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Utility Function-based Pricing Strategies in Maximizing the Information Service Provider's Revenue with Marginal and Monitoring Costs

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ABSTRACT

Previous research only focus on maximizing revenue for pricing strategies for information good with regardless the marginal and monitoring costs. This paper aims to focus on the addition of marginal and monitoring costs into the pricing strategies to maintain the maximal revenue while introduce the costs incurred in adopting the strategies. The well-known utility functions applied to also consider the consumer's satisfaction towards the service offered. The results show that the addition costs incurred for setting up the strategies can also increase the profit for the providers rather than neglecting the costs. It is also showed that the Cobb-Douglas utility functions used can enhance the notion of provider to optimize the revenue compared to quasi linear and perfect substitutes.

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1. INTRODUCTION

ISP (Internet Service Provider) provides services to access the internet to the public. The use of internet services has become an important mean for all segments of society to find a variety of information. The availability of reliable multi source information service [1], in system of city traffic information service [2] or in all aspects of university information service [3] are also vey demanding. To use internet services internet users must follow the service provided by the ISP. Currently there is a wide variety of user demand internet and various applications that make the internet provision should take into account the quality of the service (Quality of Service, QoS). Basically, QoS makes it possible to provide better service to a particular request. To demonstrate the efficiency of the ISP in service there must be interaction between price and QoS [4].

Utility functions was usually associated with a level of satisfaction that user get for the use of information services used specifically relating to maximize profits in achieving specific objectives which can be written with $U = f(x_1, x_2, \dots, x_n)$ meaning that $x_1, x_2, \dots, x_n$ contribute utility-users indicating objective satisfaction [5], [6].

Many assumptions of utility function used of which are often used by researchers, namely as a function of bandwidth where the value of loss and delay are fixed and follow the rules that the marginal utility as a function of bandwidth decreases with increasing bandwidth [7], [8]. Assumptions about the selection of utility functions, one of which is [9] by assuming that these functions should be easily simplified its derivation and easily analyzed its homogeneity and heterogeneity that influence the choice of the pricing structure for the company. One possibility that some research already mentioned for applying fuzzy system [10] and its approximation is the attempt to also include fuzzy utility function to model of pricing strategies.

Research on the theory of the pricing plan has been widely available, but only a few pricing plan involving utility functions as an indicator of consumer satisfaction. Wu and Banker [9] has analyzed the pricing scheme Internet by using one of the utility function is Cobb-Douglas modified utility function to maximize benefits to the ISP. In their research, the usage of three schemes of pricing for information services namely flat fee, usage-based, and the two-part tariff pricing schemes. Results of the analysis showed that the pricing scheme of the flat fee and a two-part tariff generate more optimal solution than the usage-based pricing schemes.

Then, further research on the pricing scheme internet has now will be involving other utility functions such as original Cobb-Douglas, quasi linear, perfect substitute utility functions that are used in three types of pricing schemes for information services that are flat fee, usage based and two-part tariff both analytically [11], and numerically with the help of LINGO 11.0 software applications [12], [13]. Based on these results, a new method of searching information services by considering the function of the precise utility function have proven to generate huge profits for ISPs to adopt this type of pricing schemes, are available, but the study only on the selection of utility functions that can maximizing profits for ISPs and ignore the marginal costs and monitoring costs. Other research also consider pricing scheme of information services with regard to perfect substitute [14] and Cobb-Douglas utility functions [15] where by using three pricing strategies, and considering marginal and monitoring costs, the optimal case for each consumer can be obtained. Based on that, the authors attempt to proceed with other utility function to be analyzed to flat fee, usage based pricing strategies.

In general, the marginal costs are defined as the costs adjusted to the level of production of goods which is resulting differences in fixed costs due to the addition of the number of units produced, while the cost of monitoring is the cost incurred by the company to monitor and control the activities carried out by the agency in managing company. In fact, the marginal cost and the cost of monitoring is also an important issue in the development of information services primarily affects the maximum objective function for three pricing schemes are flat fee, usage-based and two-part tariff. To that end, it is necessary study on marginal costs and the cost of monitoring the pricing schemes involving information services utility functions that are often used, which is perfect substitute utility function, quasi linear utility function, and Cobb-Douglas function.

Then the main controbution of this paper is basically to extend the application of the pricing scheme of pricing strategies of information service with regard to marginal and monitoring costs to enable provider to have other insight on the advantage of applying marginal and monitoring costs to information service pricing scheme and with the usage of quasi linear utility function.

2. RESEARCH METHOD

Steps conducted in this research are as follows.

1. Determine the information service pricing scheme models according to quasi linear, utility functions with flat fee, usage-based, dan two-part tariff pricing scheme for homogeneous and heterogeneous consumers.
   a. For flat fee pricing scheme, $P_X = 0$, $P_Y = 0$ and P adalah is positive.
   b. For usage-based scheme, P_X and P_Y are positive and P = 0.
   c. For two-part tariff scheme, P, P_X and P_Y are positive.
2. Formulate quasi linear utility function according to flat fee, usage-based, dan two-part tariff pricing schemes for homogeneous and heterogeneous consumers with paying attention to marginal and monitoring cots.
5. Process mail from local server.
6. Apply the optimal pricing scheme of local data server of mail traffic data.
7. Compare the pricing scheme models to each utility function previously described in previous research proposed by [14], [15].
8. Conclude and obtain the best solution of information service pricing scheme.

3. RESULTS AND ANALYSIS

This section discusses other utility function that is also well kwown namely quasi linear utility function. Original Cobb-Douglas [15], perfect substitute [16] and modified Cobb-Douglas utility functions [9], [17] are already discussed in previous research, but the comparison from all these utility functions are showed to explain the best utility function that can maximize the profit of ISP.

3.1. Model of Pricing Scheme Based Quasi linear Utility Function

The general form of utility function based on the quasi linear $U(X, Y) = a\,X + f(Y)$, where $f(Y) = Y^{b}$ Here, quasi linear utility function analyzed for homogeneous and heterogeneous consumers (high-end and low-end) as well as heterogeneous (high-demand and low-demand) consumers are based on three strategies of pricing schemes that pricing schemes flat-fee, pricing schemes usage -based, two-part pricing scheme tariff.

3.2. Homogeneous Consumer

Consumer Optimization Problems will be as follows.

$$\mathrm{Max}_{X,Y,Z}\; a\,X + f(Y) - P_x X - P_y Y - PZ - (X+Y)c \tag{1}$$

Subject to

$$X \le \bar{X}Z \tag{2}$$

$$Y \le \bar{Y}Z \tag{3}$$

$$a\,X + f(Y) - P_x X - P_y Y - PZ - (X+Y)c \ge 0 \tag{4}$$

$$Z = 0 \text{ atau } 1 \tag{5}$$

Optimization Problems of the providers will be as follows.

$$\mathrm{Max}_{P,P_X,P_Y} \textstyle\sum_i (P_X X^* + P_Y Y^* + PZ^*) \tag{6}$$

with (X^*,Y^*,Z^*) = argmax $a\,X + f(Y) - P_x X - P_y Y - PZ - (X+Y)c$

Subject to

$$X \le \bar{X}Z$$

$$Y \le \bar{Y}Z$$

$$a\,X + f(Y) - PxX - PyY - PZ - (X+Y)c \ge 0$$

$$Z_i = 0 \text{ or } 1$$

For usage-based pricing scheme and a two-part tariff:
Consumer Optimization Problems:

$$\mathrm{Max}_{X,Y,Z}\; a\,X + f(Y) - P_x X - P_y Y - PZ - (c+t)X - (c+t)Y \tag{7}$$

with constraints :

$$X \le \bar{X}Z \tag{8}$$

$$Y \le \bar{Y}Z \tag{9}$$

$$a\,X + f(Y) - P_x X - P_y Y - PZ - (c+t)X - (c+t)Y \ge 0 \tag{10}$$

$$Z = 0 \text{ atau } 1 \tag{11}$$

Optimization Problems of Providers:

$$\mathrm{Max}_{P,P_X,P_Y} \textstyle\sum_i (P_X X^* + P_Y Y^* + PZ^*) \tag{12}$$

with (X^*,Y^*,Z^*) = argmax $a\,X + f(Y) - P_x X - P_y Y - PZ - (c+t)X - (c+t)Y$

with constraints:

$$X \leq \bar{X}Z$$

$$Y \leq \bar{Y}Z$$

$$a\,X + f(Y) - PxX - PyY - PZ - (c+t)X - (c+t)Y \geq 0$$

$$Z_i = 0 \text{ atau } 1$$

Case 1. If the ISP is using flat-fee pricing scheme by setting $P_x = 0, P_y = 0$ and $P > 0$. Optimization problems consumers for flat-fee pricing scheme be :

$$\text{Max } aX + f(Y) - (0)X - (0)Y - P(1) - (X+Y)c$$
$$= \text{Max } aX + f(Y) - P - (X+Y)c$$

By using Constraint (4), then

$$aX + f(Y) - P_xX - P_yY - PZ - (X+Y)c \geq 0$$
$$\Leftrightarrow aX + f(Y) - (0)X - (0)Y - P(1) - (X+Y)c \geq 0 \Leftrightarrow P \leq aX + f(Y) - (X+Y)c$$

Optimization of provider became:

$$\underset{P,P_X,P_Y}{\text{Max}} \sum_i (P_XX^* + P_YY^* + PZ^*) = \underset{P,P_X,P_Y}{\text{Max}} \sum_i \big((0)X^* + (0)Y^* + P(1)\big)$$
$$= \text{Max}_{P,P_X,P_Y} \textstyle\sum_i \big((0)X^* + (0)Y^* + aX + f(Y) - (X+Y)c\big)$$
$$= \underset{P,P_X,P_Y}{\text{Max}} \sum_i aX + f(Y) - (X+Y)c$$

This means that if the ISP provides this price, the level of consumer spending into $X = \bar{X}$ and $Y = \bar{Y}$ with maximum utility, consumers can get $a\bar{X} + f(\bar{Y}) - (\bar{X}+\bar{Y})c$. ISP optimal price used is $\bar{X} + f(\bar{Y}) - (\bar{X}+\bar{Y})c$, the maximum benefit is $\sum_i[a\bar{X} + f(\bar{Y}) - (\bar{X}+\bar{Y})c]$.

Based on this case Lemma 1 can be stated as follows.

Lemma 1: If the ISP is using flat-fee pricing scheme, the optimal price is $a\bar{X} + f(\bar{Y}) - (\bar{X}+\bar{Y})c$ and the maximum profit to be $\sum_i[a\bar{X} + f(\bar{Y}) - (\bar{X}+\bar{Y})c]$

Case 2. If ISPs use usage-based pricing scheme by setting $P_x > 0, P_y > 0$ and $P = 0$. Optimization problems consumers on usage-based pricing scheme be:

$$\underset{X,Y,Z}{\text{Max}}\, a\,X + f(Y) - P_xX - P_yY - (c+t)X - (c+t)Y \tag{13}$$

To maximize Eq. (13), do differentiation of the *X* and *Y*:

$$\frac{\partial U}{\partial X} = 0 \Leftrightarrow \frac{\partial(a\,X + f(Y) - P_XX - P_yY - (c+t)X - (c+t)Y)}{\partial X} = 0 \text{ then}$$

$$a - (c+t) = P_x \Leftrightarrow X^* = \bar{X} \tag{14}$$

and

$$\frac{\partial U}{\partial X} = 0 \Leftrightarrow \frac{\partial(a\,X + f(Y) - P_XX - P_yY - (c+t)X - (c+t)Y)}{\partial \bar{Y}} = 0 \text{ and}$$

$$f'(\bar{Y}) - (c+t) = P_y \Leftrightarrow Y^* = \bar{Y} \tag{15}$$

Optimization of production problems became:

$$\begin{aligned}&\underset{P,P_X,P_Y}{\text{Max}}\sum_i(P_XX^*+P_YY^*)=\sum_i[P_X(\bar{X})+P_Y(\bar{Y})]\\&=\textstyle\sum_i\big[(a-(c+t))\bar{X}+\big(f'(\bar{Y})-(c+t)\big)\bar{Y}\big]\\&=\sum_i[a\bar{X}+\bar{Y}f'(\bar{Y})-(c+t)\bar{X}-(c+t)\bar{Y}]\end{aligned}$$

To maximize the function optimization problem providers, ISPs should minimize P_x and P_y. If known P_x and P_y decline, then X* and Y* increase, if X and Y are restricted, then X*=$\bar{X}$ and Y*=$\bar{Y}$. P_x and P_y yang optimal into $P_x = a-(c+t)$ and $P_y = f'(\bar{Y})-(c+t)$ with maximum profit $\sum_i[a\bar{X}+\bar{Y}\,f'(\bar{Y})-(c+t)\bar{X}-(c+t)\bar{Y}]$.

Then proceed to next lemma as follows.

Lemma 2: If ISPs use usage-based pricing scheme, the optimal price is $P_x = a-(c+t)$ and $P_y = f'(\bar{Y})-(c+t)$, the maximum benefit is:$\sum_i[a\bar{X}\ +\bar{Y}f'(\bar{Y})-(c+t)\bar{X}-(c+t)\bar{Y}]$.

Case 3. If the ISP uses a two-part pricing scheme tariff by setting $P_x > 0$, $P_y > 0$ and $P > 0$. By using Eq. (8)-(9). If these equation are substituted into Eq. (10) and to maximize the objective function (7), then:

For two-part tariff will be

$$\begin{aligned}&aX\ +f(Y)-P_xX-P_yY-PZ-(c+t)X-(c+t)Y\ \ \geq 0\\&\Leftrightarrow aX\ +f(Y)\ -(a-(c+t))\bar{X}-\big(f'(\bar{Y})-(c+t)\big)\bar{Y}-P-(c+t)X-(c+t)Y\ \ \geq 0\\&P\leq aX\ -a\bar{X}+f(Y)-\bar{Y}f'(\bar{Y})+(c+t)\bar{X}+(c+t)\bar{Y}-(c+t)X-(c+t)Y\end{aligned}$$

By substituting the value to the objective function (7)), optimization problems of providers being:

$$\begin{aligned}&\underset{P,P_X,P_Y}{\text{Max}}\sum_i(P_XX^*+P_YY^*)=\sum_i[P_X(\bar{X})+P_Y(\bar{Y})+P]\\&=\sum_i[(a-(c+t))\bar{X}+(f'(\bar{Y})-(c+t))\bar{Y}\\&\qquad\qquad+(aX\ -a\bar{X}+f(\bar{Y})-\bar{Y}f'(\bar{Y})+(c+t)\bar{X}+(c+t)\bar{Y}-(c+t)X-(c+t)Y)]\\&=\sum_i[a\bar{X}+f(\bar{Y})-(c+t)\bar{X}-(c+t)\bar{Y}]\end{aligned}$$

To maximize the function optimization problem providers, ISPs should minimize P_x and P_y. if known P_x and P_y decline, then X* and Y* increase, if X and Y are bounded, then X*≤$\bar{X}$ and Y*≤$\bar{Y}$. In other words, P_x and P_y optimal will be $P_x = a-(c+t)$ and $P_y = f'(\bar{Y})-(c+t)$. The maximum gain is achieved $\sum_i[a\bar{X}+f(\bar{Y})-(c+t)\bar{X}-(c+t)\bar{Y}]$; .

Based on this case, next lemma was obtained.

Lemma 3: If the ISP uses a two-part tariff rates, then best P_X and P_Y be $P_X = a$ and $P_Y = f'(\bar{Y})$. Maximum profit,$\sum_i[a\bar{X}+f(\bar{Y})-(c+t)\bar{X}-(c+t)\bar{Y}]$; i is consumer.

If it is assumed $\bar{Y}\,f'(\bar{Y}) > f(\bar{Y})$; $\bar{Y} > 0$ and function $f(\bar{Y})$= Y^b is a non linear function, then $a\bar{X}+\bar{Y}\,f'(\bar{Y})-(c+t)\bar{X}-(c+t)\bar{Y} > \sum_i[a\bar{X}\ +f(\bar{Y})-(\bar{X}+\bar{Y})c] > a\bar{X}+f(\bar{Y})-(c+t)\bar{X}-(c+t)\bar{Y}$, usage-based pricing schemes generate greater profits than the flat-fee and two-part tariff pricing schemes for homogeneous consumer.

3.3. Heterogeneous Consumer

Suppose that there are m high-end consumer (the upper class) (i = 1) and n low-end consumer (lower class) (i = 2). To find heterogeneous consumers' willingness to pay a given price scheme affect881881881 service providers, it is assumed every consumer in both segments have an upper limit on the same X and Y at peak hours, $a_1 > a_2$ and $b_1 > b_2$.

For flat-fee pricing scheme

Consumer Optimization Problems :

$$\underset{X_i,Y_i,Z_i}{\text{Max}}\ a_i\,X_i+f(Y_i)-\ P_XX_i-\ P_YY_i-PZ_i-(X_i+Y_i)c \qquad (16)$$

Subject to

$$X_i \leq \bar{X} Z_i \tag{17}$$

$$Y_i \leq \bar{Y} Z_i \tag{18}$$

$$a_i X_i + f(Y_i) - P_X X_i - P_Y Y_i - PZ_i - (X_i + Y_i)c \geq 0 \tag{19}$$

$$Z_i = 0 \text{ or } 1 \tag{20}$$

Optimization Problems of Providers :

$$\underset{P,P_X,P_Y}{\text{Max}}\ m\,(P_X \mathrm{X}_1{}^* + P_Y \mathrm{Y}_1{}^* + P\mathrm{Z}_1{}^*) + n\,(P_X \mathrm{X}_2{}^* + P_Y \mathrm{Y}_2{}^* + P\mathrm{Z}_2{}^*) \tag{21}$$

with $(X_i{}^*,\ Y_i{}^*, Z_i{}^*) =$ argmax $a_i X_i + f(Y_i) - P_X X_i - P_Y Y_i - PZ_i$
subject to

$$X_i \leq \bar{X} Z_i$$

$$Y_i \leq \bar{Y} Z_i$$

$$a_i X_i + f(Y_i) - P_X X_i - P_Y Y_i - PZ_i \geq 0$$

$$Z_i = 0 \text{ or } 1$$

For usage-based pricing schemes and two-part tariff
Consumer Optimization Problems:

$$\underset{X_i,Y_i,Z_i}{\text{Max}}\ a_i X_i + f(Y_i) - P_X X_i - P_Y Y_i - PZ_i - (c+t)X_i - (c+t)Y_i \tag{22}$$

Subject to

$$X_i \leq \bar{X} Z_i \tag{23}$$

$$Y_i \leq \bar{Y} Z_i \tag{24}$$

$$a_i X_i + f(Y_i) - P_X X_i - P_Y Y_i - PZ_i - (X_i + Y_i) \geq 0 \tag{25}$$

$$Z_i = 0 \text{ or } 1 \tag{26}$$

Optimization Problems of Providers:

$$\underset{P,P_X,P_Y}{\text{Max}}\ m\,(P_X \mathrm{X}_1{}^* + P_Y \mathrm{Y}_1{}^* + P\mathrm{Z}_1{}^*) + n\,(P_X \mathrm{X}_2{}^* + P_Y \mathrm{Y}_2{}^* + P\mathrm{Z}_2{}^*) \tag{27}$$

with $(X_i{}^*,\ Y_i{}^*, Z_i{}^*) =$ argmax $a_i X_i + f(Y_i) - P_X X_i - P_Y Y_i - PZ_i$
subject to

$$X_i \leq \bar{X} Z_i$$

$$Y_i \leq \bar{Y} Z_i$$

$$a_i X_i + f(Y_i) - P_X X_i - P_Y Y_i - PZ_i \geq 0$$

$$Z_i = 0 \text{ or } 1$$

Steps to get the maximum profit on any pricing scheme used by ISP.

Case 4. If the ISP using flat-fee pricing scheme by setting $P_X = 0, P_Y = 0$ and $P > 0$, where the price used by the ISP has no effect on the time of peak hours use or off-peak hours, then consumers choose the maximum consumption $X_1 = \bar{X}, X_2 = \bar{X}, Y_1 = \bar{Y},$ and $Y_2 = \bar{Y}$. Thus, any high-end consumer cost no more than $a_1\bar{X} + f(\bar{Y}) - (\bar{X} + \bar{Y})c$ and low-end consumer is not over $a_2\bar{X} + f(\bar{Y}) - (\bar{X} + \bar{Y})c$. Case 4 is a flat-fee pricing scheme so that P is equivalent for both types of heterogeneous consumers. If it is established $a_1 > a_2$ then for the provision of high-end consumer costs will follow the price for the cost of low-end consumer so $a_1\,(m) < a_2\,(m+n) \Leftrightarrow a_1 < \frac{a_2\,(m+n)}{m}$.

This means that if the consumer is charged at $a_1\bar{X} + f(\bar{Y}) - (\bar{X} + \bar{Y})c$, then only the high-end consumer who can follow this service. If consumers are charged a fee of $a_2\bar{X} + f(\bar{Y}) - (\bar{X} + \bar{Y})c$, then both types of consumers can follow this service, namely the consumers of high-end and low-end consumer. To maximize benefits, ISP charge $a_2\bar{X} + f(\bar{Y}) - (\bar{X} + \bar{Y})c$. In this case for Optimization Problems of Providers:

$$\begin{aligned}\operatorname*{Max}_{P} m(PZ_1{}^*) + n\,(PZ_2{}^*) &= m\{a_2\bar{X} + f(\bar{Y}) - (\bar{X} + \bar{Y})c\} + n\,\{a_2\bar{X} + f(\bar{Y}) - (\bar{X} + \bar{Y})c\} \\ &= (m+n)\,[a_2\bar{X} + f(\bar{Y}) - (\bar{X} + \bar{Y})c]\end{aligned}$$

Maximum profit yang obtainable produsen is $(m+n)\,[a_2\bar{X} + f(\bar{Y}) - (\bar{X} + \bar{Y})c]$. Based on this, the lemma was obtained.

Lemma 4: If ISPs use pricing schemes *flat-fee*, then harga yang dikenakan adalah $a_2\bar{X} + f(\bar{Y}) - (\bar{X} + \bar{Y})c$ with maximum profit obtained amounted $(m+n)\,[a_2\bar{X} + f(\bar{Y}) - (\bar{X} + \bar{Y})c]$.

Case 5. If ISPs use pricing schemes *usage-based* by setting $P_X > 0, P_Y > 0$ and $P = 0$ then :

Optimization problems for high-end heterogeneous consumers:

$$\max_{X,Y,Z} a_1X_1 + f(Y_1) - P_XX_1 - P_YY_1 - (c+t)X_1 - (c+t)Y_1$$

To maximize functionality on Consumer Optimization Problems for heterogeneous high-end consumer, do differentiation agains X_1 and Y_1 :

$$\frac{\partial U}{\partial X_1} = 0$$

$$a_1 - (c+t) = P_X \Leftrightarrow X_1{}^* = \bar{X} \tag{28}$$

and

$$\frac{\partial U}{\partial Y_1} = 0$$

$$f'(Y_1) - (c+t) = P_Y \Leftrightarrow Y_1{}^* = \bar{Y} \tag{29}$$

Optimization problems for heterogeneous low-end consumer:

Functions in Consumer Optimization Problems:

$$max_{\,X,Y,Z}\; a_2X_2 + f(Y_2) - P_XX_2 - P_YY_2 - (c+t)X_2 - (c+t)Y_2$$

To maximize functionality on Consumer Optimization Problems for heterogeneous low-end consumers, do differentiation against X_2 and Y_2:

$$\Leftrightarrow \frac{\partial(\,a_2X_2 + f(Y_2) - P_XX_2 - P_YY_2 - (c+t)X_2 - (c+t))}{\partial X_2} = 0, \text{ then}$$

$$a_2 - (c+t) = P_X \Leftrightarrow X_2{}^* = \bar{X} \tag{30}$$

and $\frac{\partial U}{\partial Y_1} = 0$

$$f'(Y_2) - (c+t) = P_Y \Leftrightarrow Y_2{}^* = \bar{Y} \tag{31}$$

Optimization Problems of Providers is as follows.

$$\underset{P_X,P_Y}{\text{Max}}\, m(P_X \mathrm{X_1}^* + P_Y \mathrm{Y_1}^*) + n\,(P_X \mathrm{X_2}^* + P_Y \mathrm{Y_2}^*) = m[P_X(\bar{X}) + P_Y(\bar{Y})] + n\,[P_X(\bar{X}) + P_Y(\bar{Y})]$$
$$= m[a_1\bar{X} + \bar{Y}f'(\bar{Y}) - (c+t)\bar{X} - (c+t)\bar{Y}\,] + n\,[a_2\bar{X} + \bar{Y}f'(\bar{Y}) - (c+t)\bar{X} - (c+t)\bar{Y}]$$

If applied to the problem at peak hours, to maximize the function, the ISP must minimize P_X and hence best price can not be greater than P_X can not be greater than $a_1 - (c+t)$. On the other hand, if the ISP set prices below $a_2 - (c+t)$, profit is not optimal. If applied to problems in off-peak hours, the best prices $P_Y \leq \bar{Y}f'(\mathrm{Y_1}) - (c+t)$. On the other hand, if the ISP set prices below $\bar{Y}f'(\mathrm{Y_2}) - (c+t)$, then profit is not optimal when $Y_1{*}\leq \bar{Y}$ and $Y_2{*}\leq \bar{Y}$. Because by the, price P_Y is best $f'(\mathrm{Y_2}) - (c+t) \leq P_Y \leq f'(\mathrm{Y_1}) - (c+t)$. If the price is this interval, the demand from high-end consumer remains on $\bar{X}$ and $\bar{Y}$, Thus the optimal price is given for the rush hour is $P_X = a_2 - (c+t)$ and optimal prices in off-peak hours is $P_Y = f'(\bar{Y}) - (c+t)$ maximum profit is $(a_2\bar{X} + \bar{Y}f'(\bar{Y}) - (c+t)\bar{X} - (c+t)\bar{Y})$.

Based on this case the lemma was obtained.

Lemma 5: If ISPs use usage-based price, then the optimal price is given for the rush hour is$P_X = a_2 - (c+t)$ and optimal prices in off-peak hours is $P_Y = f'(\bar{Y}) - (c+t)$ with maximum profit is

$$(m+n)(a_2\bar{X} + \bar{Y}f'(\bar{Y}) - (c+t)\bar{X} - (c+t)\bar{Y}).$$

Case 6. If ISPs use pricing schemes *two-part tariff*, then $P_X > 0$, $P_Y > 0$, and $P > 0$. where there is a cost incurred if the consumer chooses to join the service and the prices charged during peak hours and off-peak hours, the first order condition for equality Consumer Optimization Problems of high-end and low-end consumer. If it is established $a_1 > a_2$ then it can be assumed that $a_1\,(m) < a_2\,(m+n) \Leftrightarrow a_1 < \frac{a_2\,(m+n)}{m}$. This means that if the consumer is charged at $P_X = a_1 - (c+t)$ and $P_Y = f'(Y_1) - (c+t)$ and $P = a_1X - a_1\bar{X} + f(Y) - \bar{Y}f'(\bar{Y}) + (c+t)\bar{X} + (c+t)\bar{Y} - (c+t)X - (c+t)Y$ then only the high-end consumer who can follow this service. If consumers are charged a fee of $P_X = a_2 - (c+t)$ and $P_Y = f'(\bar{Y}) - (c+t)$ then both types of consumers can follow the service, namely the consumers of high-end and low-end consumer. ISPs may choose to declinekan dibecausekan cost many consumers intokan subscription fees as a barrier so that it can attract more consumers, ISPs can provide prices $P_X = a_2 - (c+t)$, $P_Y = f'(Y_2) - (c+t)$, and minimize the cost of subscription.

Optimization Problems Providers into:

$$\underset{P_X,P_Y}{\text{Max}}\; m(P_XX_1{}^* + P_YY_1{}^* + PZ_1{}^*) + n\,(P_XX_2{}^* + P_YY_2{}^* + PZ_2{}^*)$$
$$= (m+n)[a_2\bar{X} + f(\bar{Y}) - (c+t)\bar{X} - (c+t)\bar{Y})]$$

Thus the maximum profit is achieved $(m+n)(a_2\bar{X} + f(\bar{Y}) - (c+t)\bar{X} - (c+t)\bar{Y})$

Based on this case was obtained.

Lemma 6: If the ISP uses a two-part tariff rates, P_X and P_Y optimal respectively are $P_X = a_2 - (c+t)$and $P_Y = f'(\bar{Y}) - (c+t)$, and with maximum profit obtainable is$(m+n)[a_2\bar{X} + f(\bar{Y}) - (c+t)\bar{X} - (c+t)\bar{Y}]$.

If it is assumed that $\bar{\mathrm{Y}}\,\mathrm{f}'(\bar{\mathrm{Y}}) > f(\bar{\mathrm{Y}})$, with $\bar{\mathrm{Y}} > 0$ and $\mathrm{f}(\bar{\mathrm{Y}})$is non linear function, $\mathrm{f}(\bar{\mathrm{Y}}) = \mathrm{Y^b}$, $\mathrm{X} \geq 0$ and $\mathrm{Y} \geq 0$, then $(\mathrm{m+n})\big(\mathrm{a_2\bar{X}} + \bar{\mathrm{Y}}\mathrm{f'}(\bar{\mathrm{Y}})\big) > (\mathrm{m+n})(\mathrm{a_2\bar{X}} + \mathrm{f}(\bar{\mathrm{Y}}))$. Because by then, usage-based pricing scheme is better than the flat-fee pricing schemes and two-part tariff for heterogeneous high-end and low-end consumer issues.

Using similar proof for next three lemma, these results are achieved.

Lemma 7: If ISPs use pricing schemes flat-fee, then $P = a\bar{X}_2 + f(\bar{Y}_2) - (\bar{X}_2 + \bar{Y}_2)c$ with maximum profit is achieved$(m+n)[a\bar{X}_2 + f(\bar{Y}_2) - (\bar{X}_2 + \bar{Y}_2)c]$

Lemma 8: If ISPs use pricing schemes usage-based, then optimal price at peak hours is $P_X = a - (c+t)$ and optimal prices in off-peak hours is $P_Y = f'(\bar{Y}_2) - (c+t)$ with maximum profit is

$$(m+n)\;[a\bar{X}_2 + \bar{Y}_2\, f'(\bar{Y}_2) - (c+t)\bar{X}_2 - (c+t)\bar{Y}_2\,]$$

Lemma 9:If the ISP uses a two-part tariff rates, then $P_X = a - (c+t)$ and $P_Y = f'(\bar{Y}_2) - (c+t)$, $P = f(\bar{Y}_2) - f'(\bar{Y}_2)\bar{Y}_2$, P is subscription fee equal to consumer surplus from consumers evels are low, with maximum profit $m[a\bar{X}_1 + f'(\bar{Y}_2)(\bar{Y}_1 - \bar{Y}_2) + f(\bar{Y}_2)\,] + n\,[a\bar{X}_2 + f(\bar{Y}_2) - (c+t)\bar{X}_2 - (c+t)\bar{Y}_2\,]$ greater than the price of flat-fee or usage-based.

X ≥ 0 and Y ≥ 0, $m[a\bar{X}_1 + f'(\bar{Y}_2)(\bar{Y}_1 - \bar{Y}_2) + f(\bar{Y}_2)] + n[a\bar{X}_2 + f(\bar{Y}_2) - (c+t)\bar{X}_2 - (c+t)\bar{Y}_2] > (m+n)[a\bar{X}_2 + \bar{Y}_2 f'(\bar{Y}_2) - (c+t)\bar{X}_2 - (c+t)\bar{Y}_2] > (m+n)[a\bar{X}_2 + f(\bar{Y}_2) - (\bar{X}_2 + \bar{Y}_2)c]$, means that a two part tariff scheme is more optimal than usage-based pricing schemes and flat-fee for heterogeneous consumer based on high-demand and low-demand.

Based on case 1- case 9 analysis for quasi linear utility function, the comparison among utility functions that have already discussed in previous research conducted by [9], [15], [16] can be performed. Table 1-3 are the recapitulation of pricing scheme for each type of consumer in mail data traffic by comparing with previous research. In the perfect substitute utility function, in the case of homogeneous and heterogeneous consumers (high-end and low-end) the optimal pricing scheme is contained in the flat-fee pricing schemes, and for heterogeneous (high-demand and low-demand) consumers, usage- based and two-part tariff pricing schemes are the most optimal. In the quasi linear utility function, in the case of consumers homogeneous and consumer heterogeneous (high-end and low-end) pricing schemes optimally located on the usage-based pricing scheme, and for consumers heterogeneous (high-demand and low-demand) pricing scheme the most optimal is a two-part tariff scheme.

Table 1. Homogeneous Consumer for Three Utility Functions in Mail Traffic Data

Scheme	*Perfect Substitute[16]*	*Quasi linear*	*Cobb-Douglas[15]*
Flat-fee	$\sum_i (1150.391a + 1008.789b - 2159.18c)$	$\sum_i [1150.391a + (1008.789) - 2159.18c]$	$\sum_i [1150.391^a 1008.789^b - (1150.391 + 1008.789)c]$
Usage-based	$\sum_i (1150.391a + 1.008.789b - 2159.18(c+t))$	$\sum_i [1150.391a + b(1008.789)^b - 2159.18(c+t)]$	$\sum_i \{(a+b)[1150.391^a 1008.789^b] - (c+t)\,1150.391 - (c+t)\,1008.789\}$
Two-part tariff		$\sum_i [1.150.391a + (1008.789)^b - 2159.18(c+t)]$	$\sum_i [1150.391^a 1008.789^b - (c+t)\,1150.391 - (c+t)\,1008.789]$

Table 2. Heterogeneous Consumer High-End and Low-End) for Three Utility Functions in Mail Traffic Data

Scheme	Perfect Substitute[16]	Quasi linear	Cobb-Douglas[15]
Flat-fee	$(m+n)(1150.391a_2 + (1008.789)b_2 - 2159.18c)$	$(m+n)(1150.391a_2 + (1008.789)^{b_2} - 2159.18c)$	$(m+n)\,[1150.391^{a_2} 1008.789^{b_2} - (1150.391 + 1008.789)\,c]$
Usage-based	$(m+n)(1150.391a_2 + (1008.789)b_2 - 2159.18(c+t))$	$(m+n)\left(1150.391a_2 + b_2(1008.789)^{b_2} - 2159.18(c+t)\right)$	$(m+n)\left[(a_2+b_2)\left(1150.391^{a_2} 1008.789^{b_2} - (c+t)1150.391 - (c+t)1008.789\right)\right]$
Two-part tariff		$(m+n)\left(1150.391a_2 + b_2(1008.789)^{b_2} - 2159.18(c+t)\right)$	$(m+n)\,[1150.391^{a_2} 1008.789^{b_2} - (1150.391 + 1008.789)\,c]$

Table 3. Heterogeneous Consumer High-Demand and Low-Demand for Three Utility Functions in Mail Traffic Data

Scheme	Perfect Substitute[16]	Quasi linear	Cobb-Douglas[15]
Flat-fee	$(m+n)(228.516a + 343.75b - 572.266c)$	$(m+n)[228.516a + (343.75)^b - 572.266c]$	$(m+n)(228.516^a 343{,}75^b - (228.516 + 343.75)\,c)$
Usage-based	$m(1150.391a + 1.008.789b - 2159.18(c+t)) + n(228.516a + 343.75b - 572.266(c+t))$	$(m+n)\,[228.516a + b(343.75)^b - 572.266(c+t)]$	$(m+n)[(a+b)(228.516^a 343.75^b) - (c+t)\,228.516 - (c+t)343.75]$
Two-part tariff		$m[1150.391a + b(343.75)^{b-1}(665.039) + (343{,}75)^b] + n\,[228.516a + (343.75)^b - 572.226(c+t)]$	$m[(a228.516^{a-1}343.75^b)(1150.391 - 228.516) + (b228.516^a 343.75^{b-1})(1008.789 - 343{,}75) - (1150.391 + 1008.789)(c+t)] + n[(228.516 + 343.75)(c+t)] + (m+n)\,(228.516^a 343.75^b)$

Based on the interpretations of Table 1 to Table 3, by applying data obtained from local server, the original Cobb-Douglas utility function [15] is more optimal than the perfect substitute discussed previously [16] and quasi linear utility function, based on the type of pricing scheme usage-based, which is used to obtain the maximum profit with optimal price for homogeneous and heterogeneous (high-end and low-end) consumers; for heterogeneous consumers (high-demand and low-demand), the type of two-part tariff pricing scheme obtain maximum profit with optimal price.

The fact of these results is due to the general form of Cobb-Douglas function is in exponential form. It is nonlinear function, in nature. If quasi linear or perfect substitues functions which are linear or half linear functions are compared, this graph of Cobb-Douglas function has many possible local optimal in certain range. Also, comparing to previous research conducted [9] that stated for homogenous case, usage based reached highest optimal solution, the original Cobb-Douglas has also the same results. But the original Cobb-Douglas is still powerfull with the highest optimal solution compared to modified Cobb-Douglas function. This is again, due to the fact that exponential form of function has more peaks on certain range of local optimal solutions. Other aspect that can be showed that, also in previous research [9], [17] marginal and monitoring cost for heterogeneous consumers, are not really discussed, the research mostly focused on pricing strategies for information service without marginal and monitoring cost.

4. CONCLUSION

From the results, it can be concluded that by applying Cobb-Douglas utility function, will impact on higher revenue for ISP on usage based pricing strategies for homogeneous and heterogeneous consumers for high end and low end users. Two part tariff pricing strategy is best strategy to be applied in heterogeneous case of high and low demand users.

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