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Decision-Making Model for Student Assessment by Unifying Numerical and Linguistic Data

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ABSTRACT

Learning assessment deals with the process of making a decision on the quality or performance of student achievement in a number of competency standards. In the process, teacher's preferences are provided through both test and non-test, generally in a numeric value, from which the final results are then converted into letters or linguistic value. In the proposed model, linguistic variables are exploited as a form of teacher's preferences in non-test techniques. Consequently, the assessment data set will consist of numerical and linguistic information, so it requires a method to unify them to obtain the final value. A model that uses the 2-tuple linguistic approach and based on matrix operations is proposed to solve the problem. This study proposed a new procedure that consists of four stages: preprocessing, transformation, aggregation and exploitation. The final result is presented in 2-tuple linguistic representation and its equivalent number, accompanied by a description of the achievement of each competency. The α value of 2-tuple linguistic in the final result and in the description of each competency becomes meaningful information that can be interpreted as a comparative ability one student has related to other students, and shows how much potential is achieved to reach higher ranks. The proposed model contributes to enrich the learning assessment techniques, since the exploitation of linguistic variable as representation preferences provides flexible space for teachers in their assessments. Moreover, using the result with respect to students' levels of each competency, students' mastery of each attribute can be diagnosed and their progress of learning can be estimated.

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1. INTRODUCTION

Decision-making in education have evolved to help people in many purposes, such as for instructional decision, curricular decision, selection decision, placement or classification decision and personal decision [1]. An example of educational decision is selection of courses in the higher education [2]. These purposes are implemented in many types of educational decision-making; one of them is Credentialing and Certification Decisions that decides whether or not students have met certain standards [3]. This opinion is supported by [4] which states that the assessment of education related to the process of giving a decision on the quality or performance of student's achievement. This decision-making is done through a process of learning assessment on a number of competency standards.

Generally, assessment method is designed to evaluate many competencies but the final result is merely a grade without any description about student's achievement in every competency. The method can be unfair if not considering many specializations of a specific competency. Mossin *et al* [5] have proposed a model of evaluation method based on fuzzy sets which can determine the capabilities and the deficiencies of a student in different areas of knowledge in industrial automation.

The assessment process is conducted through various techniques, both test and non test, and usually the result is given in numerical value which is then interpreted into a letter or linguistic variable. Linguistic variable is a variable whose value is not numbers but words or sentences describing the competency and the words are characterized by fuzzy sets defined in the universe defined set [6]. Valuation in non-test assessment techniques such as assignments and observations is quite possible or even more appropriate if presented using linguistic variables. Sometimes, some assignments and observations would be easier to assessby means of linguistic variables because such valuations cannot be ascertained by numeric scores. Considering that possibility, it is proposed to use linguistic variables not to represent qualitative aspect, but to represent teacher's preferences in the non-test assessment techniques. Thereby, teachers can assess using linguistic variables, in the case that has been done using numerical value.

As a consequence of using the linguistic variables, assessment data set will consist of numerical and linguistic information, so it needs a procedure to combine the two types of data to obtain the final result. There have been studies, which are initiated by Herrera and Martinez [7] that combines numeric and linguistic variables and represents unification results in 2-tuple linguistic approach. This 2-tuple linguistic approach is better than other linguistic approaches to overcome the problem of combining linguistic and numerical values. Unification result of other linguistic approaches usually does not exactly match any of the initial linguistic terms, and needs an approximation process to express the result in the initial expression domain. This consequently produces the loss of information and hence causes the lack of precision, but it can be well handled by the 2-tuple linguistic approach.

Considering some problems described above, it is important to develop a robust assessment method which can accommodate the use of linguistic variables in some assessment techniques in such a way that the final result can describe students' strong and weak points in every competency. Some ideas implemented in the previous studies are combined to define solution for the problem.

The aim of this paper is to extend the concept of solving Multi Criteria Decision Making (MCDM) problems under linguistic environment, to solve the problems of learning competency evaluation. The extension includes information about determining weights of learning competency, using linguistic variables to value students' performance in some assessment techniques, combining numeric and linguistic data and informing the student's excellence in a specific competency, but did not succeed in another competency. In order to do this, the remaining part of this paper is organized as follows: In Section 2, some basic definitions of the 2-tuple fuzzy linguistic approach and some aggregation operators are briefly reviewed. Section 3 describes some basic definitions to integrate numeric and linguistic. The proposed method to solve the problems based on unifying numeric and linguistic is presented in Section 4. Section 5 presents result and analysis and finally the paper is concluded in Section 6.

2. THE 2-TUPLE FUZZY LINGUISTIC REPRESENTATION

Computational techniques for dealing with linguistic terms can be classified into three categories [8], i.e. extension principle, symbolic method, and 2-tuple fuzzy linguistic representation model. In the first two approaches, the results usually do not exactly match any of initial linguistic terms, and then an approximation process must be developed to express the result in the initial expression domain. This consequently produces a certain loss of information and hence results in the lack of precision. Herrera and Martínez [7-9], proposed the third approach, namely the 2-tuple fuzzy linguistic representation model to overcome these limitations, through2-tuples (s, α), which iscompiledby thelinguistic termss while αassessedthe numericalvaluein the interval [-0.5, 0.5].

Definition 1. The symbolic translation of a linguistic term $s_i \in S = \{s_0, \dots, s_g\}$ consists of a numerical value $\alpha_i \in [-0.5, 0.5)$ that supports the "difference of information" between a counting of information β assessed in $[0, g]$ obtained after a symbolic aggregation operation (acting on the order index of the labels) and the closest value in $\{0, \dots, g\}$ that indicates the index of the closest linguistic term in $S(s_i)$.

The linguistic representation model defines a set of functions to make transformation between linguistic terms and 2-tuples.

Definition 2. Let $s_i \in S$ be linguistic term, then its equivalent 2-tuple representation is obtained by means of the function Φ as:

$$\begin{aligned} &\Phi: S \to (S \times [-0.5,0.5)) \\ &\Phi(s_i) = (s_i, 0), \quad s_i \in S \end{aligned} \tag{1}$$

A crisp value$\beta \in [0, g]$ can be transformed into the 2-tuple linguistic variable using the following definition:

Definition 3. Let $S = \{s_0, \ldots, s_g\}$ be a linguistic term set,$\beta \in [0, g]$be a number value representing the symbolic aggregation result of linguisticterm. Then the 2-tuple that expresses the equivalent information to β is obtained using the following function:

$$\begin{aligned} &\Delta: [0, g] \to S \times [-0.5,0.5] \\ &\Delta(\beta) = (s_i, \alpha) \\ &\text{where} \begin{cases} s_i, & i = round(\beta) \\ \alpha = \beta - i & \alpha \in [-0.5,0.5) \end{cases} \end{aligned} \tag{2}$$

where round is the usual rounding operation, s_i has the closest index label to β and α is the value of the symbolic translation.

Definition 4. Let $S = \{s_0, \ldots, s_g\}$ be a linguistic term set and (s_i, α) is 2-tuple linguistic information, then there exists a function Δ^{-1} which is able to transform 2-tuple linguistic information into its equivalent numerical value $\beta \in [0, g] \subset R$:

$$\begin{aligned} &\Delta^{-1}: S \times [-0.5,0.5] \to [0, g] \\ &\Delta^{-1}(s_i, \alpha) = i + \alpha = \beta \end{aligned} \tag{3}$$

3. COMBINING NUMERICAND LINGUISTIC USING LINGUISTICAPPROACH

Let $x \in [0,1]$ is a numerical value and $S = \{s_0, \ldots, s_g\}$ a set of term linguistic. To combine numerical and linguistic values, it takes several functions that transform these values into a 2-tuple linguistic representation. Herrera and Martinez [7] have defined the function, which includes two steps, i.e. converting x into fuzzy set in S and transforming the fuzzy set into 2-tuple linguistic model assessed in S.

Definition 5. Let$x \in [0,1]$ is a numerical value and $S = \{s_0, \ldots, s_g\}$ a set of term linguistic. Transforming $x$into fuzzy set in S using τfunction defined as follows:

$$\begin{aligned} &\tau: [0,1] \to F(S) \\ &\tau(x) = \{(s_0, \theta_0), \ldots, (s_g, \theta_g)\}, s_i \in S \ and \ \theta_i \in [0,1] \end{aligned}$$

such that

$$\theta_i = \mu_{s_i}(x) = \begin{cases} 0 & if x \notin support\ (\mu_{s_i}(x) \\ \dfrac{x - a_i}{b_i - a_i} & if a_i \le x \le b_i \\ 1 & if b_i \le x \le d_i \\ \dfrac{c_i - x}{c_i - d_i} & if d_i \le x \le c_i \end{cases} \tag{4}$$

The semantic of the membership function μ_{s_i} is given b trapezoidal parametric function whose parameters are (a_i, b_i, c_i, d_i). The result will be transformed into linguistic 2-tuple using definition below.

Definition 6. Let $\tau(x) = \{(s_0, \theta_0), \ldots, (s_g, \theta_g)\}$ be a fuzzy set that represents numerical value $x \in [0,1]$ over the linguistic set $S = \{s_0, \ldots, s_g\}$. To obtain a numerical value that represents information from the fuzzy set assessed in [0,g] by means of functionχ

$$\chi: F(S) \to [0, g]$$
$$\chi(\{(s_j, \theta_j) | j = 0, \dots, g\}) = \frac{\sum_{j=0}^{g} j\theta_j}{\sum_{j=0}^{g} \theta_j} = \beta \quad (5)$$

Valueβ is transformed into 2-tuple linguistic by using the function Δ as in Equation 2.

Once obtained the transformation results in 2-tuple linguistic model, unifying process of the information is conducted using 2-tuple linguistic aggregation operator. Some aggregation operators for 2-tuple linguistic variables defined, such as arithmetic mean, weighted average, and linguistic weighted average operator [10-11]. The operators are defined as follows.

Definition 7. Let$x = \{(s_1, \alpha_1), (s_2, \alpha_2), \dots, (s_\mathrm{n}, \alpha_\mathrm{n})\}$ be a 2-tupel linguistic set, then the arithmetic mean is

$$(\bar{s}, \bar{\alpha}) = \Delta\left(\frac{1}{n}\sum_{j=1}^{n} \Delta^{-1}(s_j, \alpha_j)\right), \bar{s} \in S, \bar{\alpha} \in [-0.5, 0.5) \quad (6)$$

Definition 8. Let $\boldsymbol{x} = \{(\boldsymbol{s_1}, \boldsymbol{\alpha_1}), (\boldsymbol{s_2}, \boldsymbol{\alpha_2}), \dots, (\boldsymbol{s_\mathrm{n}}, \boldsymbol{\alpha_\mathrm{n}})\}$ be a 2-tuple linguistic set, and $W = \{w_1, w_2, \dots, w_n\}$ be their associated weights. The 2-tuple weighted average $\bar{x}^w$is

$$\bar{x}^w = \Delta\left(\frac{\sum_{i=1}^{n} \Delta^{-1}(\boldsymbol{s_i}, \boldsymbol{\alpha_i}). \boldsymbol{w_i}}{\sum_{i=1}^{n} \boldsymbol{w_i}}\right) \quad (7)$$

The results in 2-tuple linguistic should can be converted into an appropriate numerical value. There are 2 steps to convert a value of 2-tuple linguistic into a value of [0,1].

Definition 9. Let (s_k, α) be2-tuple linguisticbased onsymbolic translation, where $s_k \in S = \{s_0, \dots, s_g\}$ and $\alpha \in [-0.5, 0.5)$ whose equivalent numerical value is $\Delta^{-1}((s_k, \alpha) = \beta$ with $\beta \in [0, g]$. Function δ computes two 2-tuples based on the membership degree, from the initial 2-tuple linguistic, that support the same counting of information:

$$\delta: [0, g] \to \{S_T \times [0,1]\} \times \{S_T \times [0,1]\}$$
$$\delta(\beta) = \{(s_h, 1 - \gamma), (s_{h+1}, \gamma)\} \quad (8)$$

where$h = trunc(\beta);\ \gamma = \beta - h$

Definition 10. Let $(s_h, 1 - \gamma)\ and\ (s_{h+1}, \gamma)$ be two 2-tuple linguistic sets based on membership degree, the equivalent numerical value assessed in [0,1] is obtained using function κ

$$\kappa: \{S_T \times [0,1]\} \times \{S_T \times [0,1]\} \to [0,1]$$
$$\kappa\{(s_h, 1 - \gamma), (s_{h+1}, \gamma)\} = CV(s_h)(1 - \gamma) + CV(s_{h+1})(\gamma) \quad (9)$$

CV (.) is a function providing characteristic value. The result is a crisp value that summarize the information given by a fuzzy set $\mathrm{v_i}$, one of them is maximum value (MV).

Definition 11. If given label $\mathrm{s_i}$with the membership degree $\mu_{\mathrm{ys_i}}(\mathrm{v}), \mathrm{v} \in \mathrm{V} = [0,1]$, *height* is defined as $height(s_i) = Sup\{\mu_{\ ysi}(v), \forall v\}$. Therefore CV (.) of maximum value is defined as $MV(s_i) = \{v | \mu_{(si)}(v) = height(s_i)\}$[12].

4. PROPOSED MODEL

In the proposed model, the weight is assigned to the learning competencies, not to each of assessment techniques, and determined using a specified method. This model proposes to exploit linguistic variable to assess student's performance in multiple valuation techniques such as assignment, daily tests, daily observations (participation), presentations and portfolios. For this purpose, the representation of the linguistic variable must be defined.

The set of linguistic variables is defined on the basis of exposure to Herrera and Herrera-Viedma [13]. In view of this, a linguistic term set, $S = \{s_o, s_1, s_2, s_3, s_{4,}, s_5, s_6\}$ with seven labels used in the proposed model can be defined as follows and the semantic is described in Table 1.

Table 1. Linguistic Terms and their Semantics

Symbol	Abb.	Linguistic term	Triangular Fuzzy Number
s_0	VP	Very poor	(0,0,0.17)
s_1	P	Poor	(0, 0.17, 0.33)
s_2	A	Average	(0.17, 0.33, 0.5)
s_3	AA	Above average	(0.33, 0.5, 0.67)
s_4	G	Good	(0.5, 0.67, 0.83)
s_5	VG	Very good	(0.67, 0.83, 1)
s_6	E	Excellent	(0.83,1,1)

The cardinality of linguistic term set S in a limited number of grades must be defined appropriately. It could be small enough but does not reduce the precision of the value and it should be rich enough to allow discrimination of the performances of each criterion. Based on study, the psychologists recommended the use of 7±2 labels, less than 5 being not sufficiently informative, more than 9 being too much for a proper understanding of their differences [14]. Moreover, in educational measurement seven categoriesscaleat the same distance is consistent with Thurstone'scale and has a good reliability [1].

The set of semantic linguistic terms are represented by triangular fuzzy numbers (TFN). TFN is a simple method and easily understood to represent an assessment of decision maker, and fuzzy arithmetic operations in TFN are very easy to do [15]. In addition, the membership function of TFN is considered quite reliable in showing the uncertainty of linguistic assessmentwhich usually represents estimated subjective assessment of decision makers [16]. Uncertainty is one source of measurement errors, which in educational measurement is considered and represented as shown in the Equation

$$X = T + e$$

The Equation shows that observed score (X) consists of true score (T) and measurement error (e) [1].

Learning assessment is conducted to assess a number of competency standards. For example, Table 2 shows the math competency standards for high school of class X in the first semester.

Table 2. Competency Standardsof Math class X semester 1

Standard of Competency	Basic Competency
1. Solve problems related to the power, roots, and logarithms	1.1. Using the rules of power, roots, and logarithm 1.2. Performing algebraic manipulations in computation involving power, roots, and logarithms
2. Solve problems related to the functions, equations and quadratic functions and quadratic inequality	2.1 Understanding the concept of a function 2.2 Drawing a graph of simple algebra and quadratic functions 2.3 Using the properties and rules of quadratic equations and inequalities 2.4 Performing algebraic manipulation in computation related to quadratic equations and inequalities 2.5 Designing a mathematical model of a problem related to equality and / or a quadratic function 2.6 Solving mathematical model of a problem related to equality and / or quadratic functions and their interpretation
3. Solve problems associated with linear equations system and one variable inequalities	3.1 Solving of 2-variables linear equations system and 2-variables mixed linear and quadratic equations system 3.2 Designing a mathematical model of a problem associated with linear equations system 3.3 Solving mathematical models of a problem related to linear equations systems and its interpretation 3.4 Completing one-variable inequality that involves algebraic fractions 3.5 Designing a mathematical model of problems associated with one-variable inequality 3.6 Solving mathematical models of problems associated with one -variable inequality and its interpretation

Let $A=\{a_1,...a_m\}$ be the set of students who will be assessed based on several standard of competencies $C=\{c_1,...,c_n\}$ which are described into some basic competencies $B=\{b_1,...,b_k\}$. Learning assessment is conducted using several techniques $T=\{t_1,...,t_h\}$ in which the number of the type (h) varies according to the basic competency. For example t_1 denotes for test, $t_2$for observation, and t_3 for assignment.

The value in each t_i may be either numerical $x \in [0,1]$ or linguistic $S=\{s_0,\ldots,s_g\}$. For this purpose, the numerical value assigned by the teacher are commonly presented in [0,10] or [0,100] and must then be transformed into [0,1]. This scheme is illustrated in Table 3.

Table 3. Scheme of assessment problem with numeric and linguistic values

	Competency Standard c_1												Competency Standard c_n											
Alternative (a_i)/ students	Basic Competencyb_1				Basic Competencyb_2				Basic Competency…				Basic Competency…				Basic Competencyb_{k-1}				Basic Competencyb_k			
	t_1	t_2	…	t_h	t_1	t_2	…	t_h	t_1	t_2	…	t_h	t_1	t_2	…	t_h	t_1	t_2	…	t_h	t_1	t_2	…	t_h
a_1	x_{11}	s_{12}	…																					
⋮																								
a_m																								

Steps for determining the final value of the assessment data set are prepared using 2-tuple linguistic approach and based on the matrix operations. Set of values within each basic competency (marked by a red box) can be presented into a decision matrix $R^B = (\mathrm{r_{ij}})_{\mathrm{m\times h}}$, where $\mathrm{r_{ij}} \in [0,1]$ if using numerical values and $\mathrm{r_{ij}} \in s_i = \{s_0, s_1, \ldots, s_g\}$ if using linguistic values.

$$R^B = \begin{bmatrix} \mathrm{r_{11}} & \cdots & \mathrm{r_{1h}} \\ \vdots & \vdots & \vdots \\ \mathrm{r_{m1}} & \cdots & \mathrm{r_{mh}} \end{bmatrix}, \mathrm{B} = 1,2,\ldots,\mathrm{k} \tag{10}$$

where h varies between each basic competency.

There are two main phases of a common decision resolution scheme, i.e. the aggregation phase that aggregates the values provided by the experts to obtain a collective assessment for the alternatives; and exploitation phase of the collective assessments to rank, sort the best one/samong the alternatives [17]. For the proposed model, the stagesare carried outin a2-tuple linguistic forthe evaluation of learning competency is developed based on the common phase with several improvements. Figure 2 shows a basic scheme of this approach, whose steps are further described below.

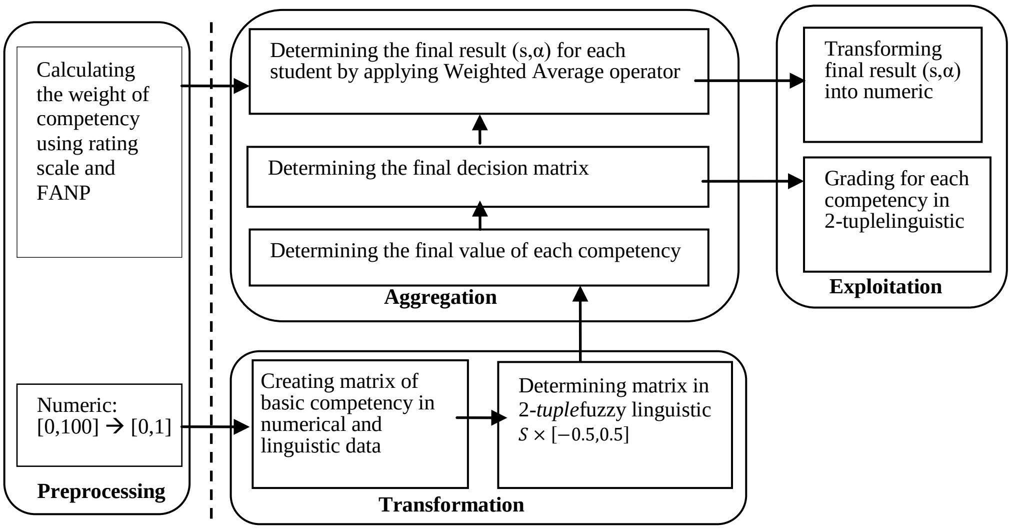

Figure 2. Steps of the proposed method

4.1. Preprocessing

There are two processes carried out at this stage, i.e. transforming numerical value into [0,1] and determining the weight of learning competencies. In general, the numerical value assigned by the teacher presented in [0,10] or [0,100]. For the proposed model, the value must be transformed into [0,1]. On the other hand, the weights of the learning competencies are determined using a combination of rating scale [18] with Fuzzy Analytic Network Process (FANP). This combined method is developed based on some research, i.e.

application of FANP in requirements in the house of quality [19], An IT project selection [20], Total Quality Management [21], and selection of facility location [22]. This combined method also considers some drawbacks of ANP [23-25] and enhance linguistic scale which is used in a pair wise comparison matrix for FANP based on the previous one [25], [27].

4.2. Transformation: Transforming Numerical and Linguistic Information Into 2-tuple Fuzzy Linguistic

Preferences given by the teacher for the student consisting of numerical and linguistic information are presented in the form of a decision matrix $\mathrm{R^B} = (\mathrm{r_{ij}})_{\mathrm{m\times h}}$. Each element in the matrix is then transformed into 2-tuple linguistic to obtain matrix $R_{L2T}^{B} = ((\mathrm{s},\alpha)_{\mathrm{ij}})_{\mathrm{m\times h}}$. The transformation is performed using Equation 1 for linguistic information; and Equation 4, 5 and 2 for numerical information.

4.3. Aggregation

4.3.1. Determining the final value of each basic competency for every student

The final value of each basic competency for every student is determined by calculating the average of each row in the 2-tuple linguistic decision matrix $R_{L2T}^{B} = ((\mathrm{s},\alpha)_{\mathrm{ij}})_{\mathrm{m\times h}}$ using Equation 6 in order to obtain a matrix column $\bar{R}_{L2T}^{B}$

$$\bar{R}_{L2T}^{B} = [(\bar{\mathrm{s}},\bar{\alpha})_{\mathrm{m\times 1}}]^{\mathrm{B}} = \begin{bmatrix} (\bar{\mathrm{s}},\bar{\alpha})_1 \\ \vdots \\ (\bar{\mathrm{s}},\bar{\alpha})_{\mathrm{m}} \end{bmatrix}^{\mathrm{B}}$$

$$\text{where } (\bar{\mathrm{s}},\bar{\alpha})_{\mathrm{i}} = \Delta\left(\frac{1}{\mathrm{h}}\sum_{\mathrm{j=1}}^{\mathrm{h}} \Delta^{-1}(\mathrm{s_{ij}},\alpha_{\mathrm{ij}})\right), \mathrm{i} = 1,\dots,\mathrm{m}\,; \quad \bar{\mathrm{s}} \in \mathrm{S}; \quad \bar{\alpha} \in [-0.5,0.5)$$

4.3.2. Determining the final decision matrix

The final decision matrix is a set of final value of each basic competency. Therefore it is composed by merging k column matrix obtained in step (c).

$$\bar{\bar{R}} = \bar{R}_{L2T}^{1} \cup \bar{R}_{L2T}^{2} \cup \dots \cup \bar{R}_{L2T}^{k}$$

$$\bar{\bar{R}} = \begin{bmatrix} (\bar{\mathrm{s}},\bar{\alpha})_1^1 & (\bar{\mathrm{s}},\bar{\alpha})_1^2 & \cdots & (\bar{\mathrm{s}},\bar{\alpha})_1^{\mathrm{k}} \\ \vdots & \vdots & \cdots & \vdots \\ (\bar{\mathrm{s}},\bar{\alpha})_{\mathrm{m}}^1 & (\bar{\mathrm{s}},\bar{\alpha})_{\mathrm{m}}^2 & \cdots & (\bar{\mathrm{s}},\bar{\alpha})_{\mathrm{m}}^{\mathrm{k}} \end{bmatrix}$$

4.3.3. Aggregating the information and the degree of importance of evaluation competency using weighted average operators to obtain the final results

At this stage the 2-tuple linguistic information for all of the attributes obtained by any alternative would be aggregated into a single value, which means aggregating each row in the final decision matrix. Since each attribute (basic competency) in the assessment data has an important weight, and then the weight is taken into account in the process of aggregation by using a weighted average operator (Equation 7).

4.4. Exploitation

4.4.1. Describing the achievement level of each competency

Each column inthe final decision matrix $\bar{\bar{R}}$shows the value of *m* students for each basic competency. Ma and Zhou [28] have proposed a method called fuzzy grading system by transforming numerical values into corresponding letter grades, based on membership degree of a fuzzy function. Moss in *et al* [5] uses a classification system based on fuzzy rule to assess each competency. Thereby, the final decision matrix $\bar{\bar{R}}$ represents a combination of the two ideas. Thus, each cell in $(\bar{\mathrm{s}},\bar{\alpha})_{\mathrm{i}}^{\mathrm{B}}, \mathrm{i} = 1,\dots,\mathrm{m}; \mathrm{B} = 1,\dots\mathrm{k}$ in the final decision matrix shows achievement of student $\mathrm{i^{th}}$ in basic competency $\mathrm{b^{th}}$ and will be elaborated as the achievement level of each student of each basic competency.

4.4.2. Transforming the final results into the final pertinent numerical values

Numerical final score which is equivalent with final result in 2-tuple linguistic is still needed in a final report. Therefore, to complete the process, the last stage is converting the final 2-tuple linguistic into numeric information using Equation 3, 8 and 9.

5. RESULTS AND ANALYSIS

Supposea teacher will assess students' competencies in a course which has 14 basic competencies as depicted in Table 2 which will be assessed by means of three kinds of evaluation techniques, i.e. test, assignments and observation. Each of the techniques can be conducted once or more. If, for example, there are six students, $A_i, i = 1, \ldots, 6$, from the assessment data set of fourteen basic competencies as in Table 2. The linguistic term set S used in the assessment is defined as in Table 1.

The decision matrix which is used to represent the assessment value of six students for each basic competency is $\mathrm{R}^{\mathrm{B}} = \left(\mathrm{r_{ij}}\right)_{6xh} (B = 1,2,\ldots,14)$, where $r_{ij} \in [0,1]$, or $r_{ij} = s_{ij} \in S, i = 1,..,6;$ $j = 1,..,$h. The fourteen decision matrices can be obtained for the fourteen relevant basic competencies of the six students. For example, the first three decision matrices are shown below.

$$\mathrm{R}^1 = \begin{vmatrix} 0.8 & E & G \\ 0.6 & A & A \\ 1 & E & VG \\ 0.7 & G & A \\ 0.6 & G & A \\ 0.8 & VG & G \end{vmatrix} \quad \mathrm{R}^2 = \begin{vmatrix} 0.7 & G & VG & VG \\ 0.5 & P & AA & A \\ 0.9 & G & G & VG \\ 0.7 & G & AA & G \\ 0.6 & G & A & A \\ 0.7 & AA & A & G \end{vmatrix} \quad \mathrm{R}^3 = \begin{vmatrix} 0.8 & G & E \\ 0.7 & A & A \\ 0.9 & G & G \\ 0.7 & G & AA \\ 0.7 & A & P \\ 0.9 & G & E \end{vmatrix}$$

Each of the decision matrices is then transformed into2-tuple linguistic matrix. The transformation results for the three example matrices above are shown below.

$$R_{L2T}^1 = \begin{vmatrix} (VG,-0.19) & (E,0) & (G,0) \\ (G,-0.41) & (A,0) & (A,0) \\ (E,0) & (E,0) & (VG,0) \\ (G,0.19) & (G,0) & (A,0) \\ (G,-0.41) & (G,0) & (A,0) \\ (VG,-0.19) & (VG,0) & (G,0) \end{vmatrix} \quad R_{L2T}^2 = \begin{vmatrix} (G,-0.12) & (G,0) & (VG,0) & (VG,0) \\ (AA,-0.29) & (P,0) & (AA,0) & (A,0) \\ (VG,0.41) & (G,0) & (G,0) & (VG,0) \\ (G,0.19) & (G,0) & (AA,0) & (G,0) \\ (G,-0.41) & (G,0) & (A,0) & (A,0) \\ (G,0.19) & (AA,0) & (A,0) & (G,0) \end{vmatrix} \quad R_{L2T}^3 = \begin{vmatrix} (VG,0.06) & (G,0) & (E,0) \\ (G,0.19) & (A,0) & (A,0) \\ (VG,0.41) & (G,0) & (G,0) \\ (G,0.19) & (G,0) & (AA,0) \\ (G,0.19) & (A,0) & (P,0) \\ (VG,0.41) & (G,0) & (E,0) \end{vmatrix}$$

For every 2-tuple linguistic decision matrix, the average of each row will be determined using Equation 6, and results a column matrix. There are fourteen column matrices which are later merged into the final decision matrix, whose results are presented in Table 4.

Table 4. The Final Decision Matrix

b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9	b_{10}	b_{11}	b_{12}	b_{13}	b_{14}
(VG,-0.06)	(G,0.47)	(VG,0.02)	(G,0)	(G,0.14)	(G,-0.17)	(AA,0.21)	(VG,0.43)	(VG,-0.33)	(E,-0.27)	(G,0.06)	(G,0.37)	(VG,0.43)	(VG,-0.2)
(AA,-0.47)	(A,0.18)	(AA,-0.27)	(AA,0.33)	(A,0.06)	(A,-0.04)	(A,-0.43)	(AA,0.49)	(AA,-0.37)	(G,-0.22)	(A,0.02)	(A,0.47)	(AA,0.41)	(G,-0.14)
(E,-0.33)	(VG,-0.4)	(G,0.47)	(VG,0.43)	(E,-0.29)	(VG,-0.24)	(E,-0.2)	(G,0.45)	(VG,-0.06)	(VG,-0.44)	(VG,-0.17)	(G,0.27)	(G,0.27)	(VG,0.04)
(AA,0.4)	(G,-0.2)	(G,-0.27)	(AA,-0.17)	(AA,0.06)	(G,0.33)	(VG,0)	(G,-0.2)	(VG,-0.33)	(G,-0.33)	(AA,-0.14)	(A,0.2)	(AA,-0.02)	(G,0)
(AA,0.2)	(AA,-0.1)	(A,0.4)	(A,0.29)	(A,0)	(G,0.33)	(VG,-0.33)	(G,0.25)	(VG,-0.33)	(AA,-0.14)	(AA,0.2)	(G,-0.06)	(AA,-0.37)	(G,0.29)
(VG,-0.4)	(AA,0.3)	(VG,0.14)	(VG,-0.4)	(VG,-0.33)	(VG,0)	(VG,0)	(VG,0.25)	(G,0.27)	(VG,-0.33)	(VG,-0.27)	(G,0.27)	(G,0.06)	(VG,0.43)

Table 5. The final results of student's assessment

Stu#	Numeric score	2-TL score	Description
1	77.86	(VG,-0.32)	Very good, although still needs 32% more to achieve this rank
2	44.28	(AA,-0.34)	Above average, although still needs 34% more to achieve this rank
3	83.53	(VG,0.03)	Very good, has a potential of 3% to achieve higher ranks
4	59.95	(G,-0.41)	Good, although still needs 41% more to achieve this rank
5	55.82	(AA,0.34)	Above average, has a potential of 34% to achieve higher ranks
6	75.44	(VG,-0.47)	Very good, although still needs 47% more to achieve this rank

Suppose the weight of fourteen competencies obtained from the calculation using combination of rating scale and FANP methods is w = {0.257 0.136 0.048 0.043 0.043 0.043 0.043 0.056 0.061 0.061 0.061 0.045 0.045 0.061}. Finally, the final results of student's assessment in the form of the 2-tuple linguistic are determined using Equation 7. To accomplish the results, the final 2-tuple linguistic scoreis converted into numeric using Equation 3, 8 and 9. The meaning of the score can be described referring to the definition of α as stated in Definition 1. The results are shown in Table 5.

Based on Table 5, the student rankings are: student #3> student #1> student #6> student #4> student #5> student #2.Table 6 shows the detail result of each competency for student#1. The score of each competency can be seen in the first row of Table 4.

From Table 6, the final value of student #1 is 77.86 and (VG, -032). The result shows that student #1 is in category of Very Good (VG), although she/he still needs 32% to reach the grade VG, indicated by a value of α = -0.32.

Table 6. Detailed result for student#1

Stu#	Numeric score	2-TL score	2-TL based membership degree	Description
1	77.86	(VG,-0.32)	(G,0.32) , (VG,0.68)	Very good, although it still needs 32% more to achieve this rank
Details:				
Competency#		2-TL score	Description	
1. C1.1		(VG, -0.06)	Very good, although still needs 6% more to achieve this rank	
2. C1. 2		(G,0.47)	Good, has potential of 47% to achieve higher ranks	
3. C1.3		(VG,0.02)	Very Good, has potential of 2% to achieve higher ranks	
4. Etc.				

The final results using a 2-tuple linguistic provide meaningful information about student's achievement. Value α on the linguistic 2-tuples in the final value of a student can be interpreted as a comparison of ability with other students if they are in the same category, and show how much potential the student has to reach higher ranks.

Generally, the final result of learning process is determined by the arithmetic average method. The method is a statistical indicator that represents the central tendency of data. The computation is simple and commonly used in student learning assessment problem. However, this method is strongly influenced by outliers and cannot deal with quantitative and qualitative data simultaneously [29]. Therefore this study proposes a model that uses 2-tuple linguistic approach to cover the weaknesses of the method.

According to [9], the 2-tuples linguistics model has been applied in many fields.However it has not been used in the assessment of learning. In [29], a problem of higher education students' selection is solved using the 2-tuples linguistics approach and it shows that this approach provides information about students' rank more accurately and reasonably, furthermore, it does not lose any valuable information from the commentator.

The results of this study in-line with the results of the previous one. In addition, this study also offers several significant findings, namely: 1) a new procedure for determining the final result which combines numerical and linguistic data, 2) the opportunity of using linguistic variable instead of numeric for some technical evaluations which consider psychological factors, so that it is possible to integrate the attitude assessment into cognitive one which is usually conducted separately, 3) the meaningfulness of the results since value α of the final results inform the degree of ability and to reach higher ranks, 4) the richness of the results since it evaluates students with respect to their levels of competence in each attribute such as knowledge or skills. Using this result, students' mastery of each attribute can be diagnosed and their progress of learning can be estimated. According to [30], the result supports the purpose of Cognitive Diagnosis Theory, which is to provide students, teachers, or parents-individual feedback regarding to student's mastery of eachcompetency measured by the assessment.

6. CONCLUSION

The modeling and handling of linguistic information are crucial in the assessment of learning since there are qualitative aspects included in the assessment. Therefore a method using 2-tuple linguistic representations to compute the final score of assessment that involves numerical and linguistic information has been proposed.

Value α of the 2-tuple linguistic in the final result and in the description of each competency becomes meaningful information that can be interpreted as a comparative ability one student has related to other students, and shows how much potential is achieved to reach higherranks. The proposed model contributes to enrich the learning assessment techniques, due to the exploitation of linguistic variable as representation preferences, providing flexible space for teachers in their assessments. Moreover, the model gives evaluation result with respect to students' levels of competence in each attribute such as knowledge or skills. Using this result, students' mastery of each attribute can be diagnosed and their progress of learning can be estimated.

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