International Journal of Electrical and Computer Engineering (IJECE)
Vol. 5, No. 1, February 2015, pp. 102~110
ISSN: 2088-8708

Robust Synchronization of the Unified Chaotic System

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Article Info

Article history:

Received Oct 6, 2014
Revised Nov 31, 2014
Accepted Dec 20, 2014

Keyword:

Compound Matrices
Robustness
Synchronization
Unified Chaotic System

ABSTRACT

This paper investigates the synchronization problem of the unified chaotic system. The case of identical, but unknown, master and slave unified chaotic systems is considered. Based on compound matrices formalism, a unified synchronization control scheme is proposed independently of the unknown system parameter. Simulation results are provided to show the effectiveness of the presented scheme.

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1. INTRODUCTION

Chaos synchronization is an attractive phenomenon involved in a variety of real-life processes. In 1990, Pecora and Carrol proved [1] that two chaotic systems can synchronize. This means that one system (slave system), can follow the trajectories of another one (master system), when an appropriate control law is applied. Since then, many synchronization schemes have been proposed [2], [3], [4], [5] such as nonlinear control [6], nonlinear observer [4], [7], [8] adaptive control [9], [10], [11] active control [12], [13], [14], fuzzy control [15], [16], and backstepping control [17], [18]. More recently, in 2002, Lü and Chen et al. [19] investigated some specific chaotic systems and described them in a unified form known as the unified chaotic system. This system plays a very important role in the study of the generalized Lorenz system family [20]. Different results related to the unified chaotic system are reported in literature [21], [22], [23], [24], [25].

In this paper, we propose a synchronization control scheme based on the concept of compound matrices, in order to synchronize two identical but unknown unified chaotic systems. Compound matrices [26], [27], have interesting spectral properties making of them a powerful tool for stability study [26], [28]. In [27], existence of Hopf Bifurcation in dynamical systems analysis and stability of matrices are investigated using the compound matrices formalism.

The paper is organized as follows. In Section 2, we introduce briefly the unified chaotic system and the theoretical tool used in this work, namely the compound matrix method. In Section 3, robust synchronization control scheme is proposed for identical but unknown master and slave unified chaotic systems. Obtained results are tested through numerical simulations, in Section 4.

2. PROBLEM STATEMENT

The unified chaotic system [19] can be expressed by the following differential equations:

$$\begin{cases} \dot{x}_1 = (25\theta+10)(x_2 - x_1) \\ \dot{x}_2 = (28-35\theta)x - x_1x_3 + (29\theta-1)x_2 \\ \dot{x}_3 = x_1x_2 - (1/3)(8+\theta)x_3 \end{cases} \tag{1}$$

where x_1, x_2 and x_3 are state variables and θ a constant parameter.

For θ varying continuously in [0, 1], the whole family of systems is chaotic [29]. It includes, in particular, the canonical Lorenz [30], Chen [31] and Lü [29] chaotic systems respectively for θ = 0, 1 and 0.8.

Let system (1) be the master system and define the slave system as

$$\begin{cases} \dot{y}_1 = (25\theta+10)(y_2 - y_1) + u_1 \\ \dot{y}_2 = (28-35\theta)y - y_1y_3 + (29\theta-1)y_2 + u_2 \\ \dot{y}_3 = y_1y_2 - (1/3)(8+\theta)y_3 + u_3 \end{cases} \tag{2}$$

where y_1, y_2 and y_3 are state variables of the slave system, θ the parameter introduced for the master system. Given the error vector $e = (e_1, e_2, e_3)^T$ defined by

$$e_i = y_i - x_i \ , \ i = 1..3 \ , \tag{3}$$

u_1, u_2 and u_3 are the control laws to be designed such that the following error dynamical system (4) is stable

$$\begin{cases} \dot{e}_1 = (25\theta+10)(y_2 - y_1) - (25\theta+10)(x_2 - x_1) + u_1 \\ \dot{e}_2 = (28-35\theta)y_1 - y_1y_3 + (29\theta-1)y_2 - (28-35\theta)x_1 + x_1x_3 - (29\theta-1)x_2 + u_2 \\ \dot{e}_3 = y_1y_2 - (1/3)(8+\theta)y_3 - x_1x_2 + (1/3)(8+\theta)x_3 + u_3 \end{cases} \tag{4}$$

Define the extended state vector $\varphi = (\varphi_i)$ as

$$\varphi = (x_1, x_2, x_3, y_1, y_2, y_3)^T \tag{5}$$

and the matrices T and N by

$$T = \begin{pmatrix} -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{pmatrix} \tag{6}$$

$$N(.) = \begin{pmatrix} 25\theta+10 & -25\theta-10 & 0 & -25\theta-10 & 25\theta+10 & 0 \\ -28+35\theta & -29\theta+1 & x_1 & 28-35\theta & 29\theta-1 & -y_1 \\ 0 & -x_1 & \frac{8}{3}+\frac{1}{3}\theta & 0 & y_1 & -\frac{8}{3}-\frac{1}{3}\theta \end{pmatrix} \tag{7}$$

such that the error vector (3) and the dynamical error chaotic system (4) can be expressed by

$$e = T\varphi \tag{8}$$

$$\dot{e} = N(.)\varphi + u \tag{9}$$

with

$$u = K(.)\varphi \tag{10}$$

and $K(.) = (k_{ij}(.)) \in \mathbf{R}^{3\times 6}$ is a non constant control gain matrix to be calculated such that (9) is stable.

Assume, furthermore, that there exists a matrix $A(.) = (a_{ij}(.)) \in \mathbf{R}^{3\times 6}$ such that

$$N(.) + K(.) = A(.)T \tag{11}$$

Therefore, the dynamical error system (4) can be reduced to

$$\dot{e} = A(.)e \tag{12}$$

Our aim consists on expressing matrix $A(.)$ entries, which depend on those of matrix $K(.)$, then calculating the gain matrix $K(.)$ such that system (12) is stable. In the sequel, we use, for simplicity, the notation k_{ij} instead of $k_{ij}(.)$.

3. PROPOSED ROBUST SYNCHRONIZATION SCHEME OF THE MASTER-SLAVE UNIFIED CHAOTIC SYSTEM

3.1. Basic Idea

Synchronization between the master system (1) and the slave system (2) is equivalent to the stability of the dynamical error system (12). The stability study of the the characteristic matrix $A(.)$ of system (12) is performed based on the compound matrix method. Related preliminary notions are introduces in the following.

Let $M_n(\mathbf{R})$ be the linear space of matrices of size *n* x *n* with entries in $\mathbf{R}$ and let *A* be a matrix in $M_n(\mathbf{R})$ and *k* an integer in [1, *n*]. We note by $\wedge$ the exterior product in $\mathbf{R}^n$.

Definition 1 [26], [27]: The additive compound matrix $A^{[k]}$ of A, with respect to the canonical basis in the k^{th} exterior product space $\Lambda^k\mathbf{R}^n$ is a linear operator on $\Lambda^k\mathbf{R}^n$ and can be defined on a decomposable element $v_1 \wedge v_2 \wedge ... \wedge v_k$ by

$$A^{[k]}(v_1 \wedge ... \wedge v_k) = \sum_{i=1}^{k} v_1 \wedge ... \wedge Av_i \wedge ... \wedge v_k \text{ , } \forall v_1 \wedge ... \wedge v_k \in \mathbf{R}^n \tag{13}$$

Relations between entries (a_{ij}) of *A* and those of $A^{[k]}$ ($\tilde{a}_{ij}$) are linear.

Let *i* be an integer in $[1, C_n^k]$. If we note by (*i*) = ($i_1,...,i_k$) the i^{th} member in the lexicographic ordering of integer k-tuples such that $1 \le i_1 < ... < i_k \le n$, we can obtain the additive compound matrix entries from the following result.

Proposition 1 [26], [27]:

$$\tilde{a}_{ij} = \begin{cases} a_{i_1 i_1} + ... + a_{i_k i_k}, & if \ (i) = (j), \\ (-1)^{r+s} a_{j_r i_s}, & if \ exactly \ one \ entry \ i_s \ of \ (i) \ does \ not \ occur \ in \ (j) \\ & and \ j_r \ does \ not \ occur \ in \ (i), \\ 0 & if \ (i) \ differs \ from \ (j) \ in \ two \ or \ more \ entries. \end{cases} \tag{14}$$

In particular, we have $A^{[1]} = A$, $A^{[n]} = trace(A)$ and for $A \in M_3(\mathbf{R})$

$$A^{[2]} = \begin{pmatrix} a_{11}+a_{22} & a_{23} & -a_{13} \\ a_{32} & a_{11}+a_{33} & a_{12} \\ -a_{31} & a_{21} & a_{22}+a_{33} \end{pmatrix} \tag{15}$$

Definition 2 [27]: Let $|.|$ a vector norm on $M_n(\mathbf{R})$ and A a matrix in $M_n(\mathbf{R})$.

The Lozinskiĭ measure (logarithmic measure) μ of A with respect to $|.|$ is defined by

$$\mu(A) = \lim_{h\to 0^+} \frac{|I+hA|-1}{h} \tag{16}$$

As examples, Lozinskiĭ measure of a matrix A with respect to the three common vector norms $|x|_1 = \sum_i |x_i|$, $|x|_2 = \sqrt{\sum_i |x_i|^2}$ and $|x|_\infty = \sup_i |x_i|$ are

$$\mu_1(A) = \sup_j (a_{jj} + \sum_{i,i\neq j} |a_{ij}|),\ \mu_2(A) = s(\frac{A+A^T}{2}) \text{ and } \mu_\infty(A) = \sup_i (a_{ii} + \sum_{j,j\neq i} |a_{ij}|) \tag{17}$$

where $s(A)$ denotes the maximum real part of the eigenvalues of A.
Compound matrices present a powerful tool for the stability study of matrices. The following result will be us ed in the sequel.
Theorem 1 [27]: if $(-1)^n \det(A) > 0$ then A is stable if and only if there exists a Lozinskiĭ measure μ on $M_m(\mathbf{R})$ such that $\mu(A^{[2]}) < 0$, $m = C_n^2$.
According to theorem 1, the stability of the characteristic matrix $A(.)$ of system (12) can be studied through its determinant and its second compound matrix.

3.2. Dynamical Error System Stability Study Based on Compound Matrices

By solving equation (11), we obtain the characteristic matrix $A(.)$ of the dynamical error system

$$A(.) = \begin{pmatrix} -25\theta - 10 - k_{11} & 25\theta + 10 - k_{12} & -k_{13} \\ 28 - 35\theta - k_{21} & 29\theta - 1 - k_{22} & -x_1 - k_{23} \\ -k_{31} & x_1 - k_{32} & -\frac{8}{3} - \frac{1}{3}\theta - k_{33} \end{pmatrix} \tag{18}$$

from which is deduced the second compound matrix as expressed in (14)

$$A^{[2]}(.) = \begin{pmatrix} 4\theta - 11 - k_{11} - k_{22} & -x_1 - k_{23} & k_{13} \\ x_1 - k_{32} & -\frac{76}{3}\theta - \frac{38}{3} - k_{11} - k_{33} & 25\theta + 10 - k_{12} \\ k_{31} & 28 - 35\theta - k_{21} & \frac{86}{3}\theta - \frac{11}{3} - k_{22} - k_{33} \end{pmatrix} \tag{19}$$

In addition, we obtain relations between entries of matrix $K(.)$ which is a block interdependent matrix

$$\begin{cases} k_{14} = -k_{11} & k_{15} = -k_{12} & k_{16} = -k_{13} \\ k_{24} = -k_{21} & k_{25} = -k_{22} & k_{26} = y_1 - x_1 - k_{23} \\ k_{34} = -k_{31} & k_{35} = x_1 - y_1 - k_{32} & k_{36} = -k_{33} \end{cases} \tag{20}$$

Referring to the compound matrix entries and the determinant of matrix $A(.)$, we propose, by the use of theorem 1, the following results.
Theorem 2: Global synchronization is achieved between unified chaotic systems described by (1) and (2) independently of the parameter θ, if the following control law is applied

$$\begin{cases} u_1 = (\alpha + |x_1|)(x_1 - y_1) \\ u_2 = (1.16\alpha - 2.28 + 1.16|x_1| + \beta)(x_2 - y_2) + (y_1 - x_1)y_3 \\ u_3 = (x_1 - y_1)y_2 \end{cases} \tag{21}$$

where $\alpha > 15.33$ and $\beta > 44.50$.

Proof:

All diagonal elements of the compound matrix $A^{[2]}$ depend on k_{11} and k_{22}. Let look for a gain matrix involving only k_{11} and k_{22} and consequently k_{14} and k_{25}. By substituting all other k_{ij} elements in A by 0, matrices A and $A^{[2]}$ become

$$A(.) = \begin{pmatrix} -25\theta - 10 - k_{11} & 25\theta + 10 & 0 \\ 28 - 35\theta & 29\theta - 1 - k_{22} & -x_1 \\ 0 & x_1 & -\frac{8}{3} - \frac{1}{3}\theta \end{pmatrix} \tag{22}$$

$$A^{[2]}(.) = \begin{pmatrix} 4\theta - 11 - k_{11} - k_{22} & -x_1 & 0 \\ x_1 & -\frac{76}{3}\theta - \frac{38}{3} - k_{11} & 25\theta + 10 \\ 0 & 28 - 35\theta & \frac{86}{3}\theta - \frac{11}{3} - k_{22} \end{pmatrix} \tag{23}$$

The determinant of matrix $A(.)$ is given by

$$\begin{aligned} \det(A(.)) = {} & -50\theta^3 + (\frac{29}{3}k_{11} - \frac{25}{3}k_{22} - 195)\theta^2 + \\ & (-25x_1^{\,2} + 1730 - \frac{1}{3}k_{11}k_{22} + 77k_{11} - 70k_{22})\theta \\ & -10x_1^{\,2} - k_{11}x_1^{\,2} + 720 - \frac{80}{3}k_{22} - \frac{8}{3}k_{11}k_{22} - \frac{8}{3}k_{11} \end{aligned} \tag{24}$$

By applying theorem 1, using the Lozinskiĭ measure with respect to $|.|_1$, system (11) is stable if the following inqualities are satisfied

$$4\theta - 11 - k_{11} - k_{22} + |x_1| < 0 \tag{25a}$$

$$|x_1| - \frac{76}{3}\theta - \frac{38}{3} - k_{11} + |-28 + 35\theta| < 0 \tag{25b}$$

$$-\frac{11}{3} + |25\theta + 10| + \frac{86}{3}\theta - k_{22} < 0 \tag{25c}$$

$$\begin{aligned} & -50\theta^3 + (\frac{29}{3}k_{11} - \frac{25}{3}k_{22} - 195)\theta^2 \\ & +((-25x_1^{\,2} + 1730 - \frac{1}{3})k_{11}k_{22} + 77k_{11} - 70k_{22})\theta \\ & -10x_1^{\,2} - k_{11}x_1^{\,2} + 720 - \frac{80}{3}k_{22} - \frac{8}{3}k_{11}k_{22} - \frac{8}{3}k_{11} < 0 \end{aligned} \tag{25d}$$

Inequalities (25a), (25b) and (25c) are sufficient conditions guarantying that and (25d) is related to the determinant of matrix $A(.)$.

Left-hand sides of inequalities (25a), (25b) and (25c), can be majorated given that $0 \le \theta \le 1$. Furthermore, polynomial inequality (25d) can be satisfied when all monomials are non positive.

Conditions (25a), (25b), (25c) and (25d) can be therefore reduced to

$$\begin{cases} k_{11} > 46/3 + |x_1| & (26a) \\ k_{22} > 60 & (26b) \\ 29k_{11} - 25k_{22} < 57 & (26c) \end{cases}$$

The gain matrix entry k_{11} can be chosen in the form

$$k_{11} = |x_1| + \alpha \text{ with } \alpha > \frac{46}{3} = 15.33 \quad (27)$$

Substituting (26c) in (27), it comes

$$k_{22} > \frac{29}{25}|x_1| + \frac{29}{25}\alpha - \frac{57}{25} = 1.16|x_1| + 1.16\alpha - 2.28 \quad (28)$$

and a possible choice of the gain matrix entry k_{22} is

$$k_{22} = 1.16|x_1| + 1.16\alpha - 2.28 + \beta \text{ with } \beta > 0 \quad (29)$$

Given the constraint on the parameter α and the new expression of k_{22}, (26c) holds for every $\beta > 44.50$. Finally, by calculating the other entries of the gain matrix $K(.)$, according to (20), and using the relation the control law expression of theorem 2 is retrieved.

$$u_i = \sum_{j=1}^{6} k_{ij}\varphi_j \quad (30)$$

Note that α and β represent tuning parameters for the designed controller used to enhance system performances. An optimal choice of these parameters is done through trial and error process.

4. SIMULATION RESULTS

In this section, 3 cases are considered to show the effectiveness of the proposed method: $\theta = 0$ (Lorenz chaotic system), $\theta = 0.8$ (Lü chaotic system) and $\theta = 1$ (Chen chaotic system). Corresponding simulation results are represented respectively in figure 1, 2 and 3.

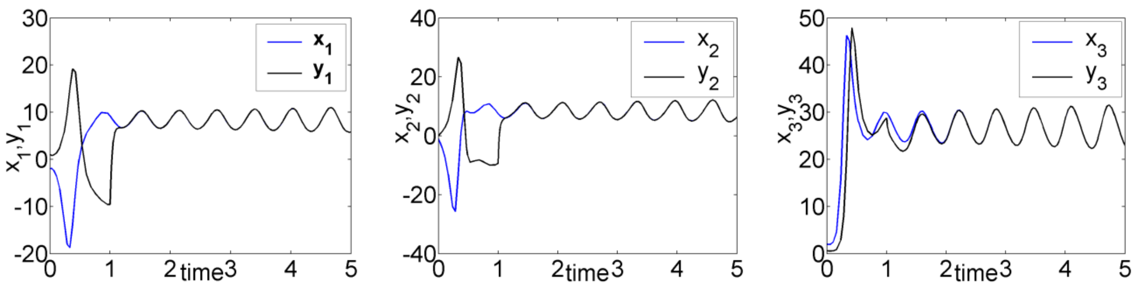

Figure 1. State trajectories of master and slave systems for $\theta = 0$. Control is activated at time t=1.

Differential equations are solved using the fourth-order Runge–Kutta method with a time step size equal to 0.001.

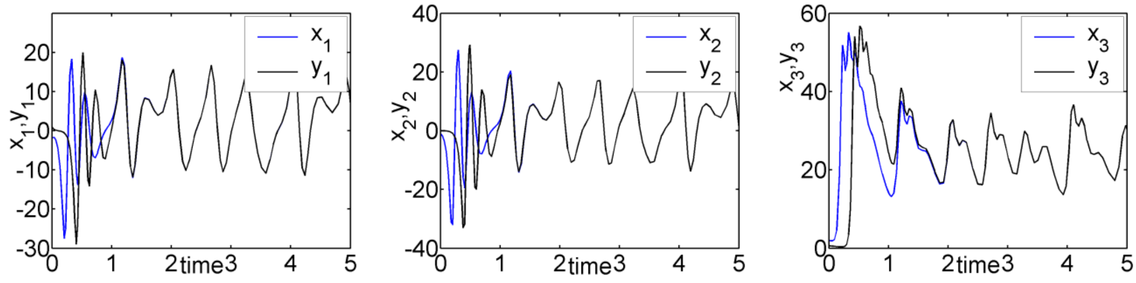

Figure 2. State trajectories of master and slave systems for θ= 0.8. Control is activated at time t=1.

In all simulations, the constant parameters α and β are respectively chosen equal to 15.5 and 55. Different initial conditions are considered for the master and the slave systems and are respectively fixed to (-2, -1, 2) and (1, 0, 0.6). In the three cases, we can notice that the trajectories of the controlled slave system synchronize with those of the master system. Numerical simulations have shown the effectiveness of the proposed method.

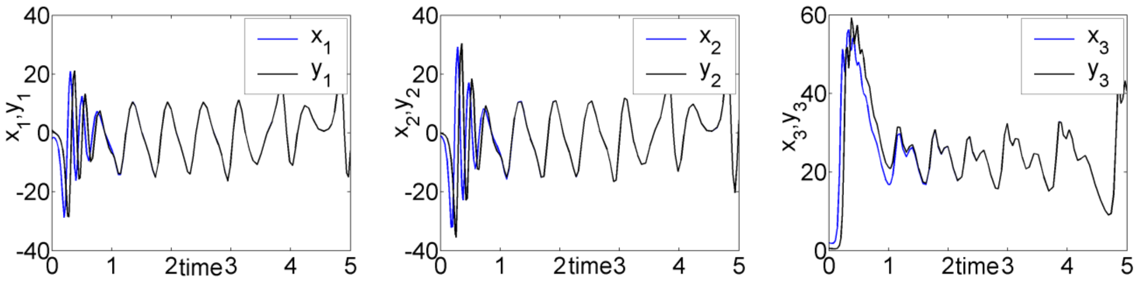

Figure 3. State trajectories of master and slave systems for θ= 1. Control is activated at time t=1.

Unlike other reported results, as in [24], [32] and [33], the control law designed in this work is independent of the chaotic system parameter θ. For this reason it's qualified as robust. Moreover, for the specific cases of Lorenz, Chen and Lü chaotic systems, the performed simulations indicate that synchronization is achieved faster than in other previous works [23], [24], [32].

5. CONCLUSION

In this paper, is investigated the synchronization of identical, but unknown, master and slave unified chaotic systems. The proposed synchronization scheme is based on compound matrices formalism. The obtained control law is independent of the unknown system parameter and is consequently efficient for all the family of considered chaotic systems. Numerical simulations are provided to illustrate the capability of the proposed method which can be applied to a large class of chaotic systems, with or without uncertainties. A possible extension of this work is the synchronization of two different unknown unified chaotic systems.

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