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Robust Model Predictive Control Based on MRAS for Satellite Attitude Control System

Fateme Pirouzmand
Departement of Control Engineering, Science and Research Branch, Islamic Azad University, Boroujerd, Iran

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ABSTRACT

In this paper, an improved robust model predictive controller (RMPC) is proposed based on model reference adaptive system (MRAS). In this algorithm, using the MRAS a combinational RMPC controller for three degree freedom satellite is designed such that the effect of moment of inertia uncertainty and external disturbance is compensated on the stability and performance of closed loop system. Control law is a state feedback which its gain is obtained by solving a convex optimization problem subject to several linear matrix inequalities (LMIs). To avoid the actuators saturation an input constraint is incorporated as LMI in the mentioned optimization problem. In addition to, using the MRAS system the effect of input disturbance is rejected on the system. The advantages of this algorithm are needless to exact information from system's model, robustness against model uncertainties and external disturbance. Results from the simulation of the system with the proposed algorithm are presented and compared to generalized incremental model predictive control (GIPC). The results show that the suggestive controller is more robust than the GIPC method.

Corresponding Author:

Fateme Pirouzmand
Departement of Control Engineering, Science and Research Branch
Islamic Azad University, Boroujerd, Iran
Email: pirouzmand.f@gmail.com

1. INTRODUCTION

Recently, model predictive control (MPC), due to its various applications in constrained, multivariable and complex systems, has been received many attentions. Since the uncertainty is not explicitly investigated in the structure of predictive controllers, the performance and robustness of the system may not be satisfied in the presence of the uncertainty, though this method has many advantages. Since 1990, many studies have been devoted to considering robustness in the model predictive control algorithm. In [1], based on the dynamic state-feedback control law, an RMPC controller is designed for uncertain linear systems. This algorithm has a high computational burden. In [2], based on the output feedback control law, an off-line RMPC controller is designed for systems with two types of uncertainty, i.e. norm-bounded and polytopic uncertainties. An off-line RMPC controller with larger applicable area is designed for uncertain nonlinear systems, in [3]. Although the off-line RMPC algorithms greatly reduce the online computational burden, its optimality is largely degraded as compared with online RMPC. In [4], an off –line RMPC controller is designed for linear systems with bounded state disturbance and measurement noise in which, noise and disturbance belong to a convex set. Practical examples of these algorithms in process control are given in [5] and [6]. In the mentioned algorithms, design of RMPC controller is given as infinite horizon objective function minimization for a set of uncertain models. In order to solve this problem, an upper bound is assumed for the objective function, and a state feedback or output feedback control law is achieved by minimizing objective function subject to some LMIs which guarantee the robustness of the system. Using invariant ellipsoid concept, input constraints are also embedded as LMI in the mentioned optimization problem, in order to prevent saturation of the actuators. Although the disturbance issue is discussed in [4], this problem exists in another them. In this paper, this problem is solved by combination of MRAS system with RMPC algorithm. During recent years, model predictive control has been significantly used and developed in other industries such as aerospace industry. In [7], a predictive controller is designed to control the spacecraft which is tracking and chasing the target spacecraft. During launching satellites, launch vehicles impart high levels of vibration to satellite. In order to isolate the whole satellite from vibration, a model predictive control is proposed in [8]. Moreover, in [9], an explicit MPC is designed for satellite attitude control system.

In this paper, using an on-line RMPC controller as well as MRAS system, a three-axis combinational RMPC controller is proposed for satellite attitude control system (ACS) in the presence of moment of inertia uncertainty, external disturbance and input constraint. The regarded actuator is reaction wheels with input bound of 1 N.m. This paper is prepared in 5 sections as follow: After introduction, in section 2, the problem design of combinational RMPC controller is proposed. In section 3, the linear and nonlinear models of satellite are achieved. Section 4 presents the simulation results of this algorithm. Moreover, simulation results of the combinational RMPC algorithm are compared with that of GIPC algorithm [13]. Finally, a summary of results is provided in section 5.

Note 1. Symbol * denotes symmetric terms in a symmetric matrix.

2. ROBUST MODEL PREDICTIVE CONTROL BASED ON MRAS

2.1 RMPC Formulation

Consider the linearized time-variant system as:

$$x(k+1) = A(k)x(k) + B(k)u(k)$$
$$y(k) = Cx(k) \tag{1}$$

$$[A(k)\ B(k)] \in \Omega$$
$$\Omega = Co\{[A_1\ B_1][A_2\ B_2]\ldots[A_L\ B_L]\} \tag{2}$$

Where $u \in R^{n_u}$, $x \in R^{n_m}$ and $y \in R^{n_y}$ denote the control signal, state vector and system output, respectively. Ω is a predetermined set, polytope set, which includes the system uncertainties. Co is called the convex hull. This means that if $[A\ B] \in \Omega$ then, for $0 < \lambda_i < 1$, we have:

$$[A\ B] = \sum_{i=0}^{L} \lambda_i [A_i\ B_i] \tag{3}$$

Assumption: suppose that the pair (A,B) is stablizable by state feedback control law (it means that there exist a matrix F so that A+BF is a stable matrix).

Let $x(k+i\,|\,k)$ and $u(k+i\,|\,k)$ be the system state and control action at sampling time k. then consider a cost function as:

$$J(k) = \sum_{i=0}^{\infty} \left[x(k+i\,|\,k)^T Q_1 x(k+i\,|\,k) + u(k+i\,|\,k)^T R u(k+i\,|\,k) \right] \tag{4}$$

Where k is the current time.Q_1 and R are symmetric positive definite weighting matrices. In the RMPC algorithm, the goal is to obtain optimal control sequence $u(k\,|\,k), u(k+1\,|\,k), \ldots$ to minimize a robust performance objective function subject to model uncertainties as:

$$\min_{u(k+i|k),\, i=0,1,\ldots,m} \ \max_{[A(k+i)\ B(k+i)] \in \Omega,\, i \geq 0} \tag{5}$$

This problem is a min-max optimization problem. The maximization operation is finding $[A(k+i)\, B(k+i)] \in \Omega$ based on which the largest $J_\infty(k)$, that is called worst case of $J_\infty(k)$, is computed. Although this problem is convex for finite m, it is not computationally tractable. To solve this problem an upper bound on the worst case is derived. Then it is minimized by adopting the following control law:

$$u(k+i\,|\,k)=F_k x(k+i\,|\,k) \tag{6}$$

Where the state feedback gain F_k is obtained to the mentioned convex optimization problem. The closed loop system is shown in Figure 1.

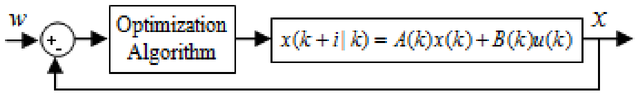

Figure 1. Block diagram of the RMPC method

In the following, at first an important lemma which is used in the next parts is introduced. Then the state feedback gain F_k is computed.

Lemma 1. (Schure complement lemma). [10] Suppose *Q(x)* and *R(x)* are symmetric positive definite matrix functions and matrix *s(x)* is linear into parameter *x*. the following inequalities are equivalent:

$$\begin{bmatrix} Q(x) & S(x) \\ S^T(x) & R(x) \end{bmatrix} > 0 \Leftrightarrow \begin{cases} R(x)>0, Q(x)-S(x)R^{-1}(x)S^T(x)>0 \\ \quad or \\ Q(x)>0, R(x)-S(x)Q^{-1}(x)S^T(x)>0 \end{cases} \tag{7}$$

Consider a Lyapunov function $V(x)=x^T(k)Px(k), P=P^T \succ 0$ of the state $x(k\,|\,k)=x(k)$ of the system (1). Suppose that V satisfies inequality (8) and it guarantees the robust stability of system (1) with the uncertainty set (2).

$$\begin{aligned} &V(x(k+i+1\,|\,k))-V(x(k+i\,|\,k))\le -[x(k+i\,|\,k)^T \\ &\quad \times Q_1 x(k+i\,|\,k)+u(k+i\,|\,k)^T Ru(k+i\,|\,k)] \end{aligned} \tag{8}$$

According toLyapunov theory if $x(\infty\,|\,k)=0$ then $V(x(\infty\,|\,k))=0$. By summing (8) from $i=0$ to $i=\infty$ and using the Lyapunov stability theory, we have:

$$\max_{[A(k+i)\;\; B(k+i)]\in\Omega, i\ge 0} j_\infty(k)\le \max V(x(k\,|\,k))\le\gamma \tag{9}$$

Using lemma 1, the relation (9) can be rewritten as:

$$\begin{bmatrix} 1 & x(k\,|\,k)^T \\ x(k\,|\,k) & Q \end{bmatrix}\ge 0, Q>0 \tag{10}$$

Where $Q=\gamma P^{-1}>0$. The robust stability condition is satisfied for system (1) with uncertain set (2) if the following inequality is satisfied:

$$\begin{bmatrix} Q & * & * & * \\ A_jQ+B_jY & Q & * & * \\ Q_1^{1/2}Q & 0 & \gamma I & * \\ R^{1/2}Y & 0 & 0 & \gamma I \end{bmatrix}\ge 0,\; j=1,\dots,L \tag{11}$$

Proof. Substitute the relations (1), (6) and variables $p=\gamma Q^{-1}$ and $Y=FQ$ in (8). Then Multiply both side of the result by Q and use lemma 1 to obtain the LMI (11).

Remark 1. To apply the physical limitations on the control input the following LMI can be added to the LMI sets (10) and (11):

$$\begin{bmatrix} u_{\max}^2 I & * \\ Y^T & Q \end{bmatrix} \geq 0 \tag{12}$$

2.2 Tracking Problem

Suppose that the goal is to track a reference signal $w_r(k) = Cx_r(k+i \mid k)$. In this case, to track the reference signal w(k), a cost function in (13) can be minimized.

$$\begin{aligned} J(k) = \sum_{i=0}^{\infty} & [Cx(k+i \mid k) - Cx_r(k+i \mid k)]^T \\ & \times Q_1[Cx(k+i \mid k) - Cx_r(k+i \mid k)] \\ & + [u^T(k+i \mid k) R u(k+i \mid k)] \end{aligned} \tag{13}$$

Where $\tilde{Q} = C^T Q_1 C$ is a positive definite matrix and $X = x(k+i \mid k) - x_r(k+i \mid k)$ is a shifted state. Therefore, without waste of generality problem the control law is obtained by $u(k+i \mid k) = FX(k+i \mid k)$.

2.3 Disturbance Rejection

In most of the time, the state feedback control law cannot reject disturbance while the presence of disturbance is unavoidable. In this paper, an online MRAS system is used to solve this problem. In the MRAS algorithm, the desired performance of the system is specified by a reference model. Then the input disturbance is estimated by the adaptive system and the parameter of the controller is adjusted based on the error which is the difference between the outputs of the system and model. The key problem with MRAS is to specify the adjustment mechanism so that a stable system is obtained and the mentioned error brings to zero. In the following, the adjustment mechanism is derived by using the MIT rule.

Lemma 2. (MIT rule) [11]. Consider a closed loop system in which the controller has one adjustable parameter θ. Let y, y_m and e denote system output, model output and the error between them, respectively. Here, the parameter θ is adjusted by minimizing a loss function as:

$$J(\theta) = \frac{1}{2} e^2 \tag{14}$$

To make J small, it is reasonable to change the parameters in the direction of the negative gradient of J as:

$$\frac{d\theta}{dt} = -\gamma \frac{\partial J}{\partial \theta} = -\gamma e \frac{\partial e}{\partial \theta} \tag{15}$$

Figure 2 shows the block diagram of the RMPC method based on MRAS algorithm.

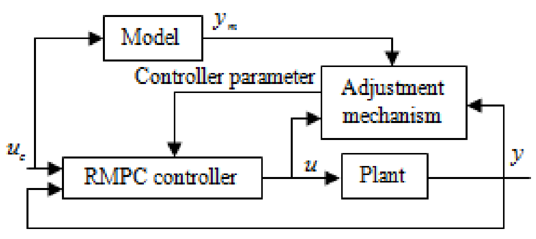

Figure 2. Block diagram of the RMPC method based on MRAS algorithm

3. THREE DEGREE OF FREEDOM SATELLITESTATESPACE MODEL

In this section, three degree of freedom rigid satellite model is presented. A microsatellite is shown in Figure 3 [12]. Axes X_B, Y_B and Z_B are satellite's body axis frame. The angles roll (φ), pitch (θ) and yaw (ψ) are defined by successive rotations around the body axes X_B, Y_B and Z_B. Parameters p, q and r are the angular rate.

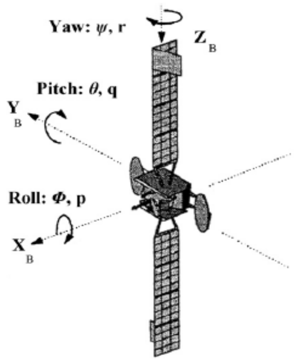

Figure 3. Microsatellite reference and body coordinates [12]

The nonlinear state model of the satellite is according to the following relation [12]:

$$\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \\ \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} \dfrac{M_x - qI_{zz}r + rI_{zz}q}{I_{xx}} \\ \dfrac{M_y - rI_{xx}p + pI_{zz}r}{I_{yy}} \\ \dfrac{M_z + pI_{xx}q - qI_{yy}p}{I_{zz}} \\ p + \dfrac{(q\sin\phi + r\cos\phi)\sin\theta}{\cos\theta} \\ q\cos\phi - r\sin\phi \\ \dfrac{q\sin\phi + r\cos\phi}{\cos\theta} \end{bmatrix} \tag{16}$$

Where M_x, $M_y$and M_z are the input torques, Parameters I_x, I_y and I_z are the moment of inertia around the body axes.The linearized state space model can be obtained by the Jacobian method and the satellite parameters in Table 1[12].

Table 1. Satellite parameters [12]

Parameters	Description	Value
I_{xx}	Moment of inertia (x-axis)	1.928 kgm^2
I_{yy}	Moment of inertia (y-axis)	1.928 kgm^2
I_{zz}	Moment of inertia (z-axis)	4.953 kgm^2
M_x, M_y, M_z	Input to satellite	1 N.m
φ_0	Initial roll Euler angle	0.362 rad
θ_0	Initial pitch Euler angle	0.524 rad
ψ_0	Initial yaw Euler angle	-0.262 rad
p	Body roll rate	0 rad/s
q	Body pitch rate	0 rad/s
r	Body yaw rate	0 rad/s

$$A=\frac{df}{dx}=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & \sin\phi\tan\theta & \cos\phi\tan\theta & 0 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi & 0 & 0 & 0 \\ 0 & \frac{\sin\phi}{\cos\theta} & \frac{\cos\phi}{\cos\theta} & 0 & 0 & 0 \end{bmatrix}, \quad (17)$$

$$B=\frac{df}{dx}=\begin{bmatrix} \frac{1}{I_{xx}} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{I_{yy}} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{I_{zz}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (18)$$

Where *A=Jacobian (f(x),x)* and *B=Jacobian(f(x),u)* are Jacobian matrices of nonlinear system with respect to state vector $x=[p \ \ q \ \ r \ \ \varphi \ \ \theta \ \ \psi]^T$ and input vector $u=[M_x,.M_y,.M_z]$, respectively. The discrete-time state space model is obtained by discretizing the linearized system at sampling time 10 ms.

4. NUMERICAL SIMULATION

In this section, the performance and robustness of the proposed RMPC and GIPC algorithms for ACS is evaluated. The detail of GIPC algorithm is described in [13].

The moment of inertia variation of the microsatellite is an important problem in ACS design. The ACS must be robust enough against the moment of inertia uncertainty. It is considered to be 20% perturbation with respect to their nominal values as:

$$\begin{aligned} \bar{I} &= \begin{bmatrix} \bar{I}_{xx} & 0 & 0 \\ 0 & \bar{I}_{yy} & 0 \\ 0 & 0 & \bar{I}_{zz} \end{bmatrix}, \\ \bar{I}_{xx} &= I_{xx,nom} \pm 20\% I_{xx,nom}, \\ \bar{I}_{yy} &= I_{yy,nom} \pm 20\% I_{yy,nom} \\ \bar{I}_{zz} &= I_{zz,nom} \pm 20\% I_{zz,nom} \end{aligned} \quad (19)$$

In this paper, the attitude control hardware includes three reaction wheels oriented at body axes. The reaction wheel dynamic is:

$$\begin{aligned} x_a(k+1) &= 0.86x_a(k) + 0.0093u_a(k) \\ y_a(k) &= 14.28x_a(k) \end{aligned} \quad (20)$$

4.1 The Performance of Controllers without Constraint

Figures 4.a and 4.b demonstrate the step response and control effort of ACS for the linear model using the RMPC and the GIPC methods without constraint.

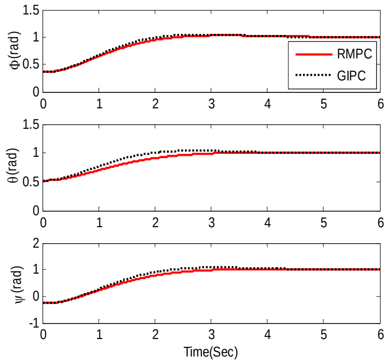

Figure 4.a. Euler angles comparison of RMPC and GIPC algorithms without constraint

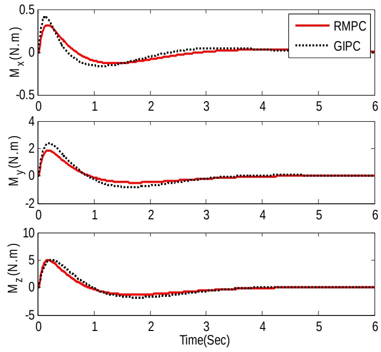

Figure 4.b. Control actions comparison between the RMPC and GIPC algorithms without constraint

The simulation results show that a large overshoot occurwhich it is led to actuator saturation.

4.2 The Performance of the Controllers with Input Constraint

In the previous subsection, we did not consider the control constraint due to the robustness analysis. Now, the input constraint is explicitly considered in optimization problem of controller design procedure. Suppose that the control action is limited by $|u| \leq 1$. According to Figs 5.a and 5.b, in this condition the system remains stable and the attitude control signalsare significantly reduced for two both of controllers, since the overshoot is reduced due to input constraint.

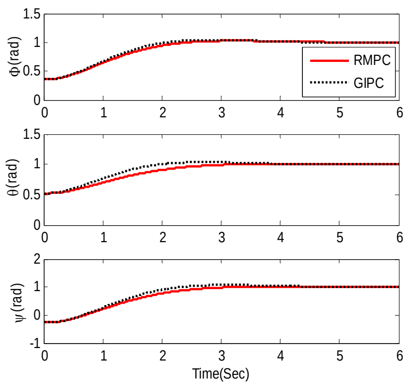

Figure 5.a. Euler angles comparison of RMPC and GIPC algorithms with presence of constraint

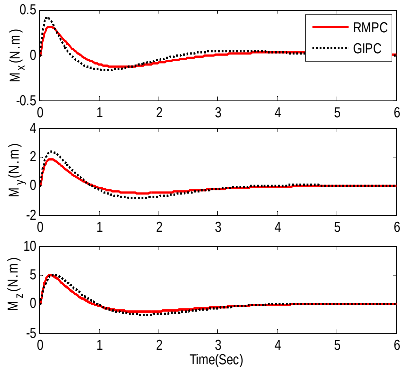

Figure 5.b. Control actions comparison between the RMPC and GIPC algorithms with presence of constraint

4.3 The Performance of the Controllers against Nonlinearity

The linear and nonlinear models of the ACS have different behaviors. Figures 6 and 7 show the step response and control effort of RMPC and GIPC algorithms. According to the simulation results, although both controllers have a good performance, the control torques in the RMPC algorithm is smoother than the GIPC.

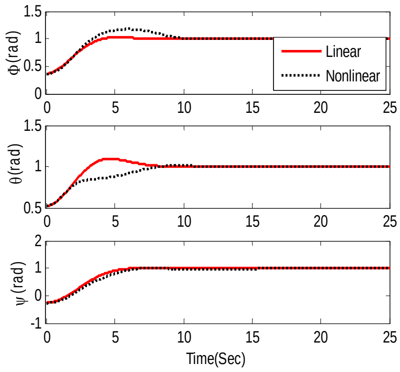

Figure 6.a. Comparing the step response of ACS for the linear and nonlinear model using the RMPC method

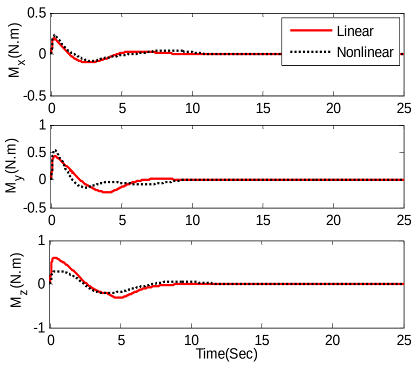

Figure 6.b. Comparing the control actions of ACS for the linear and nonlinear model using the RMPC method

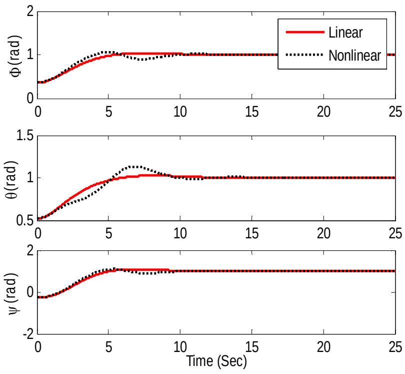

Figure 7.a. Comparing the step response of ACS for the linear and nonlinear model using the GIPC method

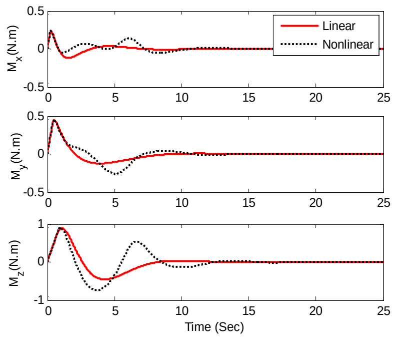

Figure 7.b. Comparing the control actions of ACS for the linear and nonlinear model using the RMPC method

4.4 The Effects of Disturbance on the Controllers Performance

In this subsection, the capability of disturbance attenuation of the RMPC and the GIPC algorithms is evaluated by input disturbance $T(k) = 0.5[u(t-10)]$ on the nonlinear model. According to Figures 8 and 9, unlike the RMPC algorithm, the GIPC method rejects this disturbance.

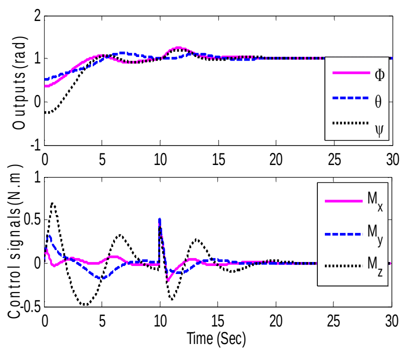

Figure 8. The GIPC performance against nonlinearity

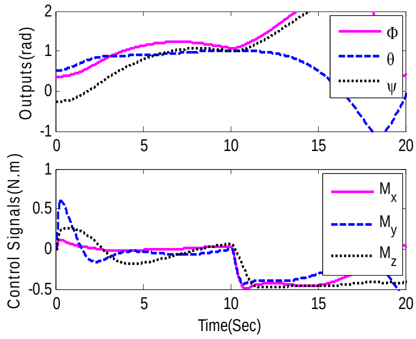

Figure 9. The RMPC performance against nonlinearity

According to the subsection C from previous part, to solve this problem the MRAS algorithm is incorporated into the RMPC strategy. Figure 10 shows the block diagram of combinational RMPC. Figure 11 illustrates the step response and control effort of the nonlinear model of ACS using the combinational RMPC method. Obviously, the effect of external disturbance on Euler angles and control actions is removed.

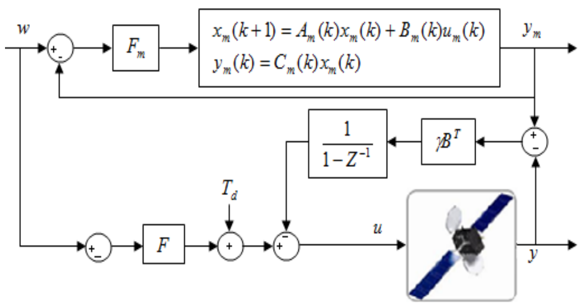

Figure 10. The block diagram of combinational RMPC

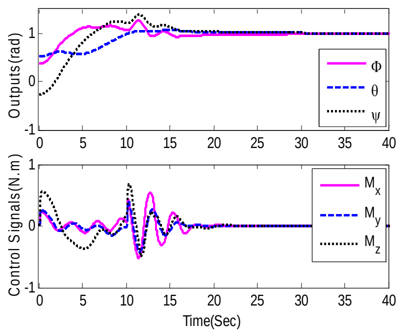

Figure 11. The combinational RMPC performance against nonlinearity

4.5 The Controllers Performance against Increasing Uncertainty

In order to study the robustness of the RMPC and GIPC, we increase the moment of inertia uncertainty in the presence of the disturbance $T(k) = 0.5[u(t-10)]$. Figures 12 and 13 show the step response and control actions of ACS using the combinational RMPC and GIPC methods in the +70% perturbed conditions and the mentioned disturbance. According to Figures 13 and 14, although the closed loop system with both controllers remains stable under the moment of inertia uncertainty and external disturbance, the control actions in the GIPC method is saturated. In contrast, in the proposed RMPC method the control actions close to zero after a little oscillation and they are not saturated.

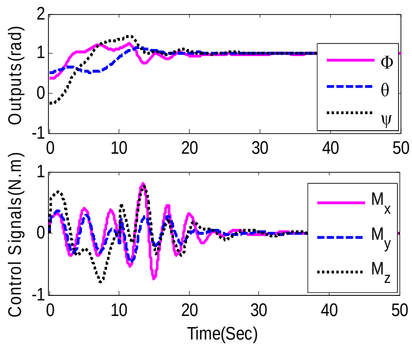

Figure 12. The step response and control effort of ACS using RMPC method in the +70% perturbed condition

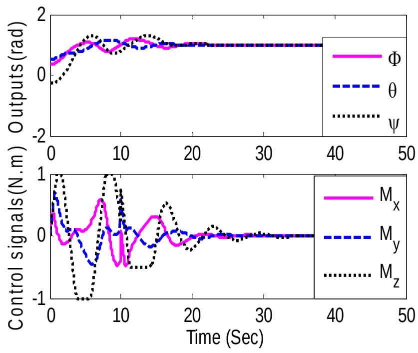

Figure 13. The step response and control effort of ACS using GIPC method in the +70% perturbed condition

5. CONCLUSION

In this paper, a three axis combinational RMPC controller is designed for ACS system so that the closed loop system is robust against the moment of inertia uncertainty, input constrain and external disturbance. The control law is a state feedback that its gain is obtained by solving a convex optimization problem subject to several LMIs which they guarantee the asymptotic stability of system. To avoid the undesirable effect of reaction wheels' saturation the input constrains are added to the mentioned LMIs. Moreover, to access disturbance attenuation of the RMPC strategy, the MRAS algorithm is incorporated into the RMPC algorithm. The effectiveness of the proposed controller was evaluated by means of extensive simulations using the nonlinear model of ACS. The experiments in this paper show that although the RMPC algorithm in the absence of the disturbance has a high robust performance, in the presence of disturbance the controller cannot stabilize the closed loop system. For this reason, the more robust version RMPC by using the MRAS algorithm is proposed in this paper. Furthermore, although both controllers combinational RMPC and GIPC have a good performance against uncertainty and disturbance, the proposed RMPC has the better attenuation capability for disturbance and uncertainty parameters. This means that the robustness in the combinational RMPC is more than the GIPC.

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BIOGRAPHY OF AUTHOR

Fatem Pirouzmand received the BSc degree in electronic engineering from the Azad University in 2007. She received MSc degree in Science and research university in control field in 2013. Her research interests include robust model predictive control and optimal control. She is now working in the instrumentation field in the oil refinery.