-convex optimization, however, relaxation techniques such as semi-definite programming (SDP) are used to relax the optimization. To solve this problem, all constraints should be considered simultaneously and the solution complexity order is $O\left(n^2\right)$ where n is the number of sensors. The complexity of SDP prevents solving large size problems. Therefore, it is necessary to reduce the problem size in large and distributed WSNs. In this paper, we propose a two stage optimization to reduce the solution time, while provide better accuracy compared with original SDP method. We first select some sensors that have the maximum connection with anchors and perform the localization. Then, we select some of these sensors as virtual anchors. By adding the virtual anchors, we decrease the number of constraints. We propose an algorithm to select virtual anchors so that the total solution complexity and time decrease considerably, while improving the localization accuracy.

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1. INTRODUCTION

Wireless sensor networks (WSN) provide fast, quite cheap and reliable solutions to a large number of industrial, commercial and military applications, ranging from surveillance [1] and tracking to disaster management, robotics and other tasks [2, 3, 4] . Knowing the correct positions of sensor nodes is essential to many applications in next-generation sensor networks. Location awareness refers to devices that can passively or actively determine their location. Location awareness without the active participation of the device is known as non-cooperative localization. Location awareness enables new applications for ubiquitous computing systems and mobile phones.

WSNs present novel trade-offs in system design; on one hand, the low cost of the sensors facilitates massive scale and highly parallel computation. On the other hand, each sensor is likely to have limited power, limited reliability, and only local communication with a modest number of neighbors. These application contexts and potential massive scale make it unrealistic to rely on careful placement or uniform arrangement of sensors. Using globally accessible beacons or to equip sensors with GPS is not feasible considering cost or energy constraints. Therefore, ranging-base localization techniques are introduced for WSNs [5]. The goal of localization is to determine the physical coordinates of a group of sensor nodes. These coordinates can be global, meaning that they are aligned with some externally meaningful system like GPS. They can also be local, meaning that the sensor nodes only have they related position in constellation.

In localization problem using neighboring distance measurements, it is assumed that the accurate positions of some nodes are known, called anchors. Using some partial pair-wise distance measurements and the localization algorithm, it is possible to locate all sensor nodes in the network. The distance measurement between nodes could be performed in different methods. By knowing the transmitted power, using the received signal strength (RSS), and considering the effect of path loss, shadowing, and other losses the distance between two sensors could be found [6]. Both time of arrival (TOA) and time difference of arrival (TDOA) could be used to estimate the distances between sensors. However, unsynchronized sensors with multi-path effect, highly degrades the accuracy of this estimation [7]. The methods such as angle of arrival (AOA) or direction of arrival (DOA) are not applicable in pair-wise distance measurement and may be used for target localization in a direct formulation [7].

In the past, localization problems were solved algebraically and computed by least squares solution to hyperbolic equations called multi-lateration. Nowadays, the optimum solution for sensor network localization is provided using optimization. One of the first proposed convex optimization models for wireless sensor network localization is introduced [8] in which the problem is relaxed to a second order cone programming (SOCP) problem. However, this method needs a large number of anchors on the area boundary to have acceptable performance. In 2004, Biswas [9] proposed a semi-definite programming (SDP) relaxation method for WSN localization. The research shows that the problem of finding unique solution to a noiseless non-linear system describing the common point of intersection of hyper spheres in real Euclidean vector space, can be expressed as a semi-definite program via distance geometry. This method outperforms the SOCP [8] in terms of accuracy and average estimation error. Biswas also developed a solution [10] to deal with noisy corrupted data, and SDP relaxation used to transform the problem into a convex optimization scenario. This method [10] is based on maximum likelihood (ML) estimation. However, ML based methods are usually very time consuming in comparison with other estimation methods. To reduce the solution time, a method called smaller SDP (SSDP) [11] was proposed which further relaxes the original SDP. In this method, a single semi-definite matrix cone is relaxed into a set of small-size semi-definite matrix cones. The non-convexity of the problem covered by introducing a convex objective function [12]. This method is only effective for noise-free measurement cases. The research on improving the performance of SDP is still an open issue. In recent developments, researchers improve the performance of WSNs localization in terms of robustness and accuracy especially in non-line of sight (NLOS) and harsh environments [13].

However, in all SDP solutions, when the size of SDP problem increases, the dimension of matrix cone increases, simultaneously and the amount of unknown variables increases, non-linearly. It is known that the arithmetic operation complexity of the SDP is at least $O\left(n^2\right)$ to obtain an approximate solution. This complexity prevents solving large size problems. Therefore, it would be very beneficial to reduce the SDP problem size. On the other hand, the impact of noise on distance measurement and estimation error is important and this effect varies inversely with problem size.

The motivation of this work is to reduce the required time to solve the optimization problem, while increasing the localization accuracy. In this paper, we propose a two stage localization algorithm based on SDP for WSNs that is applicable in any modification of the original SDP approach. Simulation results demonstrate that the proposed method is very effective and significantly decreases the solution time and improves the average position error.

The remainder of the paper is organized as follows. In section 2, the system model is described and the problem is formulated. Section 3 proposes a solution for the problem. In section 4, simulation results are presented and section 5 concludes the paper.

2. RESEARCH MODEL

We consider a WSN with m anchors (known positions) and n sensors (unknown positions) in a two dimensional (2D) environment. The extension of this localization problem to higher dimensions is straightforward. Some notations used in this paper are as follows. $\mathbf{I}$, $\mathbf{e}$ and $\mathbf{0}$ denote the identity matrix, the vector of all ones and the vector of all zeros, respectively. The 2-norm of a vector $\mathbf{x}$ is denoted by $\|\mathbf{x}\|$. A positive semi-definite matrix $\mathbf{X}$ is represented by $\mathbf{X} \succeq \mathbf{0}$. The position of anchor nodes are presented by vector $\mathbf{V}_a = \{a_1, a_2, ..., a_m\}$ and the Euclidean distance between x_j and x_i is denoted as d_{ij} and between a_k and x_j is denoted by d_{jk} as follows:

$$d_{ij} = \|x_i - x_j\| \quad \text{and} \quad d_{jk} = \|x_j - x_k\| \tag{1}$$

3. PROPOSED METHOD

The optimization problem can be expressed as follows:

$$
\begin{aligned}
&\text{Find} \quad \mathbf{X} \in R^{2\times n} \\
&\text{S.t.} \quad Y_{ii} - 2Y_{ij} + Y_{jj} = \overline{d_{ij}}^2, \forall (i,j) \in N_s \\
&Y_{jj} - 2X_j^T a_k + {a_k}^2 = \overline{d_{jk}}^2, \forall (j,k) \in N_a \\
&N_s = \{\ (i,j)|x_i - x_j < r\ \}, \quad N_a = \{\ (j,k)|x_j - x_k < r\ \} \\
&\mathbf{Y} = \mathbf{X}^T\mathbf{X}
\end{aligned}
\tag{2}
$$

where r is the communication range, and $\mathbf{X} = [x_1, \ldots, x_n]$, $\overline{d_{ij}}$ and $\overline{d_{jk}}$ are the noisy range measurements. The problem in (2) is non-convex and may be relaxed using SDP method as presented in (3).

$$
\mathbf{Y} \succeq \mathbf{X}^T\mathbf{X} \rightarrow \mathbf{Z} \succeq 0; \mathbf{Z} = \begin{pmatrix} \mathbf{I}_2 & \mathbf{X}^T \\ \mathbf{X} & \mathbf{Y} \end{pmatrix} \tag{3}
$$

The idea behind the proposed method is simple. The nearest sensors to the anchors with most connections to the anchors have the best measurements. We first localize some of sensors with best measurements, and use these sensors with known location as virtual anchors. In the second stage, the real and virtual anchors are both used to localize the rest of sensors in the network. The proposed method is explained in Table 1.

Table 1. Proposed Algorithm

Step Number	Operation
Initialization	Anchors with known and sensors with unknown location
1	Calculate all the sensor-sensor and sensor-anchor distance using noisy measurements $\overline{d_{ij}}$, $\overline{d_{jk}}$
2	Select the sensors that are in communication range of at least two anchors
3	Estimate the location of selected sensors
4	Choose the sensors with most distance to existing anchors as virtual anchor
5	Choose the sensors that are not selected in 2
6	Calculate all the measurements $\overline{d_{ij}}$ and $\overline{d_{jk}}$ with new virtual anchors
7	Estimate the location of remained sensors

4. RESULT AND ANALYSIS

Computer simulations are used to evaluate the performance of the proposed algorithm. We consider a network with 7 anchors and 100 sensors deployed randomly in a normalized area. Simulation is performed using CVX toolbox in MATLAB software [14]. The average estimation error is calculated as follows:

$$
\text{Average Error} = \frac{1}{n}.\sum_{j=1}^{n} \parallel \overline{x_j} - a_j \parallel \tag{4}
$$

We perform the simulation for different communication ranges. We assume that the distance measurements are corrupted by noise denoted by noise factor as calculated using (5) and (6). We compare the results with original SDP used to solve the WSN localization [13]. Simulation parameters are summarized in table 2.

$$
\overline{d_{ij}} = d_{ij}.\left(1 + randn\left(1\right) \times \text{noise factor}\right) \tag{5}
$$

$$
\overline{d_{jk}} = d_{jk}.\left(1 + randn\left(1\right) \times \text{noise factor}\right) \tag{6}
$$

The simulation environment, the location of anchors, the exact and estimated locations of sensors using original SDP and proposed two-stage SDP are illustrated in figure 1 and figure 2, respectively. In figure 1 and figure 2, the communication range and noise factor are set to 0.3 and 0.15, respectively. The average execution time to solve the optimization problem, highly depends to hardware configuration. However, the relative and normalized execution time could be used as a criterion to compare the computational complexity of the proposed algorithm. In this paper, we have averaged the total execution time over 50 networks with 1000 realizations. The proposed two stage SDP

Table 2. Simulation parameters used for localization

Variable	value
Simulation Area	Normalized 1×1
Number of sensors	100
Number of Anchors	7
Communication range	$[\ 0.3 \quad 0.35 \quad 0.4 \quad 0.45\ ]$
Noise factor	$[\ 0.05 \quad 0.1 \quad 0.15 \quad 0.2 \quad 0.25\ ]$

outperforms the original SDP [10, 13], by 12% improvement in execution time. One the other hand, the average localization error shows 30% improvement on average.

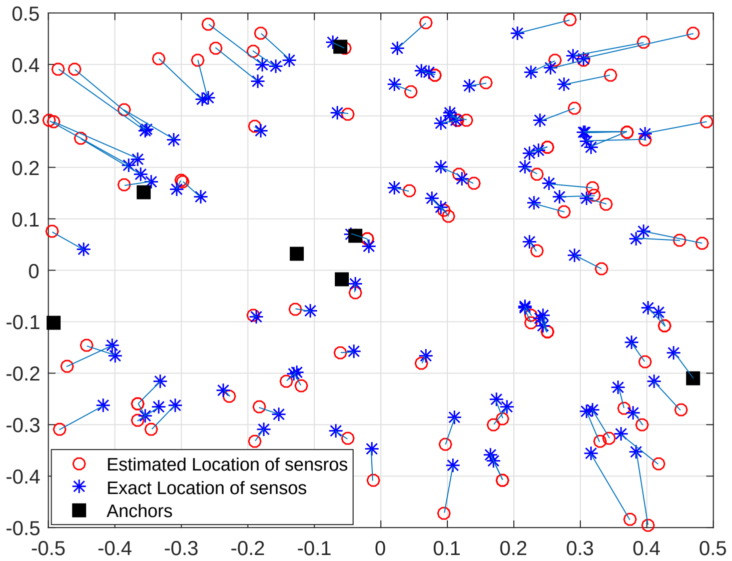

Figure 1. Simulation area, the exact and estimated location of sensors using original one-step localization

In figure 3, the effect of noise factor in execution time is evaluated. This simulation demonstrates that the increase in noise factor, highlights the difference between original SDP and the proposed algorithm in terms of execution time. In all noise factors, the solution time of the proposed algorithm is much less than the original SDP.

The effect of communication range on average localization error is presented in figure 4. As this simulation shows, increasing the communication range, decreases the localization error. The proposed two stage algorithm has better localization error in comparison with original SDP in all communication ranges. Figure 5, illustrates the effect of different noise factors on average localization error. This figure demonstrates that in all noise figures and corrupted measurements, our proposed two stage localization has reduced the average localization error in compared with one stage original SDP localization [10, 13].

5. CONCLUSION

In this paper we studied the localization problem in wireless sensor networks. The localization could be interpreted as a non-convex optimization problem. Semi-definite programming have widely been used to relax and solve the optimization problem. However, the computational complexity increases non-linearly with increasing the number of sensors. We proposed a two stage SDP solution to solve the localization problem. We first, performed the localization with some selected sensors and used some of localized sensors as virtual anchors. Simulation results confirm that the proposed algorithm improves the localization accuracy 30% on average, while decreases the average execution time by 12%.

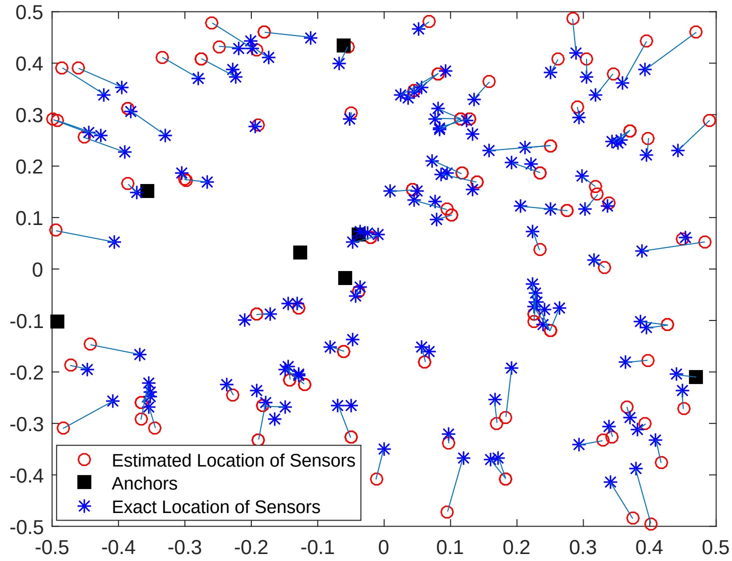

Figure 2. Simulation area, the exact and estimated location of sensors using proposed two-step localization

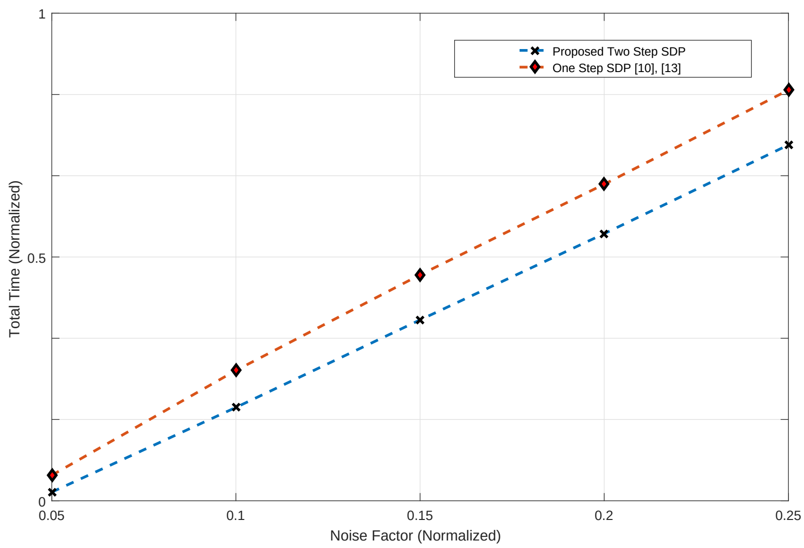

Figure 3. The effect of noise factor on the execution time of localization algorithms

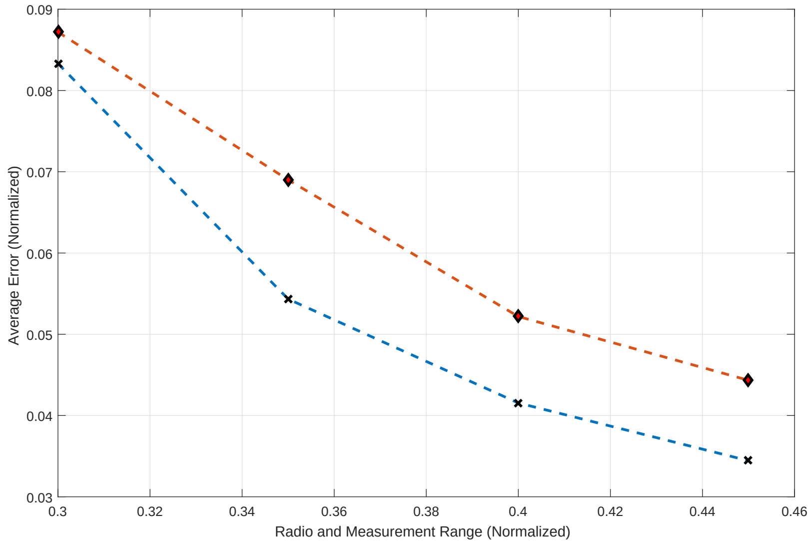

Figure 4. The effect of radio range on average localization error

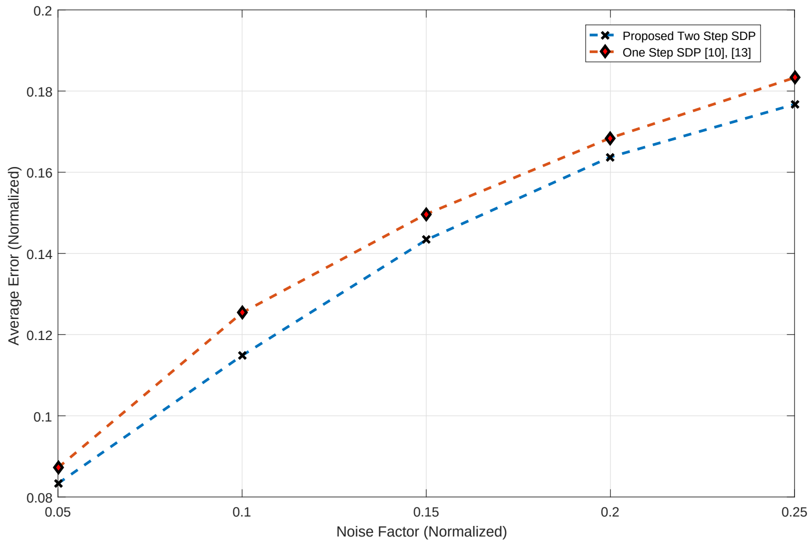

Figure 5. The effect of noise factor on average localization error

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Reza Shahbazian received the B.Sc. degree in Electrical Engineering from the Iran University of Science and Technology (IUST), Tehran, Iran, in 2008 and the M.Sc. degree (with honors) in Telecommunications Engineering from the IUST, in 2011. In past, he has been a member of the Wireless Communications Laboratory in the School of Electrical Engineering at the IUST. He is currently a member of Cognitive Radio Laboratory at the Shahid Beheshti University (SBU), Tehran, Iran, to work under supervision of Dr. Ghorashi. He is affiliated with IEEE as student member. In journal of supercomputing, sensor review, wireless personal communications, and other scientific publications, he has served as invited reviewer. Further info on his homepage: http://faculties.sbu.ac.ir/shahbazian

Seyed Ali Ghorashi received his B.Sc. and M.Sc. degrees in Electrical Eng. from the University of Tehran, Iran, in 1992 and 1995, respectively. Then, he joined SANA Pro Inc., where he worked on modelling and simulation of OFDM based wireless LAN systems and interference cancellation methods in WCDMA systems. Since 2000, he worked as a research associate at Kings College London on capacity enhancement methods in multi-layer W-CDMA systems sponsored by Mobile VCE. In 2003 He received his PhD at Kings College and since then he worked at Kings College as a research fellow. In 2006 he joined Samsung Electronics (UK) Ltd as a senior researcher and now he serves as an associate professor at Department of Telecommunications, Faculty of Electrical Engineering, Shahid Beheshti University at Tehran, Iran, working on wireless communications.