

## A Spanning Tree Approach in Placing Multi-channel and Minimum Channel PMU's for Power System Observability

Srihari Mandava, Vanishree J, Ramesh V  
School of Electrical Engineering, VIT University, India

---

### Article Info

#### Article history:

Received Nov 18, 2014

Revised Apr 8, 2015

Accepted Apr 25, 2015

---

#### Keyword:

Minimum channel

Multichannel

Observability

PMU

Spanning tree

Tree search

---

### ABSTRACT

Synchronized phasor measurements have become the measurement technique of choice for electric power systems. They provide positive sequence voltage and current measurements synchronized to within a microsecond. The objective is to use the spanning tree approach and tree search technique for optimal placement of multichannel and minimum channel synchronized phasor measurement units (PMUs) in order to have full observability of Power System. The novel concept of depth of observability is used and its impact on the number of PMU placements is explained. The spanning tree approach is used for the power system graphs and a tree search technique is used for finding the optimal location of PMUs. This is tested on IEEE-14 and IEEE-30 bus system. The same technique is modified to optimally place minimum channel PMUs on the same IEEE-14 and IEEE-30 bus systems. Matlab tool has been used for fulfilling the objective.

*Copyright © 2015 Institute of Advanced Engineering and Science.  
All rights reserved.*

---

### Corresponding Author:

Srihari Mandava,  
School of Electrical Engineering  
VIT University, India  
Email: mandavasrihari@yahoo.co.in

---

## 1. INTRODUCTION

The present world is mostly using SCADA (Supervisory Control and Data Acquisition system) for measuring power system data for knowing the state estimation. The data measured from different parts of power system network by SCADA are not synchronized and provide an inaccurate state of the system during the dynamic events on the system like load change.

Positive sequence measurements [1] are used to define the state of the power system at any instant. The use of the positive sequence voltage of the network is explained well in [2]. Synchronized phasor measurement units measure positive sequence voltages and currents synchronized in less than a microsecond with the use of Global Positioning System (GPS). The measured synchronized data is then sampled and processed using the techniques developed for computer relaying applications [3]. In addition to these, the PMUs also measure local frequency, rate of change of frequency which helps to measure harmonics, negative and zero sequence quantities. Recent spate of spectacular blackouts on power systems throughout the world has provided an added impetus to wide scale deployment of PMUs.

The main advantage of using PMUs in power systems is their capability of directly measuring the state of the system. Therefore, synchronized measurements provided by PMUs is great help to power system control systems especially in interconnected networks. While PMUs are not yet found in every substation, their utilization in substations for protection and control functions is rapidly increasing. As they become available in large numbers they can provide valuable information for energy management system applications as well. There will be a great improvement in the state estimation of power system network with the implementation of these PMUs. The great advantage of PMU is that, the PMU yields the synchronized measurements across the power system. Initial work on PMU placement is based on the assumption that PMUs will have infinite number of channels to monitor phasor currents of all branches that are incident to the

bus where a PMU will be installed [3, 4]. While there are manufacturers that produce PMUs with several channels to measure phasor currents and voltages, the number of channels is typically limited. Also, note that PMUs capture samples of phase voltages and currents that are received from the instrument transformers connected to a bus or a breaker in the substation. The sampled three phase signals are converted into positive sequence phasors at regular intervals and then telemeter to the phasor data concentrators. Phasor measurements are used by various application functions at energy control centers. One such application is state estimation which not only provides the best estimate of the system state but also acts as filter for gross errors in analog and digital measurements. Moreover, results of state estimation are used by many applications as inputs and therefore have a significant impact on the overall performance of the energy management systems. Some of the applications that rely on state estimation results include real-time contingency analysis, voltage stability assessment, transient stability assessment, real-time power flow, security constrained optimal power flow, load forecasting as well as the market applications.

### 1.1 Complete Observability

System is said to be completely observable if voltage and current phasor of every bus in network is known. Figure 1 shows a completely observed system. The voltage at buses B, E and H are directly measured by PMU-1, PMU-2 and PMU-3 respectively, while voltages at buses A, C, D, F, G and I can be calculated using the measured voltages and line currents. Let buses B, E, H are defined as PMU buses where bus voltages and line currents are directly measured. Buses A, C, D, F, G and I are defined as calculated buses because their voltages are calculated from the PMU measurements of the buses linked to them.

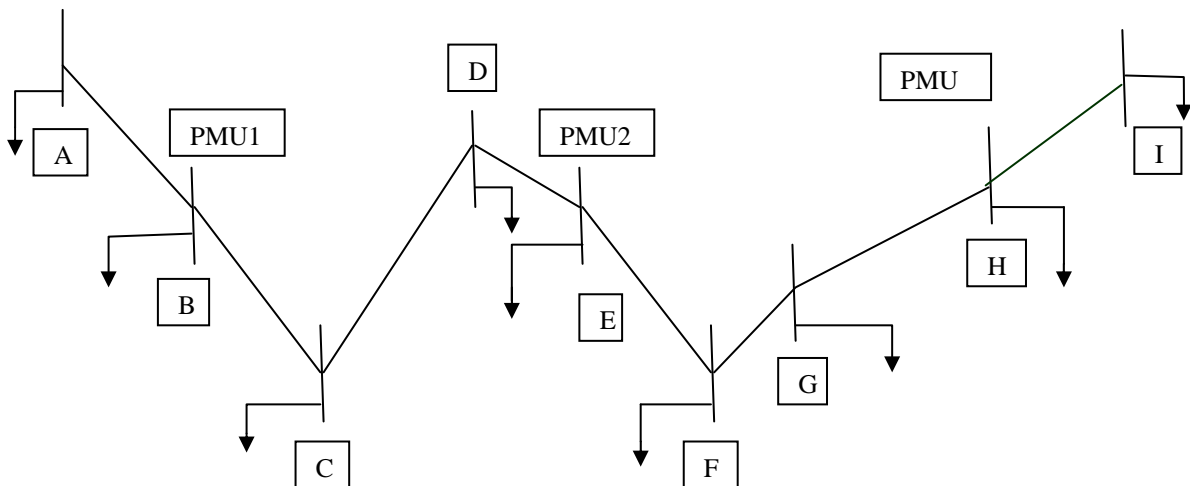


Figure 1. Complete observability using PMU's

The problem of strategic placement of PMUs in power system to have a fully observable system, has received a lot of attention from researchers. [4], [5] investigate the problem of optimal placement of PMUs to observe the network using integer programming. In [6], authors propose an exhaustive search approach to determine the minimum number and optimal placement of PMUs for state estimation. Although the proposed method in this paper yields the global optimal solution, the method is yet computationally intensive for large systems. In [3], authors propose a method to identify the strategic location for PMU installation in the system based on the dual search method using a modified bisecting search and a simulated annealing method. The modified bisecting search fixes the number of PMUs for which the simulated annealing based method then attempts to find a placement set that makes the system topologically observable. A graph theoretic approach for placing PMUs based on incomplete observability is proposed in [7] where simulated annealing method is used to solve the pragmatic communication-constrained PMU placement problem. In [8] authors represent a method to identify the optimal placement of PMU for power system state estimation based on the minimum condition number of the normalized measurement matrix. [9] Shows a technique of identifying the optimal PMU placement using the genetic algorithm (GA). Authors in [10], use the particle swarm optimization in power system, to obtain the optimal PMU placement for full observability of the system. In [11] a binary particle swarm optimization based method is used to minimize the number of required PMUs and maximize the measurement redundancy. In case of system

unobservability, a method to test the system observability, as well as identifying the observable islands is presented in [12].

Although, many researchers have investigated the problem of strategic placement of PMUs on power systems, to achieve full observability, all their studies is based on an unrealistic assumption that each PMU is capable of measuring the phasor voltage of a bus and the phasor current of all branches incident to that bus. Since number of branches incident to each bus is different, solving the optimization problem based on the assumption that each PMU can measure phasor current of all branches incident to the bus where PMU is installed is more of an idealistic assumption.

In this work, a new type of PMU is considered. This type of PMU is capable of measuring the phasor voltage of the bus, as well as the phasor current of only some branches (i.e. depends upon the number of channels with PMU) incident to the bus where PMU is installed, these types of PMUs will be referred to as “minimum channel” PMUs and number of them are already installed in utility companies system. Different methods are used by researchers to test the observability of the system. List of the most commonly used method for observability check in power systems is as follows:

- Numerical method based on nodal variable formulation
- Numerical method based on branch variable formulation
- Topological observability check method

In this paper, the spanning tree approach is used to check the observability of the system which comes under the third method mentioned above.

### 1.2 Tree Search Placement Technique

The objective is to place PMUs such that the entire system is observable. The envisioned technique consists of a series of “walks” along branches of a spanning tree and queries are made on each node if a PMU placement is possible.

- The search procedure starts at a root node and goes down the tree until it reaches a terminal node.
- At this point, it backtracks and searches for another route.
- Identify a walk by a bus pair from bus-to bus.
- A root node is specified arbitrarily; the search for PMU locations is terminated when the procedure returns to this root node.
- At this time, the spanning tree has been fully searched.

The resulting PMU placement strategy guarantees the existence of a completely observable condition for the spanning tree. It likewise guarantees the complete observability for the parent graph.

## 2. ILLUSTRATION OF SPANNING TREE APPROACH

The PMU placement technique (or alternatively the tree search) is illustrated first before the formal algorithm is presented. Consider the spanning tree in Figure 2 that is composed of 14 nodes and with 20 branches.

- Jump-start the placement process by arbitrarily designating Node 5 as the root node.
- First PMU placement should be one bus away from the root node 5 at Node 6 so as to observe the root node. Then, we take a series of forward moves along a chosen path defined by the nodal sequence 6-11-10-9-7-8 each time querying for possible PMU placement.
- The next logical placement will be at Node 9, which makes nodes 10,7 observable with depth-of-zero unobservability .Note that the PMUs are physically separated 3 buses from each other along the chosen path.
- We can now pose the following rule that: given a desired depth of unobservability, the next candidate PMU placement node must be of distance [13].

$$D_p = U + 3$$

Where -  $D_p$  is the number of buses away from the current PMU placement the next one will be;

-  $U$  is the desired level of unobservability (For complete observability  $U=0$ )

- The next move is to the terminal node 8 but PMU at this place observes only one bus so the next PMU location will be at location 7.
- By placing PMUs 6, 9, 7 the entire span 5, 6, 11, 10, 9, 7, 8 observable and also the nodes connected to the PMU buses (6, 9, 7) i.e., 12, 13, 14, 4. Is also observable. Backtrack is done until it reach a node where an unobservable path can be taken.
- Then move to nodes 1, 2, 3 where here 1 acts as sub root node and we again apply spanning tree algorithm by placing PMU at node directly connected to root node i.e., 2 which makes unobservable path 1, 2, 3 observable.
- So by placing PMU'S at 2, 6, 7, 9 entire network is observable.

- To ensure minimum number of PMU placements, it is necessary to perform another search from a different root nodes i.e., from 1 to 14.
- Compare all solutions for one with least number of PMUs

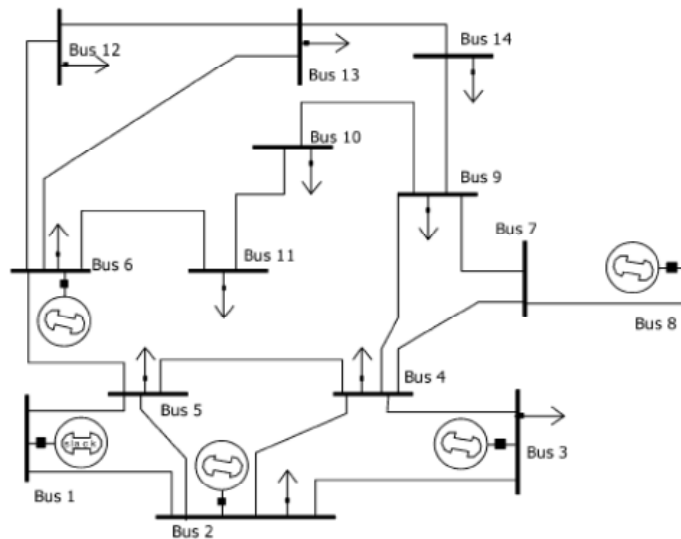


Figure 2. IEEE-14 bus system

### 3. MINIMUM CHANNEL PMU

Manufacturer specifies channel number of PMU i.e. either single channel or two channel or multichannel. The conventional techniques assume PMU as multi-channel PMU but these techniques cannot be applied when there is only single channel or two channel PMUs are present. Suppose there are only single channel or two channel PMUs are present depth of observability condition and observable buses matrix in program should be changed. In this case PMUs placement in network is obtained as follows.

In case there is only single channel the depth of observability condition  $D_p = U + 3$  changes to  $D_p = U + 2$  as shown in 4.1. Moreover, for single channel placing PMU starts from root node itself as it provide less number of PMUs than placing at one bus away from root node. If there is only single channel PMU, observable buses in this condition is just the entire span of traverse, but not the other buses that also connected to PMU buses as that in case of multichannel.

In case there is only two channel the depth of observability condition  $D_p = U + 3$  is same. If there is only two channel PMU, observable buses in this condition is just the entire span of traverse, but not the other buses that also connected to PMU buses as that in case of multichannel.

#### 3.1 Illustration of Spanning Tree Approach to Single Channel PMU

The PMU placement technique (or alternatively the tree search) is illustrated first before the formal algorithm is presented. Consider the spanning tree in Figure 2 that is composed of 14 nodes and with 20 branches.

- Jump-start the placement process by arbitrarily designating Node 12 as the root node.
- For single channel PMU first PMU placement should be root node 12. Then, we take a series of forward moves along a chosen path defined by the nodal sequence 12, 13, 14, 9, 7, 8 each time querying for possible PMU placement.
- We can now pose the following rule that: given a desired depth of unobservability, the next candidate PMU placement node must be of distance [2]  
 $D_p = U + 2$   
 Where -  $D_p$  is the number of buses away from the current PMU placement the next one will be;  
 -  $U$  is the desired level of unobservability (For complete observability  $U = 0$ )
- The next move is to place PMU at bus 14 according to  $D_p = U + 2$  rule and next will be 7 by applying same rule. Note that PMU's are separated by two buses.
- By placing PMU's 12, 14, 7 the entire span 12, 13, 14, 9, 7, 8 observable. Backtrack until we reach a node where an unobservable path can be taken.

- Then move to nodes 1, 2, 3, 4, 5, 6, 11, 10 where here 1 acts as sub root node and we again apply spanning tree algorithm by placing PMU at sub root node and applying above rule PMU placement will be at 3, 5, 11.
- So by placing PMUs at 1, 3, 5, 7, 11, 12, 14 entire network is observable.
- To ensure minimum number of PMU placements, it is necessary to perform another search from a different root nodes i.e., from 1 to 14.
- Compare all solutions for one with least number of PMUs.

For two channel PMU placement will be at locations as given by  $D_p = u + 3$ . Only buses present in span which has been traversed is taken as observable buses but not buses linked to PMU buses as that in case of multichannel. The same approach is applied to IEEE-30 bus system also.

#### 4. RESULTS

The spanning tree approach is first applied to multi-channel PMU and then it is applied to minimum channel PMU i.e to one channel PMU and two channels PMU for optimum placement of PMUs on IEEE-14 bus and IEEE-30 bus networks. The optimum location of PMUs is provided in the table 1 for 14 bus system. From the results presented in the table 1 below, it can be said that the number of PMU locations in a network with multichannel PMU is less than minimum channel PMU. For example the number of PMU locations for IEEE-14 bus network with multichannel is 4 i.e. at buses 2, 6, 7 and 9 where as with 2- channel PMU, the number of PMU locations are 5 i.e. at buses 2, 4, 6, 9 and 13 for complete observability of the system. Also the number of PMU locations required for a given network will decrease with the increase in the number of channels of PMU. With 1-channel PMU, for an IEEE-14 bus network the number of PMU locations are 8 i.e. at buses 1, 3, 5, 7, 9, 11, 12 and 14 where as with 2-channel PMU the number of PMU locations are 5 i.e. at buses 2, 4, 6, 9 and 13.

From the results shown in the table 1, it can be observed that there are more than solution for the optimum locations of PMUs for the IEEE-14 bus and IEEE-30 bus networks for multi-channel and minimum channel i.e.1-channel and 2-channel PMUs. All the optimum solutions are in bold. So, the PMUs can be located with any one of the solution and to consider which solution is best depends upon the buses which are important for observation, the availability and the cost of PMUs. In order to optimize the cost, the one channel PMU can replace the two channels PMU in some cases without the loss of complete observability of the system. This can be explained by taking the root node 3 for the IEEE-14 bus network with two channels PMU. From table 1, with root node 3 the optimal placement of PMU for complete observability of the system is 2, 4, 7, 11 and 13 bus. Here, it is considered that all the buses i.e. 2, 4, 7, 11 and 13, the PMUs are of two channels. The observability of each bus is as shown in table 2.

From table 2, it is observed that the bus 9 is observed by two PMUs placed at buses 4 and 7. So, the two channels PMU at bus 2 or at bus 7 can be replaced by one channel PMU so that the total cost can be reduced. The same thing can be applied for other root nodes of optimal solutions. The elapsed time for IEEE-14 bus system using spanning tree algorithm is 2.955993 seconds and for IEEE-30 bus system it is only 5.270514 seconds. This is very much less than other algorithms like matrix manipulation which is 391.145657 seconds [14].

Table 1. IEEE 14 bus system

Root Node	Multi Channel		One Channel		Two Channel	
	PMU bus	Observability	PMU bus	Observability	PMU bus	Observability
1	2,5,7,10,13	Complete	1,3,5,7,9,11,12,14	Complete	<b>2,5,7,10,13</b>	<b>Complete</b>
2	1,3,6,7,9	Complete	1,2,4,6,7,10,12,14	Complete	1,3,6,7,9,13	Complete
3	1,4,7,11,13	Complete	1,3,5,7,9,11,12,14	Complete	<b>2,4,7,11,13</b>	<b>Complete</b>
4	3,5,7,10,13	Complete	1,3,4,6,7,10,12,14	Complete	<b>2,5,7,10,13</b>	<b>Complete</b>
5	<b>2,6,7,9</b>	<b>Complete</b>	1,3,5,7,9,11,12,14	Complete	<b>2,4,6,9,13</b>	<b>Complete</b>
6	2,5,7,11,13	Complete	1,3,5,6,7,10,12,14	Complete	<b>2,5,7,11,13</b>	<b>Complete</b>
7	2,6,7,10,12,14	Complete	<b>1,3,5,7,9,11,13</b>	<b>Complete</b>	<b>2,5,7,10,13</b>	<b>Complete</b>
8	2,6,7,10,12,14	Complete	<b>1,3,5,7,9,11,13</b>	<b>Complete</b>	<b>2,5,7,10,13</b>	<b>Complete</b>
9	2,6,7,10,12,14	Complete	1,3,5,7,9,11,12,14	Complete	2,5,7,10,14	Complete
10	2,6,7,9,13	Complete	<b>1,3,5,7,10,11,13</b>	<b>Complete</b>	2,5,7,9,12,14	complete
11	2,6,7,10,14	Complete	1,3,5,7,9,11,12,14	Complete	<b>2,5,7,10,13</b>	<b>Complete</b>
12	2,5,7,10,13	Complete	<b>1,3,5,7,11,12,14</b>	<b>Complete</b>	<b>2,5,7,10,13</b>	<b>Complete</b>
13	2,6,7,10,12,14	Complete	1,3,5,7,9,11,12,13	Complete	<b>2,5,7,10,14</b>	<b>Complete</b>
14	2,6,7,9,13	Complete	<b>1,3,5,7,11,12,14</b>	<b>Complete</b>	2,5,7,9,10,13	Complete

Table 2. Observability of each bus

PMU at Bus	Observed Buses
2	2,1,3
4	4,5,9
7	7,8,9
11	11,6,10
13	13,12,14

## 5. CONCLUSION

The spanning tree algorithm is applied for optimum placement of PMUs for complete observability of power system network. The PMUs with ideal assumption as multichannel PMU is considered and also the realistic PMUs i.e the minimum channel PMUs (1-channel and 2-channel) are considered and are applied on IEEE-14 bus and IEEE-30 bus network. This method can be applied on any system with more number of buses and the computation time required is few seconds like 3 to 5 seconds.

## REFERENCES

- [1] A.G. Phadke, J.S. Thorp, 'Synchronized phasor measurements and their applications'.
- [2] A.G. Phadke, J.S. Thorp, M.J. Adamiak, "A new measurement technique for tracking voltage phasors, local system frequency, and rate of change of frequency", IEEE Transaction on PAS. Vol. 102, No. 5, (May 1983), pp 1025-1038.
- [3] T.L. Baldwin, L. Mili, M.B. Boisen, and R. Adapa, "Power system observability with minimal phasor measurement placement", IEEE Transactions on Power Systems, Vol. 8, No. 2, (May 1993), pp. 707-715.
- [4] X. Bei, A. Abur, "Observability analysis and measurement placement for systems with PMUs", Proceedings of the IEEE PES Power Systems Conference and Exposition, New York, NY. (October 10-12, 2004)
- [5] B. Xu, A. Abur, "Optimal placement of phasor measurement units for state estimation", project report, PSERC, (Oct. 2005).
- [6] S. Chakrabarti, E. Kyriakides, "Optimal placement of phasor measurement units for state estimation", IEEE Transaction on power systems, Vol. 23, No. 3, (Aug. 2008), pp 1433-1440.
- [7] R.F. Nuqui, A.G. Phadke, "Phasor measurement unit placement techniques for complete and incomplete observability", IEEE Transaction on power delivery, Vol. 20, No.4, (Oct 2005.) pp. 2381- 2388.
- [8] C. Rakpenthai, S. Premrudeepreechacharn, S. Uatrongjit, N.R. Watson, "An optimal PMU placement method against measurement loss and branch outage", IEEE Transaction on Power delivery, Vol. 22, (Jan. 2007) No. 1.
- [9] B. Milosevic, M. Begovic, "Non-dominated Sorting genetic algorithm for optimal phasor measurement placement", IEEE Transaction on Power systems, Vol. 18, No. 1, (Feb. 2003), pp. 69-75.
- [10] Y. Del Valle, G.K. Venayagamoorthy, S. Mohagheghi, J.C. Hernandez, R.G. Harley, "Particle swarm optimization: basic concepts, variants, and applications in powersystems", IEEE Transaction evolutionary computation, Vol. 12, No. 2, (Apr 2008.), pp. 171-195.

- [11] S. Chakrabarti, G.K. Venayagamoorthy, E. Kyriakides, "PMU Placement for power system observability using binary particle swarm optimization", Power Engineering Conference, AUPEC 08, Australian universities, (2008), pp. 1-5.
- [12] A. Monticelli and F.F. Wu, "Network Observability: Identification of Observable Islands and measurement placement", IEEE Transactions on PAS, Vol. PAS-104, No.5, (May 1985), pp. 1035- 1041.
- [13] Reynaldo F. Nuqui, Member, IEEE, and Arun G. Phadke, Life Fellow, IEEE "Phasor Measurement Unit Placement Techniques for Complete and Incomplete Observability", IEEE Transactions on Power delivery, Vol. 20, No. 4, (October 2005).
- [14] Abdul-Aziz Fish, S. Chowdhury, and S.P. Chowdhury, Optimal PMU placement in a power network for full system observability", proc IEEE , Vol.978, (Nov. 2011), 4577-1022.