

# A General Method to Parameter Optimization for Highly Efficient Wireless Power Transfer

Kazuya Yamaguchi<sup>1</sup>, Takuya Hirata<sup>2</sup>, and Ichijo Hodaka<sup>3</sup>

<sup>1</sup>Department of Control Engineering, National Institute of Technology, Nara College, Japan

<sup>2</sup>Interdisciplinary Graduate School of Agriculture and Engineering, University of Miyazaki, Japan

<sup>3</sup>Department of Environmental Robotics, Faculty of Engineering, University of Miyazaki, Japan

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## ABSTRACT

This paper proposes a new and general method to optimize a working frequency and a load resistance in order to realize highly efficient wireless power transfer. It should be noticed that neither resonant frequency nor matched impedance maximizes efficiency of wireless power transfer circuit, in general. This paper establishes a mathematical model of a commonly used wireless power transfer circuit, and derives a mathematical expression of circuit efficiency which involves a working frequency, a load resistance and the other parameters as symbols. This enables us to find the optimal working frequency and load resistance. The result of this paper is compared with results by a method based on resonance and impedance matching, and then clarified by a numerical example.

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## Corresponding Author:

Ichijo Hodaka

Department of Environmental Robotics, Faculty of Engineering, University of Miyazaki

1-1, Gakuen Kibanadai Nishi, Miyazaki, 889-2192, Japan

Email: hijhodaka@cc.miyazaki-u.ac.jp

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## 1. INTRODUCTION

Supplying electric power to electric devices generally needs electric wires. Transferring electric power without electric wires is called wireless power transfer (WPT), where we can avoid messy cable connections and reduce possibility of getting an electric shock. The method to transfer electric power from a voltage source to a load is based on electromagnetic phenomena between transmitting and receiving sides, which originates from the work[1].

Stimulated by the work[2], numerous number of researches about WPT have been reported. WPT with high efficiency expressed in terms of coupling coefficient and quality factor is used for a medical application[3]. A load resistance which realizes high efficiency is applied for a DC-DC converter in an implanted system[4]. A relay circuit between a transmitting circuit and a receiving circuit is proposed to improve efficiency of WPT[5]. An oscillation circuit is utilized as a transmitting circuit for a transmission with a DC power supply targeted to implants[6].

Many papers use the idea of impedance matching to maximize power at a load, which is reviewed in [7]. It is realized by matching impedance of a load to output impedance of a power source. Since matched impedance includes a working frequency, many papers choose a resonant frequency as the working frequency to generate a sinusoidal voltage input at the power source. This choice enables to make reactance of matched impedance zero, and match the real parts of output impedance of the power source to a resistive load. On the other hand, it is pointed out in [8] that efficiency which is defined as the ratio of the load power and the input power is not always maximized with resonance. There are a superior load and frequency of input than a load and frequency with impedance matching and resonance to obtain high efficiency. Therefore we should adjust the working frequency of power source to a frequency regardless of resonant frequency.

In this paper, the optimal load resistance based on a mathematical expression of efficiency to maximize

efficiency of a WPT circuit is newly obtained by using the state space approach which represents the behavior of circuit as a set of differential equations. This explains that methods by impedance matching and resonance in the other literature are not optimal in a sense of parameter optimization. Finally, a numerical calculation of efficiency is shown to compare our method with the other method for efficient WPT.

## 2. ANGULAR FREQUENCY AND LOAD RESISTANCE FOR MAXIMAL EFFICIENCY

In this paper, the circuit on Figure 1 is analyzed.

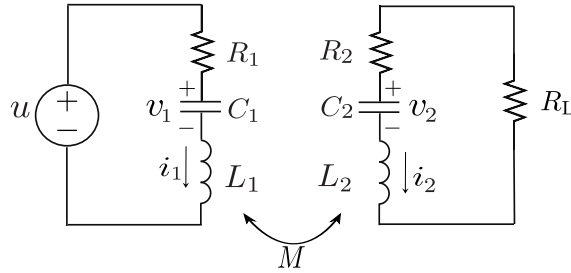


Figure 1. A simple WPT circuit

On the above circuit, the left side is called transmitting side, and the right side is called receiving side.  $u$  is a sinusoidal input for WPT.  $R_1, R_2, C_1, C_2$  are parasitic factors of the circuit.  $L_1$  and  $L_2$  are self inductances, and  $M$  is the mutual inductance between  $L_1$  and  $L_2$ .  $R_L$  is a load resistance which consumes energy.  $v_1$  and  $v_2$  are the voltage of  $C_1$  and  $C_2$ , and  $i_1$  and  $i_2$  are the current of  $L_1$  and  $L_2$ .

In this section, we attempt to maximize efficiency which is defined by the ratio of the load power and the input power. To maximize efficiency, a mathematical expression of efficiency is expressed by composing a mathematical model as follows.

$$\dot{x} = Ax + Bu, \quad x = [v_1 \quad v_2 \quad i_1 \quad i_2]^T \quad (1)$$

$$A = \frac{1}{\Delta} \begin{bmatrix} 0 & 0 & \frac{\Delta}{C_1} & 0 \\ 0 & 0 & 0 & \frac{\Delta}{C_2} \\ -L_2 & M & -R_1 L_2 & R_3 M \\ M & -L_1 & R_1 M & -R_3 L_1 \end{bmatrix}, \quad B = \frac{1}{\Delta} \begin{bmatrix} 0 \\ 0 \\ L_2 \\ -M \end{bmatrix}$$

$$\Delta = L_1 L_2 - M^2, \quad R_3 = R_2 + R_L.$$

Such model is called the state space equation, and  $v_1, v_2, i_1,$  and  $i_2$  are state variables which represent the behavior of the circuit in the model. The mathematical expression of efficiency is described by finding the state solution from the model as below.

$$\eta = \frac{R_L M^2 C_2^2 \omega^4}{(R_1 L_2^2 + R_3 M^2) C_2^2 \omega^4 + R_1 (-2L_2 + R_3^2 C_2) C_2 \omega^2 + R_1} \quad (2)$$

where  $\omega$  is the angular frequency of  $u$ . Then we assume  $\omega$  and  $L_1, L_2, C_1, C_2$  as

$$\omega = \omega_0 = \sqrt{\frac{1}{L_1 C_1}} = \sqrt{\frac{1}{L_2 C_2}}. \quad (3)$$

If the condition (3) is satisfied, the efficiency  $\eta_0$  is determined as

$$\eta_0 = \frac{R_L M^2}{R_3 (M^2 + R_1 R_3 L_2 C_2)}. \quad (4)$$

From expression (4), the load resistance  $R_{L0}$  which maximizes expression (4) is derived as below[9].

$$R_{L0} = \sqrt{R_2 \left( R_2 + \frac{(\omega_0 M)^2}{R_1} \right)}. \quad (5)$$

The maximal efficiency under the condition (3) is found by adjusting  $R_L$  to  $R_{L0}$  in (5). However the condition (3) is not always fulfilled, and therefore the standard condition which maximizes efficiency should be derived.  $\eta$  is seen to be maximized at

$$\omega = \omega_{\text{opt}} = \frac{1}{\sqrt{L_2 C_2}} \sqrt{\frac{2L_2}{2L_2 - R_3^2 C_2}}. \quad (6)$$

by solving  $\partial\eta/\partial\omega = 0$  and assuming  $2L_2 - R_3^2 C_2 > 0$ [10]. If  $2L_2 - R_3^2 C_2 \leq 0$ , efficiency increases monotonically as  $\omega$  increases, and has no maximum. If  $R_3$  approaches to zero,  $\omega_{\text{opt}}$  in (6) approaches to  $\omega_0$  in (3). Although a load is normally attached to many WPT circuits, and hence these expressions imply different values each other. Similarly the load resistance  $R_{L\text{opt}}$  which maximizes  $\eta$  is found by solving  $\partial\eta/\partial R_L = 0$  in the following.

$$R_{L\text{opt}} = \sqrt{R_2 \left( R_2 + \frac{(\omega M)^2}{R_1} \right) + \left( \omega L_2 - \frac{1}{\omega C_2} \right)^2}. \quad (7)$$

If  $\omega = 1/\sqrt{L_2 C_2}$ ,  $R_{L\text{opt}}$  in (7) becomes same as  $R_{L0}$  in (5). It has been revealed that the expressions  $\omega_0$  and  $R_{L0}$  which are adopted as an ideal angular frequency and load resistance are the limited conditions in  $\omega_{\text{opt}}$  and  $R_{L\text{opt}}$  which are found in this paper. Then we should clarify which angular frequency and load resistance are appropriate for efficient WPT.

### 3. OPTIMAL LOAD RESISTANCE TO REALIZE HIGH POWER

We discuss a condition for high power of WPT. As a method to improve the power of a load, impedance matching which is used by fitting the load impedance to the complex conjugate of output impedance of input is well known. If impedance matching is realized, the maximal power of load is obtained.

For the circuit at Figure 1, the condition for impedance matching is examined. The equivalent circuit of Figure 1 is described as below[11, 12].

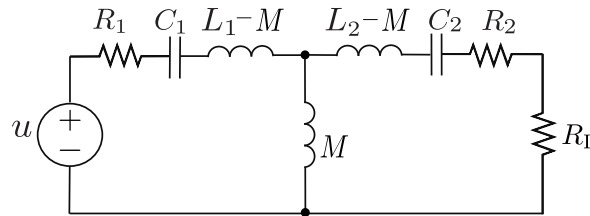


Figure 2. The equivalent circuit of Figure 1

The synthetic impedance  $Z$  seen from  $R_L$  is derived from Figure 2.

$$Z = \frac{\omega^2 C_1 C_2 (R_2 k_1 + R_1 k_2) - j\{\omega^4 M^2 C_1 C_2 + \omega^2 C_1 C_2 (R_1 R_2 - k_1 k_2) + \omega(C_1 k_1 + C_2 k_2) - k_1 - k_2\}}{\omega^2 C_1 C_2 (k_1 - jR_1)}. \quad (8)$$

$$k_1 = \left( \omega L_1 - \frac{1}{\omega C_1} \right), k_2 = \left( \omega L_2 - \frac{1}{\omega C_2} \right)$$

Then we assume  $\omega$  and  $L_1, L_2, C_1, C_2$  as (3), and  $Z$  becomes

$$Z = R_2 + \frac{M^2}{R_1 L_1 C_1}. \quad (9)$$

In terms of impedance matching, the ideal  $R_{L\text{mat}}$  which maximizes the average power of  $R_L$  is found in the following[7].

$$R_L = R_{L\text{mat}} = R_2 + \frac{M^2}{R_1 L_1 C_1}. \quad (10)$$

$R_{L\text{mat}}$  has been obtained by applying impedance matching under the condition (3). However it maximizes the power of  $R_L$  on the condition (3), and therefore  $R_{L\text{mat}}$  is not always satisfied. Moreover  $\omega_0$  in (3) has no relevance with the objective to maximize the power of load and efficiency.

#### 4. NUMERICAL CALCULATION OF EFFICIENCY

The two types of angular frequencies  $\omega_0$  and  $\omega_{opt}$ , and the two types of load resistances  $R_{L0}$  and  $R_{Lopt}$  have been derived in the previous section. Then it is examined which angular frequency and load resistance are appropriate to improve efficiency by a numerical calculation. For the numerical calculation, we set the values of elements as below.

Table 1. values of elements

$R_1$	$1\Omega$
$R_2$	$4\Omega$
$L_1, L_2$	$5.20 \times 10^{-5}\text{H}$
$M$	$1.00 \times 10^{-5}\text{H}$
$C_1, C_2$	$9.14 \times 10^{-8}\text{F}$

Then  $\omega_0 = 4.59 \times 10^5 [\text{rad/sec}]$ ,  $\omega_{opt} = 5.48 \times 10^5 [\text{rad/sec}]$ ,  $R_{L0} = 10.0 [\Omega]$ ,  $R_{Lopt} = 14.4 [\Omega]$ . On this situation, efficiency is calculated as Figure 3.

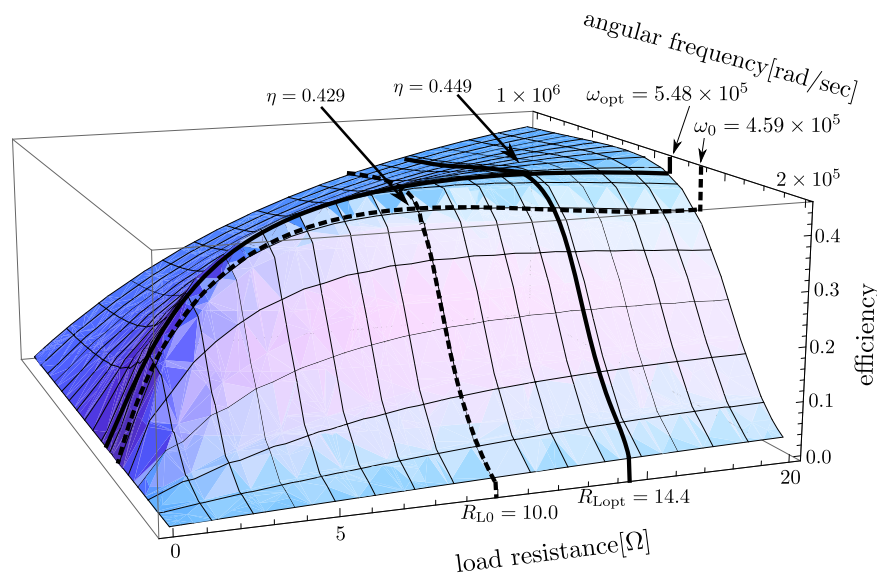


Figure 3. Efficiency by our method and the others

While efficiency at  $\omega = \omega_0$  and  $R_L = R_{L0}$  is 0.429, efficiency at  $\omega = \omega_{opt}$  and  $R_L = R_{Lopt}$  is 0.449. Many papers use  $\omega = 1/\sqrt{LC}$  as an angular frequency of input. Our result shows that we should not use  $\omega = 1/\sqrt{LC}$  but  $\omega_{opt}$ , and also, we should not use the load resistance  $R_{L0}$  but  $R_{Lopt}$  in order to obtain highest efficiency.

#### 5. CONCLUSION

In this paper, we proposed a method to accomplish the highest efficiency of wireless power transfer. We have shown efficiency by our method is indeed higher than efficiency by conventional methods using resonant phenomena and impedance matching. The key to accomplish the highest efficiency has been mathematical expressions and mathematical calculation of average powers and efficiency of wireless power transfer circuits.

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