

## Comparison of Voltage Vector Control Based on Duty Cycle Analysis in Three Phase Four Leg System of Active Filter

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### ABSTRACT

Comparison of voltage vector control in various forms of tetrahedron that result from switching combination on three-phase four-leg system of active filter is presented especially asymmetric tetrahedron shape which is a projection pqr-coordinate into  $\alpha\beta$ -coordinate. Parameter tetrahedrons such as modulation boundary-line, reference vector, switching duration time and duty cycle are described. Duty cycle analysis conducted on the Shen's model, the Zhang's model, the Perales's model and asymmetric's model are presented. The characteristic results showed that switching combination of each IGBT conductor especially its review on the neutral wire. Asymmetric tetrahedron can be proposed as control technique in three-phase four-leg system of active filter.

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## 1. INTRODUCTION

Three-phase system active filter topology has evolved from three-phase three-wire becomes three-phase four-wire and then three-phase four-leg. In three-phase four-wire system of active filter, the fourth-wire forms a neutral wire which is connected to middle place in couples of split capacitors. Meanwhile, in three-phase four-leg system the fourth-wire as a neutral wire is connected to middle place in a pair of IGBT conductor. Steps in current compensation control on three-phase four-wire system resemble as on three-phase three-wire system. Control of the compensation current is done by adjusting voltage-vector either on three-phase four-wire system or three-phase four-leg system active filter. Voltage-vector is three-dimensional space-vector where the vector resulting of on-off switching combination signal from pair of IGBT conductor. In spite of different topology, both active filter have same technique. Switching combination on three-dimensional space-vector modulation (SVM) has produce sixteen pieces of voltage-vector [1],[2]. Modulation parameters such as reference vector, duty cycle, pulse pattern, total harmonics distortion (THD) and distortion factor (DF) can be determined by sixteenth of voltage-vector in three-dimensional of tetrahedron shape and then voltage-vector control on three-phase active filter is regulated by all of them.

In the Shen's Model [1], resulting of sixteenth switching combinations in three-phase four-wire active filter can be described by the invariant Clark's transformation. Three dimensional position of voltage-vector located in  $\alpha\beta$ -coordinates consisted of two point of zero switching vector (ZSV) position at the original coordinate and fourteen points of non-zero switching vectors (NZSV) at another. The fourteenth points of NZSV can be classified into six-prisms consists the top-bottom of tetrahedrons. It shows fourteen-possible positions voltage-vector as references in the boundary of each tetrahedron. Should be noted, the

coordinate transformation is obtained from the invariant Clark's transformation which produce maximum modulation boundary-line on  $\alpha\beta 0$ -coordinate are 0.82 Vdc.

In the Zhang's model [2], coordinate transformation based on the non-invariant Clark's transformation is used to produce sixteenth switching combination positions at  $\alpha\beta\gamma$ -coordinates in three-phase four-leg power converter (active filter). It describes in one sector which has four tetrahedron of each voltage-vector occupies three-dimensional space-vector so in overall they have twenty-four tetrahedrons. Reference-vectors of tetrahedrons are combination of three pieces of voltage-vector nearest neighbors. It can be seen from the non-invariant Clark's transformation that maximum modulation boundary-line according to the Zhang's model are equal 0.67 Vdc.

Without doing Clark's transformation, in the Perales's model [3] switching combination of on-off pair of IGBT conductor in three-phase four-wire of active filter produces sixteenth voltage-vector as reference-vector like the Shen's model and the Zhang's model. Voltage-vectors in abc-coordinates of the Perales's model which is directly mapped in three-dimensional space-vector produce a dodecahedron shape with a side length as maximum modulation boundary-line are 1 Vdc. From sixteenth combinations of voltage-vectors it can be made twenty-four region pointers (tetrahedrons) as reference-vectors which make description three pieces of voltage-vector nearest neighbors.

This paper aims to create a new model of voltage-vector based on the Kim-Akagi's mapping matrices model (pqr-power theory) doing for compensation current control space vector modulation (SVM), it can be used to harmonics elimination [4],[5]. Current Compensation control in three-phase four-wire system of active filter is explained by the pqr-power theory. Reinterpretation of pqr-coordinate from mapping matrices model by the Euler angle rotation method is turned out, and it produces in a different way when it is compared from its original theory. In the Kim-Akagi's mapping matrices model, pqr-coordinate is identical to dqn-coordinate when the n-axis coincides on the 0-axis, whereas in the Euler angle rotation method pqr-coordinate is generated from twice rotation [6]. Projection of pqr-coordinate into  $\alpha\beta 0$ -coordinate produces twenty-four of asymmetric voltage-vector in three-dimensional space-vector. The maximum modulation boundary-line according to the Euler angle rotation method is equal 0.5 Vdc.

## 2. MODULATION BOUNDARY-LINE IN THREE-PHASE SYSTEM ACTIVE FILTER

Intersection of a reference signal from sources which is derived any signal that does not have periodic signal but has any amplitude there will be producing asymmetric modulation. Square wave pulse width modulation (PWM) signal which has a result of this intersection in the interval  $(0-\pi / 2)$ ,  $(\pi / 2-\pi)$ ,  $(\pi-3\pi / 2)$ , and  $(3\pi / 2-2\pi)$ , it also has random and different from each other of duty cycle. Any signal can be either total harmonics or disorder signal. Asymmetric model in this modulation can be resulted from projection of the pqr-coordinate of voltage-vector of the Euler angle rotation method into  $\alpha\beta 0$ -coordinate. Fourier analysis can not be performed because of the resulting square signal is not periodically. To keep in separate formulations, it is needed to perform a duty cycle analysis. Carrier-based and hysteresis modulation is different from asymmetric modulation.

The following step of duty cycle analysis in space-vector modulation (SVM) has done through measurement such as following [7]:

- Determine the switching combination,
- Identify the voltage-vector positions,
- Identify the reference vector,
- Determine the switching time duration,
- Calculate duty cycle,
- Determine the pulse pattern,

Three-phase three-wire system active filter has a capacitor in DC-link voltage that is equal 1 Vdc. The maximum modulation boundary-limit permissible is equal to DC-link voltage. Reference vector and duty cycle are determined by drawing a perpendicular line as the resultant between two pieces of voltage vector in line of the circle. Pythagoras formula can be determined the reference vector length is  $\sqrt{3} / 2 = 0,87$  Vdc. It can be concluded that value of reference vector between 0.87 Vdc -1 Vdc is modulation boundary-line which is a forbidden zone within the limits of the Quang's model [8]. The Perales's modulation boundary-line description is like the Quang's model.

Three-phase four-wire system and four-leg system of active filter is developed by Shen [1] and Zhang [2] which is obtained voltage vector that has a value smaller than allowable modulation boundary-line of three-phase three-wire system that is equal 0.82 Vdc and 0.67 Vdc like shown in Figure 1.

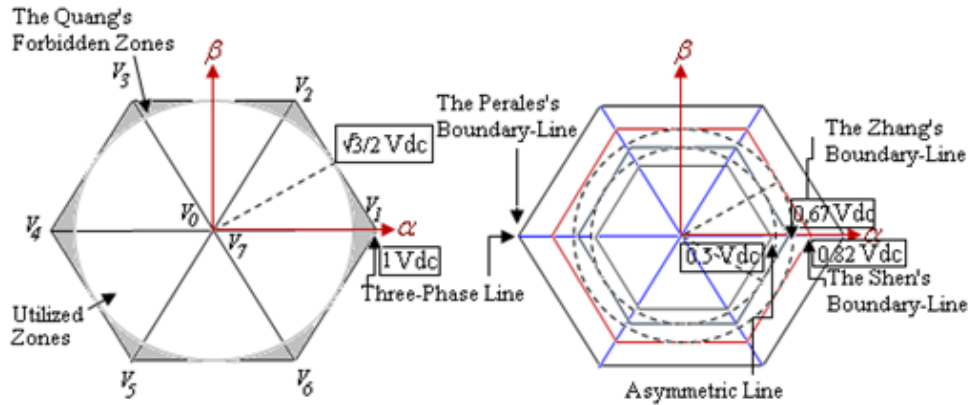


Figure 1. Modulation boundary-line in three-phase systems of active filter

**2.1. Coordinate Transformation Model**

Using three-phase four-leg system of active filter, Shen [1] explained the invariant Clark's transformation coordinate system in which variables are converted by following equation:

$$\begin{bmatrix} v_{\alpha} \\ v_{\beta} \\ v_0 \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \tag{1}$$

The sixteen possible combinations can be divided into two vector switching which has a length is equal 0.82 Vdc. In data processing, control vector-space is necessary for a conventional three-phase converter requires the reference vector.

Using the same formulation based on coordinate transformation (the non-invariant Clark's transformation), if the table transformation is made into abc-coordinate to  $\alpha\beta 0$ -coordinate like Shen [1], than a combination of switching from each voltage vector produce sixteen possible.

$$\begin{bmatrix} v_{\alpha} \\ v_{\beta} \\ v_0 \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \tag{2}$$

Zhang [2] also produce 16 switching combination on-off pair of IGBT conductor- a, b, c and neutral.

**2.2. Coordinate Rotation Method**

Rotation method of Euler angle can be done with a continuous configuration either two or more the Cartesian's coordinate. Rotation R ( $\theta_1, \theta_2, \theta_3$ ) is a relation which is done to move the stationary reference frame towards the rotating reference frame.

In mathematical notation, the Euler angles rotation method can be determined as follows;

$$\begin{aligned} R(\theta_1, \theta_2, \theta_3) &= R_x(\theta_3) R_y(\theta_2) R_z(\theta_1) \\ &= \begin{bmatrix} \cos \theta_3 & \sin \theta_3 & 0 \\ -\sin \theta_3 & \cos \theta_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \theta_2 & 0 & \sin \theta_2 \\ 0 & 1 & 0 \\ -\sin \theta_2 & 0 & \cos \theta_2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_1 & \sin \theta_1 \\ 0 & -\sin \theta_1 & \cos \theta_1 \end{bmatrix} \end{aligned} \tag{3}$$

Where in  $0 < \theta_1, \theta_2 < 2\pi$  dan  $0 < \theta_3 < \pi$ .

Principle of the Euler angle rotation method can be used to reinterpretation of the Kim-Akagi's mapping matrices model in solving problems which is related to pqr-coordinate in [4],[5]. Multiplication of twice rotation in  $\theta_1, \theta_2$  angle is the same in producing formulation as the Kim-Akagi's mapping matrices model as follows:

$$\begin{aligned}
 \begin{bmatrix} X_p \\ X_q \\ X_r \end{bmatrix} &= \begin{bmatrix} \cos \theta_2 \cos \theta_1 & \cos \theta_2 \sin \theta_1 & \sin \theta_2 \\ -\sin \theta_1 & \cos \theta_1 & 0 \\ -\sin \theta_2 \cos \theta_1 & -\sin \theta_2 \sin \theta_1 & \cos \theta_2 \end{bmatrix} \begin{bmatrix} X_g \\ X_f \\ X_e \end{bmatrix} \\
 &= \sqrt{\frac{2}{3}} \begin{bmatrix} \cos \theta_2 \cos \theta_1 & \cos \theta_2 \sin \theta_1 & \sin \theta_2 \\ -\sin \theta_1 & \cos \theta_1 & 0 \\ -\sin \theta_2 \cos \theta_1 & -\sin \theta_2 \sin \theta_1 & \cos \theta_2 \end{bmatrix} \begin{bmatrix} 1 & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} X_a \\ X_b \\ X_c \end{bmatrix} \tag{4}
 \end{aligned}$$

Here in  $X \equiv i, v$  (current or voltage). Equation 4 can be described in steps such as Figure 2 and Figure 3 for each of the Euler angle rotation.

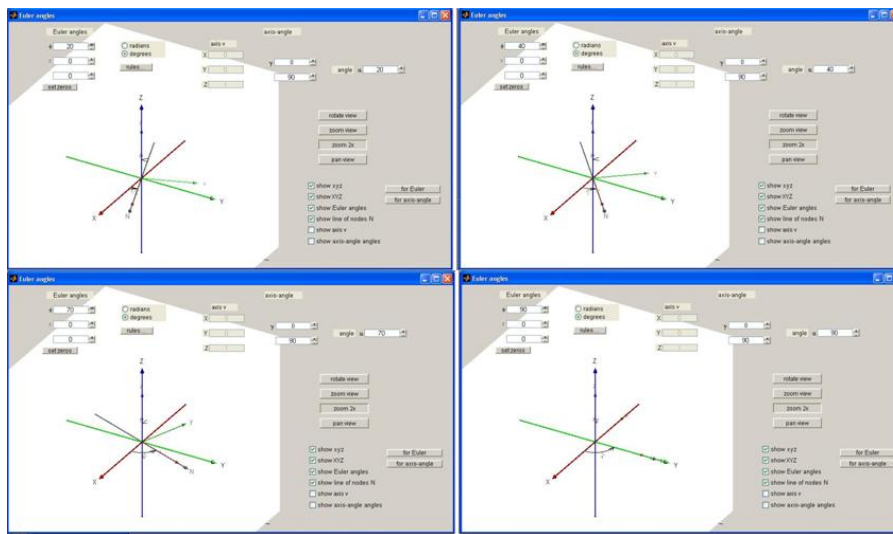


Figure 2. Change a first of angle producing rotation in the horizontal plane

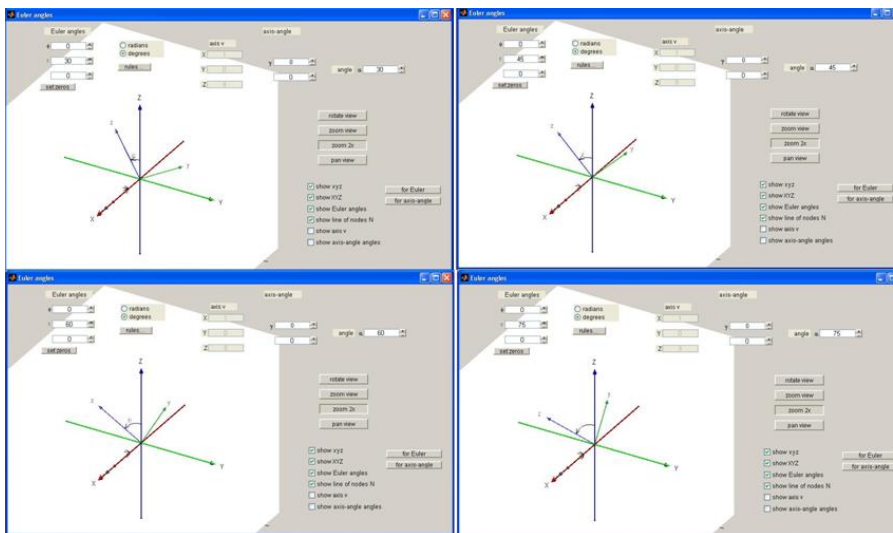


Figure 3. Change the second of angle producing rotation in the vertical plane

### 2.3. Asymmetric Tetrahedron Shape

Rotation method of two or three Euler angle becomes more like a spiral shape than a cylindrical shape. A spiral shape will be easily analyzed when it is considered as a skewed vector. Three-dimensional

space vector in three-phase four-leg system of active filter for this is a cylindrical coordinate (shape) with six prism such as Figure 4a, while based on Euler angle rotation method this result forms a stack of two pieces hexagonal as skewed vector such as Figure 4b, 4c. Rotation method of two angles Euler produces pqr-coordinate if it is done the projection in a horizontal plane (sliding side) obtaining new coordinates dqr-coordinate such as figure-4b and projection in a vertical plane (sliding oblique) obtaining  $\alpha\beta$ -coordinate such as Figure-4c.

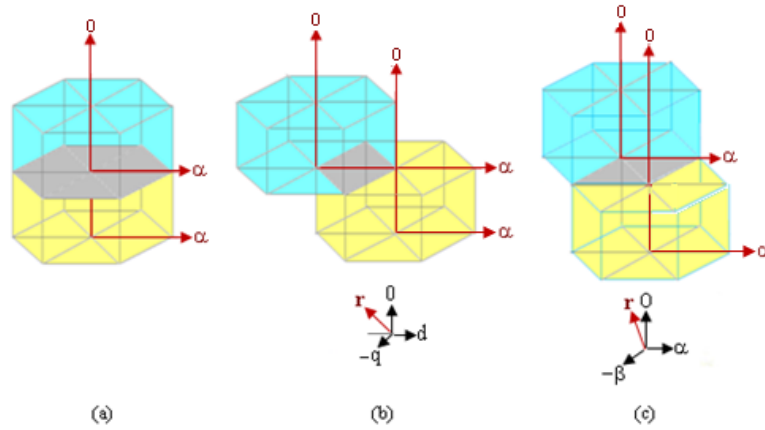


Figure 4. pqr-coordinate projection is produce; (a) a cylindrical coordinate; (b) dqr-coordinate; (c)  $\alpha\beta$ -coordinate

Two hexagonal stacked with a shift at a central point where the hexagonal of the top and the bottom are not centralized, it can be considered as a skewed vector. The center part of the composition of the two hexagonal shapes elongated hexagonal with two-piece center point, namely state-(1,1,1) and state-(0,0,0) which does not coincide with each other. Overall, three-dimensional space vector can be parsed into twelve-pairs of asymmetric tetrahedron. Asymmetric tetrahedron shape between each other of tetrahedron indicates harmonics occurrence on voltage vector in which has got 0.5 Vdc. Hexagonal piles which forms an asymmetric tetrahedron pair when are mapped in a two-dimensional horizontal plane (viewed from the top) can be obtained coordinate points which is the position voltage vector in  $\alpha\beta$ -coordinates such as Figure 5a and from vertical plane (viewed from the side) as shown following in Figure 5b.

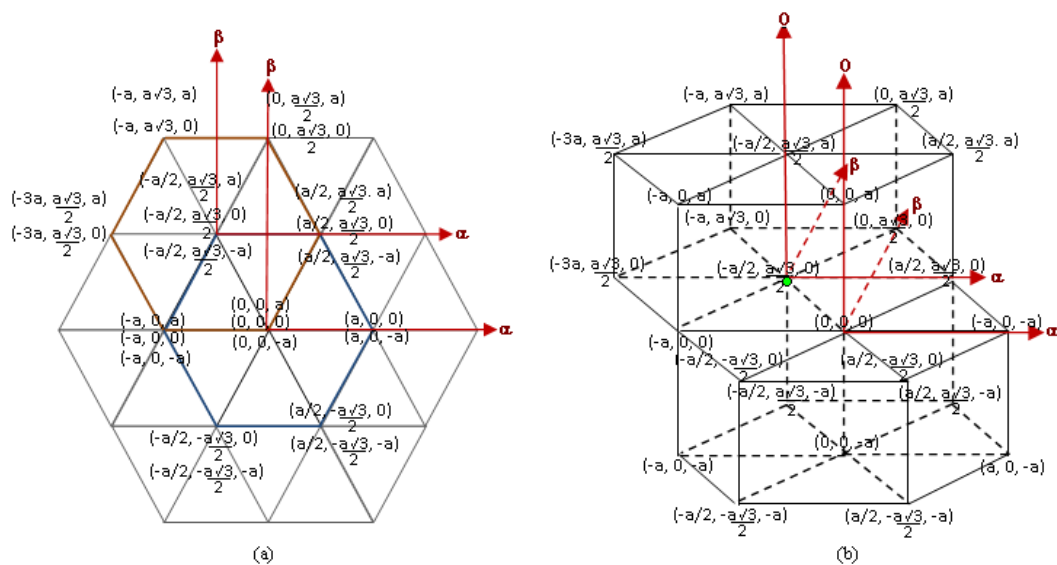


Figure 5. Asymmetric tetrahedron in  $\alpha\beta$ 0-coordinate; (a) View from the top (horizontal); (b) View from the side (vertical)

Each of these coordinate can be mapped in four-pair combination of IGBT and generates a voltage vector in space of tetrahedron. Sixteenth combination pair of IGBT conductor can be described in twenty-four coordinates of a pile of regularly hexagonal, which means they have twenty-four or twelve-pairs of asymmetric tetrahedron. The twenty-fourth coordinates can be decomposed into each of tetrahedron with separated center point, where are they both in state-(1,1,1,1) and in state-(0,0,0,0). Based on figure-5b, each sided of prism with a length of “a” is equal 0.5 Vdc for upside part of asymmetric tetrahedron and -0.5 Vdc for downside part of asymmetric tetrahedron. Table-1 will be given position of twenty-fourth voltage vector as a result of a switching combination pair of IGBT conductor on three-phase system active filter. Twenty-fourth voltage vector can be described as an asymmetrical tetrahedron pair. A skewed vector actually resembles the Perales’s model when determining voltage vector used to abc-coordinate. In this case, the difference of state-(1,1,1,1) and state-(0,0,0,0) does not coincide with each other at one position. Perales [3] get twenty-four voltage vectors without Clarke transformation. In space vector modulation (SVM) Clark’s transformation is used to obtain voltage vector.

Table 1. Voltage vector on the skewed vector model (a=0.5 Vdc)

Voltage Vector	V□□a (Vdc)	V□/a (Vdc)	V□/a (Vdc)	Voltage Vector	V□/a (Vdc)	V□/a (Vdc)	V□/a (Vdc)
V <sub>0</sub>	0	0	0	V <sub>12</sub>	0	$-\sqrt{3}/4$	0
V <sub>1</sub>	0	0	-1/2	V <sub>13</sub>	1/4	$-\sqrt{3}/4$	0
V <sub>2</sub>	0	0	1/2	V <sub>14</sub>	-1/4	$-\sqrt{3}/4$	1/2
V <sub>3</sub>	-1/4	$-\sqrt{3}/4$	-1/2	V <sub>15</sub>	-1/4	$-\sqrt{3}/4$	0
V <sub>4</sub>	1/4	$-\sqrt{3}/4$	1/2	V <sub>16</sub>	-1/2	0	1/2
V <sub>5</sub>	1/2	0	-1/2	V <sub>17</sub>	-3/4	$-\sqrt{3}/4$	1/2
V <sub>6</sub>	1/4	$\sqrt{3}/4$	-1/2	V <sub>18</sub>	-1/2	$-\sqrt{3}/4$	1/2
V <sub>7</sub>	-1/4	$\sqrt{3}/4$	-1/2	V <sub>19</sub>	0	$-\sqrt{3}/4$	1/2
V <sub>8</sub>	-1/2	0	-1/2	V <sub>20</sub>	1/4	$-\sqrt{3}/4$	1/2
V <sub>9</sub>	-1/2	0	0	V <sub>21</sub>	1/2	0	0
V <sub>10</sub>	-3/4	$\sqrt{3}/4$	0	V <sub>22</sub>	1/4	$-\sqrt{3}/4$	0
V <sub>11</sub>	-1/2	$-\sqrt{3}/2$	0	V <sub>23</sub>	-1/4	$-\sqrt{3}/4$	0

Reference vector is sum of three pieces of voltage vector nearest neighbor and its representations compares of magnitude (Vdc) from Va, Vb, and Vc. Reference vector can be obtained from each of tetrahedron. After the determination of reference vector, next steps are determined switching time duration and calculated duty cycle. In figure-6, reference vector (Vref) has described in a spherical coordinate and this reduction aims to determine switching time duration and duty cycle.

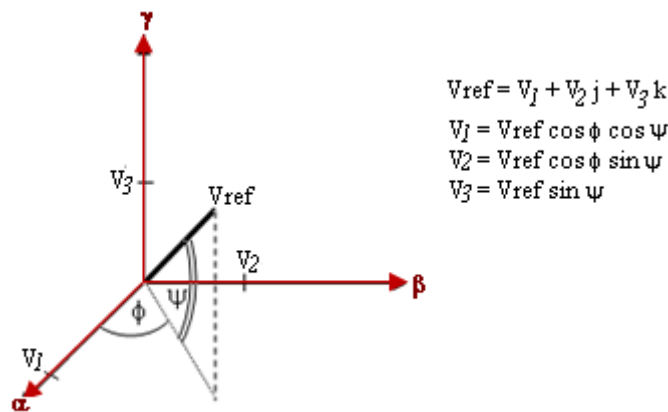


Figure 6. Reference vector (Vref) description in a spherical coordinate

To get the line to neutral voltage (V<sub>an</sub>, V<sub>bn</sub>, V<sub>cn</sub>, V<sub>nn</sub>), circuit of a,b,c-conductor and neutral conductor on the active filter based on a spherical coordinate can be obtained as follows [9]:

$$V_{an} = V_{ref} \frac{(\cos\phi\cos\psi + \sin\psi)}{2} \quad (5a)$$

$$V_{bn} = V_{ref} \frac{(-2\cos\phi\cos\psi + \sqrt{3}\sin\phi\cos\psi + \sin\psi)}{2} \quad (5b)$$

$$V_{cn} = V_{ref} \frac{(-2\cos\phi\cos\psi - \sqrt{3}\sin\phi\cos\psi + \sin\psi)}{2} \quad (5c)$$

$$V_{zn} = -V_{ref} \frac{(\cos\phi\cos\psi + \sin\psi)}{2} \quad (5d)$$

So the switching duration time of each a,b,c-conductor and neutral conductor are obtained from the following equations:

$$T_a/T_s = \frac{1}{2} + \frac{V_{an}}{V_{dc}} = \frac{1}{2} + \frac{M}{2}(\cos\phi\cos\psi + \sin\psi) \quad (6a)$$

$$T_b/T_s = \frac{1}{2} + \frac{V_{bn}}{V_{dc}} = \frac{1}{2} + \frac{M}{2}(-2\cos\phi\cos\psi + \sqrt{3}\sin\phi\cos\psi + \sin\psi) \quad (6b)$$

$$T_c/T_s = \frac{1}{2} + \frac{V_{cn}}{V_{dc}} = \frac{1}{2} + \frac{M}{2}(-2\cos\phi\cos\psi - \sqrt{3}\sin\phi\cos\psi + \sin\psi) \quad (6c)$$

$$T_z/T_s = \frac{1}{2} - \frac{V_{zn}}{V_{dc}} = \frac{1}{2} - \frac{M}{2}(\cos\phi\cos\psi + \sin\psi) \quad (6d)$$

Noted that  $T_z = \frac{T_s}{2} = \frac{1}{2f_s}$ , modulation index,  $M = \frac{V_{ref}}{V_{dc}}$ , and the boundary conditions each sector is  $0 < \phi < \frac{\pi}{3}$ ,  $0 < \psi < \frac{\pi}{4}$ . Furthermore, to determine the duty cycle of each tetrahedron can be used to the following equation.

$$\frac{T_a}{T_s} = \frac{D_1}{2} + D_2 + D_3 + D_4 \quad (7a)$$

$$\frac{T_b}{T_s} = \frac{D_1}{2} + D_2 + D_3 \quad (7b)$$

$$\frac{T_c}{T_s} = \frac{D_1}{2} + D_3 \quad (7c)$$

$$\frac{T_z}{T_s} = \frac{D_1}{2} \quad (7d)$$

Herein  $D_z = 1 - (D_1 + D_2 + D_3)$ .

Value of  $V_{ref}$  and  $\psi$ -angles if incorporated into twenty-fourth asymmetric tetrahedron can be determined time duration and duty cycle, it is required for switching combination of both a,b,c-conductor and neutral conductor.

### 3. DUTY CYCLE ANALYSIS ON VARIOUS TETRAHEDRON SHAPE

Geometrical principles are needed to analyse the reference vector on the tetrahedron shape. A cubic has a length side is 2 unit (centimeter) if it is cut in cross section than resulting a tetrahedron shape like as Figure 7. The longitude is connected a central point toward middle point, and it can be used to determine the value of reference vector of tetrahedron ( $V_{ref}$ ). The resulting angle between the longitude to the edges plane defined  $\Phi$ -angle while the resulting angle between the longitudes to the upright planes defined  $\psi$ -angle such as in equation-5 and equation-6. Weight value of the  $\alpha$ -axes,  $\beta$ -axes and  $\gamma$ -axes are represented duty cycle ( $D_1, D_2, D_3$ ).

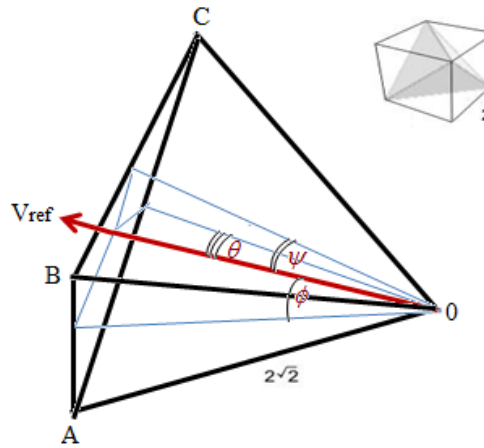


Figure 7. Geometry analysis of tetrahedron from a cubic cutting

**3.1. Duty Cycle Analysis in the Shen’s Model**

There are six non-zero vector switching (NZSV) and two zero vector switching (ZSV) in one sector (60°) which can be described as switching combination divided into four initial tetrahedron, wherein each tetrahedron is defined by three NZSV and two ZSV. Fourth of tetrahedron in each sector can be produced parameter tetrahedron such as reference vector and duty cycle. In Table 2, duty cycle calculation result in the Shen’s model is given.

Table 2. Duty cycle calculation in the Shen’s model

No	Vref	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	Dz/2	No	Vref	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	Dz/2
1	1,06014	0,80	0,22	0	0	7	0,8165	0,44	0,56	0	0
2	0,8165	0,35	0,62	0	0	8	1,06014	0,61	0,44	0	0
3	0,8165	0,44	0,56	0	0	9	1,06014	0,80	0,22	0	0
4	1,06014	0,61	0,44	0	0	10	0,8165	0,35	0,62	0	0
5	1,06014	0,80	0,22	0	0	11	0,8165	0,44	0,56	0	0
6	0,8165	0,35	0,62	0	0	12	1,06014	0,61	0,44	0	0

**3.2. Duty Cycle Analysis in the Zhang’s Model**

One state of sector (prism) in the Zhang’s model is produced four pieces of tetrahedron therefore comprising all of six prism in a cylindrical shape entirely has twenty-four pieces of tetrahedron. Each of tetrahedron has different pieces, so that analysis each of tetrahedron should be performed differently. Using equation 5, equation 6 and equation 7 in previous discussion, the parameters tetrahedron such as value of Vref, switching time of each tetrahedron and duty cycle can be determined. In Table 3, duty cycle calculation result in the Zhang’s model is given.

Table 3. Duty cycle calculation in the Zhang’s model

No	Vref	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	Dz/2	No	Vref	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	Dz/2
1	0.630185	0.37	0.48	0.03	0.1	13	0.670008	0.29	0.51	0.04	0.08
2	0.629232	0.23	0.56	0.03	0.09	14	0.737428	0.32	0.56	0.04	0.04
3	0.770022	0.39	0.53	0.06	0.01	15	0.631682	0.06	0.59	0.02	0.15
4	0.385343	0.01	0.39	0.04	0.25	16	0.770022	0.39	0.53	0.06	0.01
5	0.631172	0.18	0.55	0.01	0.12	17	0.631172	0.24	0.55	0.01	0.09
6	0.631172	0.36	0.4	0.07	0.09	18	0.386695	0.19	0.24	0.05	0.23
7	0.770685	0.33	0.57	0.05	0.02	19	0.499733	0.45	0.14	0.11	0.15
8	0.510512	0.11	0.48	0.02	0.18	20	0.770685	0.33	0.57	0.05	0.02
9	0.598461	0.14	0.55	0.02	0.13	21	0.629232	0.18	0.69	0.01	0.11
10	0.630185	0.2	0.56	0.02	0.1	22	0.630185	0.43	0.4	0.07	0.09
11	0.770022	0.4	0.53	0.06	0.01	23	0.770022	0.33	0.57	0.05	0.02
12	0.770022	0.33	0.58	0.05	0.02	24	0.770022	0.4	0.53	0.06	0.01



**3.3. Duty Cycle Analysis in the Perales’s Model**

Tetrahedron (region pointer) analysis in abc-coordinate of the Perales’s model is used quite a lot. There are sixty-four tetrahedrons that can be obtained. However, from these sixty-fourth tetrahedrons, taking twenty-four pieces of them is enough to determine value of duty cycle. It is interesting in the Perales’s model when parameter of fourth pieces of initial state of tetrahedron is known, then this value will be repeated to another. Using equation 5, equation 6 and equation 7 in previous discussion, the parameters of the fourth initial state tetrahedron in the Perales’s model such as value of Vref, the switching duration time of each phase and duty cycle can be determined. In Table 4, duty cycle calculation result in the Perales’s model is given.

Table 4. Duty cycle calculation in the Perales’s model

No	Vref	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	Dz/2	No	Vref	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	Dz/2
1	1.060137	0.22	0.88	0	0	13	1.060137	0.22	0.88	0	0
2	0.8165	0.41	0.6	0	0	14	0.8165	0.41	0.6	0	0
3	0.694025	0.16	0.67	0.07	0.09	15	0.694025	0.16	0.67	0.07	0.09
4	0.938975	0.58	0.54	0	0	16	0.938975	0.58	0.54	0	0
5	1.060137	0.22	0.88	0	0	17	1.060137	0.22	0.88	0	0
6	0.8165	0.41	0.6	0	0	18	0.8165	0.41	0.6	0	0
7	0.694025	0.16	0.67	0.07	0.09	19	0.694025	0.16	0.67	0.07	0.09
8	0.938975	0.58	0.54	0	0	20	0.938975	0.58	0.54	0	0
9	1.060137	0.22	0.88	0	0	21	1.060137	0.22	0.88	0	0
10	0.8165	0.41	0.6	0	0	22	0.8165	0.41	0.6	0	0
11	0.694025	0.16	0.67	0.07	0.09	23	0.694025	0.16	0.67	0.07	0.09
12	0.938975	0.58	0.54	0	0	24	0.938975	0.58	0.54	0	0

**3.4. Duty Cycle Analysis In Asymmetric’s Model**

Based on analysis pair of asymmetric tetrahedron shapes obtained from projection in  $\alpha\beta 0$ -coordinate like Figure 8, it can be described coordinate position on the top and the bottom of tetrahedron pair. Six sectors that are on the top and on the bottom tetrahedrons are produced the same value of duty cycle in one circle. The center point of the top tetrahedron is positioned in vector  $V_{15}$  and the center point of the bottom of tetrahedron is positioned in vector  $V_0$ . Asymmetrical tetrahedron appears as pieces of prism form pairs that are not balanced between the top and the bottom. Reference vector is determined by drawing a line connecting the center point with a cross-section of the prism. In one circle, there are twenty-four pieces of reference vector. Therefore, duty cycle can be determined. In Figure 8, tetrahedron pair is described with the center point being at the  $V_{15}$  on the top of tetrahedron.

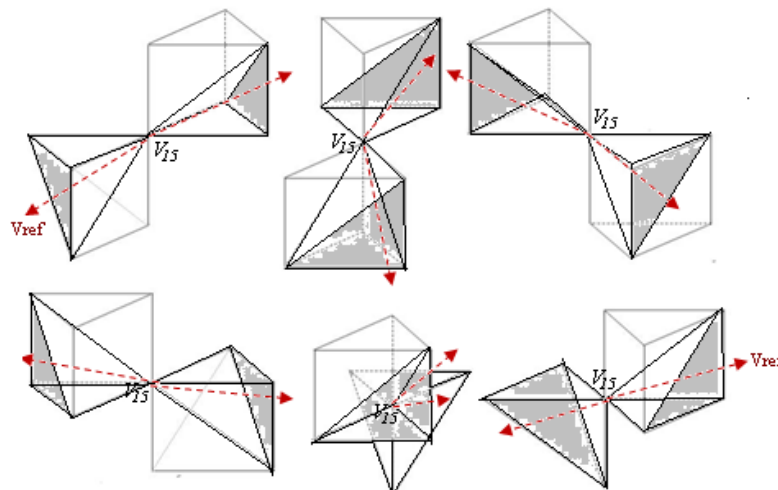


Figure 8. Tetrahedron pairs on state-(1,1,1,1) the top of center point ( $V_{15}$ )

Using equation 3, equation 4 and equation 5, the tetrahedron parameters such as value of Vref, switching time of each tetrahedron and duty cycle can be determined. Table 5 is given the results of duty cycle calculation in asymmetric models.

Table 5. Duty cycle calculation in the asymmetric model

No	Vref	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	Dz/2	No	Vref	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	Dz/2
1	0.600925	0.12	0.55	0.04	0.15	13	0.471403	0.1	0.44	0.01	0.21
2	0.166663	0.06	0.11	0.01	0.4	14	0.471399	0.1	0.44	0.01	0.21
3	0.471399	0.26	0.32	0.04	0.19	15	0.471403	0.26	0.32	0.04	0.19
4	0.471403	0.26	0.32	0.04	0.19	16	0.471399	0.26	0.32	0.04	0.19
5	0.220477	0.1	0.18	0.02	0.35	17	0.471393	0.1	0.44	0.01	0.21
6	0.471403	0.16	0.4	0.01	0.21	18	0.471399	0.1	0.44	0.01	0.21
7	0.471399	0.26	0.31	0.03	0.19	19	0.471403	0.26	0.32	0.04	0.19
8	0.471393	0.28	0.36	0.06	0.15	20	0.333326	0.36	0.11	0.23	0.15
9	0.55277	0.12	0.43	0.01	0.21	21	0.300461	0.14	0.17	0.05	0.32
10	0.471403	0.1	0.44	0.01	0.21	22	0.471399	0.1	0.44	0.01	0.21
11	0.471393	0.16	0.32	0.07	0.22	23	0.471393	0.24	0.3	0.06	0.19
12	0.333331	0.12	0.43	0.01	0.21	24	0.440957	0.09	0.41	0.02	0.23

#### 4. RESULT AND DISCUSSION

The following comparison of duty cycle analysis based on each tetrahedron model was developed. In the Shen's and the Perales's model, value of duty cycle in the c-conductor ( $D_3$ ) and the neutral conductor ( $D_z$ ) is equal zero, eventhough actually calculation produces a negative value, it indicates that the Shen's and Perales's model to current compensation control is not necessary to regulate switching combinations from the c-conductor and the neutral conductor. The Shen's and Perales's model even conducted on three-phase four-wire system of active filter resulted and indicated that it can be done by simplifying and using on three-phase three-wire system of active filter, since switching combination is only done in a,b-conductor (two-phase modulation). In the Zhang's model, it appears that value of duty cycle in neutral wire ( $D_z$ ) is smaller than value of duty cycle in asymmetric model. It means, in the Zhang's model, the period of switching in fourth-leg of IGBT module controller is shorter than asymmetric model.

In Figure 9, result comparison of duty cycle between Shen's, Zhang's, Perales's, Asymmetric model is given. The graph the Shen's and Perales's lines can not be described because the value of duty cycle ( $D_z$ ) for both of them is equal zero.

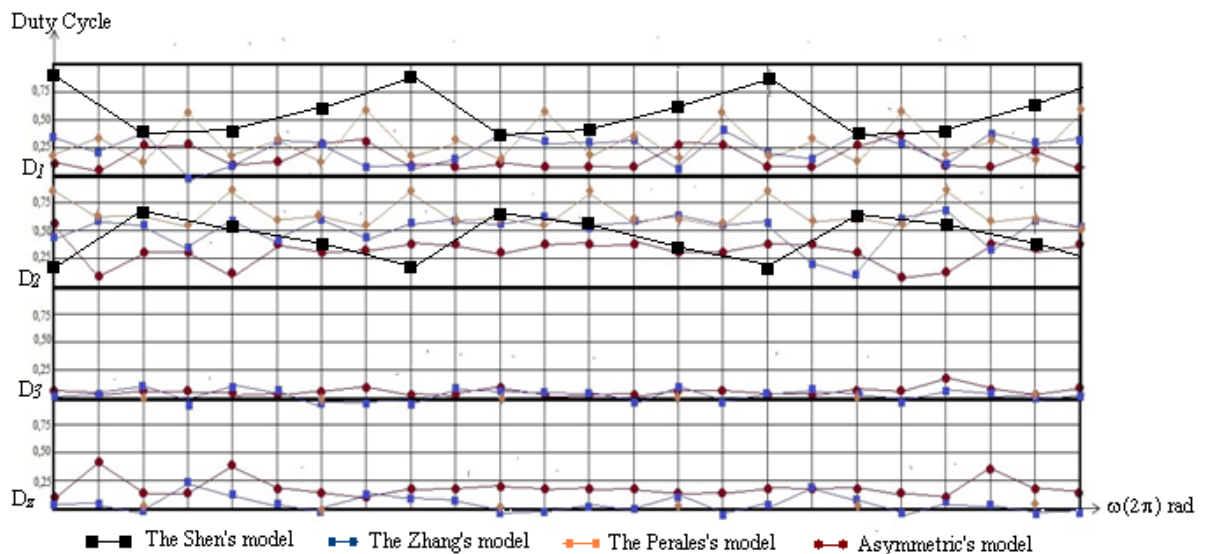


Figure 9. Result comparison of duty cycle between Shen's, Zhang's, Perales's, Asymmetric model

#### 5. CONCLUSION

Asymmetric voltage vector model as a result of pqr-coordinate projection into  $\alpha\beta$ -coordinate can be used to regulate current compensation control. A skewed vector description of asymmetric voltage vector defines the modulation boundary-line as equal 0.5 Vdc where this value is less than the model developed by Shen, Zhang and Perales.

The parameter calculation of tetrahedron and compare existing model based on duty cycle analysis in neutral wire can be concluded where asymmetric voltage vector model have value greater than another. It

suggests that switching time in the neutral conductor of three-phase four-leg system of active filter should be longer than the Shen's, Zhang's and Perales's model.

Calculation can also be concluded that in current compensation control, the Shen's and Perales's models actually quite uses a three-phase three-wire system of active filter because the modulation is done by simplifying the a, b-conductor (two-phase modulation).

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