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# Power System State Estimation with Weighted Linear Least Square

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**ABSTRACT**

Power system state estimation with conventional method, Weighted Linear Least (WLS), is performed in two steps. In the first stage the observability of system is done and then is system is observed state estimation is carried out. Otherwise estimator is disrupted and is not able to estimate the states. The another estimator, Weighted least Absolute Value (WLAV) has presented which is able to estimate the system states in all situations but this estimator have auxiliary variable which reduces the convergence rate estimator. In this paper, a new method, Weighted Linear Least Square (WLLS), is proposed for power system state estimation. The proposed method, WLLS, has fewer variables than WLAV. The objective function of the proposed method is linear therefore this estimator is able to estimate the states in unobservability situations. Case studies on IEEE 14 bus system show WLLS has good accuracy and speed and is able to estimate the states in all conditions.



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## 1. INTRODUCTION

In power systems state estimator complete received data from the SCADA system. Power system state estimation is not required, if the received data from SCADA are adequate and correct. Therefore in power systems Energy Management System (EMS) was created which firstly preserve the system in normal conditions and secondary operate in optimum economic conditions. A series data of system are required to accomplish this goal. This data are transmitted by SCADA system, to EMS. State estimation of power systems is the interface of SCADA and EMS. However, before state estimation an estimator should be test the observability of system. Then estimate states with minimum error. Remarkably observability of power system depends on the number and location of installed meters.

State estimation methods can be divided into two groups. The first group is based on mathematical methods and the second group is based on intelligent methods. Weighted Linear Least (WLS), Weighted least Absolute Value (WLAV) and Estimation with Non-Fixed Error (M-Estimator) are famous presented mathematical methods [1-10]. Among intelligent techniques can be point out Fuzzy Inference System (FIS), state estimation based on Neural Network (NT) and Adaptive Neuron Fuzzy Inference System (ANFIS) [11-15]. Intelligent methods of mathematical methods for estimating required less time, but they have not sufficient accurate. The accuracy and speed of WLS and WLAV are better than the other mathematical methods [16-17].

Accordingly, in this paper are studied two methods for power system state estimation, WLS and WLAV. Then a new method, Weighted Linear Least Square (WLLS), is proposed. The proposed method has fewer variables than WLAV and has more performance range than the WLS So that even in unobservability of system this estimator can be estimate power system states. Meanwhile, accuracy and speed of the proposed method is reasonable and acceptable. Case study on IEEE 14 bus system show the accuracy and performance range of the proposed method is acceptable.

## 2. STATE ESTIMATION WITH WLS

State estimation function is the measured amounts likeness to corresponding calculated amounts. Several methods have proposed for creating state estimation function. Most of them are based on probabilistic methods so thus, the measurement error is assumed to have a certain probability distribution.

In power systems the measurement error is generally assumed as normal probability distribution according to equation (1), which is also quite close to reality.

$$pdf(Z_{est}) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2}(\frac{Z_{meas}-Z_{est}}{\sigma})^2} \quad (1)$$

Where:
$Z_{meas}$: measured value
$Z_{est}$: The estimated value (expected $Z=E(Z)$)
$\sigma$: standard deviation of $Z$

The aim is to maximize the probability $pdf(Z_{est})$ in state estimation by probabilistic method. In general, aim is maximizing the measurements multiplied *pdf*s according to equation (2).

$$\max \ f = pdf(Z_{est,1})\ldots pdf(Z_{est,m}) \quad (2)$$

Where:
$m$: number of measurements

Equation (2) can be rewrite as a logarimic function for simplifying the optimization.

$$\begin{aligned}\max \ \log f &= \sum_{i=1}^{m} \log pdf(Z_{est,i}) \\ &= -\frac{1}{2}\sum_{i=1}^{m}[(\frac{Z_{meas,1}-Z_{est,i}}{\sigma_i})^2 \\ &\quad -\frac{m}{2}\log 2\pi - \sum_{i=1}^{m}\log \sigma_i]\end{aligned} \quad (3)$$

The second and third sentences are constant then the final objective function will be as equation (4).

$$\min \ J = \sum_{i=1}^{m}(\frac{Z_{meas,1}-Z_{est,i}}{\sigma_i})^2 \quad (4)$$

In power systems $Z_{est}$ is dependent on state variables of the system, and the objective function will depend on the state variables according to equation (6).

$$Z_{est,i} = h_i(\hat{x}) \quad (5)$$

$$J(\hat{x}) = \sum_{i=1}^{m}(\frac{Z_{meas,1}-h_i(\hat{x})}{\sigma_i})^2 \quad (6)$$

Where the $h_i(x)$ is called the i[th] measurement function. If $Z_{meas}$ and $h(x)$ are written as the matrix $J(x)$ can be simplified as equation (7).

$$J(\hat{x}) = [Z - h(\hat{x})]^T R^{-1} [Z - h(\hat{x})] \tag{7}$$

Where we have:

$$Z = \begin{bmatrix} Z_1 \\ Z_2 \\ . \\ . \\ Z_m \end{bmatrix} \tag{8}$$

So instead of $Z_{meas,i}$, $Z_i$ is used.

$$h(\hat{x}) = \begin{bmatrix} h_1(x_1, x_2 \ldots, x_n) \\ h_2(x_1, x_2 \ldots, x_n) \\ . \\ . \\ h_m(x_1, x_2 \ldots, x_n) \end{bmatrix} \tag{9}$$

$$R = \begin{bmatrix} \sigma_1^2 & & & & \\ & \sigma_2^2 & & & \\ & & . & & \\ & & & . & \\ & & & & \sigma_m^2 \end{bmatrix} \tag{10}$$

For optimization of function $J(x)$ exist several method which the usual method is Gauss-Newton (GN). This optimization method is based on iterative method and states matrix is optimized as equation (11).

$$x^{k+1} = x^k - \left[G(x^k)\right]^{-1} g(x^k) \tag{11}$$

Where $k$ is k[th] repeating stepsa and $g(x^k)$ and $G(x^k)$ are obtained from equations (12) to (14).

$$G(x^k) = \frac{\partial g(x^k)}{\partial x} = H^T(x^k) R^{-1} H(x^k) \tag{12}$$

$$g(x^k) = H^T(x^k) R^{-1} (Z - h(x^k)) \tag{13}$$

$$H(\hat{x}) = \frac{\partial h(\hat{x})}{\partial x} \tag{14}$$

$G(x)$ is called the gain matrix.

Therefore power system state estimation with WLS algorithm and GN optimization can present such as figure (1).

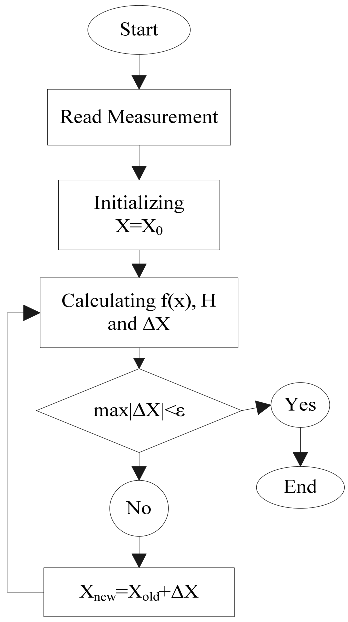


Figure 1. WLS state estimation algorithm

The value of $\varepsilon$ determines accuracy of state estimation. Whatever $\varepsilon$ is little the results are accurate and calculate time increase.

## 3. STATE ESTIMATION WITH WLAV

WLAV state estimation is similar WLS method with this difference that in WLAV method is used absolute error instead square error.

$$f(\hat{x}) = \sum_{i=1}^{m} w_i \left| Z_i - h_i(\hat{x}) \right| \tag{15}$$

Where $w_i$ is the reverse of $\sigma_i$.

$$w_i = \frac{1}{\sigma_i} \Rightarrow W = R^{-1} \tag{16}$$

Optimizing the function $f(x)$ is linear so that the Taylor approximation $h(x)$ at the point $x_0$ is used.

$$\Delta Z = H \Delta x + e \tag{17}$$

Where we have:

$$\Delta Z = Z - h(x_0) \tag{18}$$

$$H = \frac{\partial h(x)}{\partial x}\Big|_{x = x_0} \tag{19}$$

$$\Delta x = x - x_0 \tag{20}$$

It will assume absolute error ($|e_i|$) is less than a certain amount, such as $\varepsilon_i$:

$$|e_i| \le \varepsilon_i \tag{21}$$

The above equation can convert to two equations by adding two non-negative slack variables.

$$e_i - l_i = -\varepsilon_i \quad (22)$$

$$e_i + k_i = \varepsilon_i \quad (23)$$

Which we can write:

$$e_i = u_i - v_i \quad (24)$$

So that:

$$u_i = \frac{1}{2} l_i \quad (25)$$

$$v_i = \frac{1}{2} k_i \quad (26)$$

Whatever $\varepsilon_i$ is smaller, it is like the $e_i$ is minimized. So the objective function *f(x)* has changed to objective function *f´(x)*.

$$f'(\hat{x}) = \sum_{i=1}^{m} w_i|\varepsilon_i| = \sum_{i=1}^{m} w_i(u_i + v_i) \quad (27)$$
$$s.t \qquad \Delta Z^k = H(x^k)\Delta x^k + u - v$$

The function *f´(x)* can be optimized by linear programming (LP). Therefore the algorithm for optimization function *f´(x)* can be presented as follows.

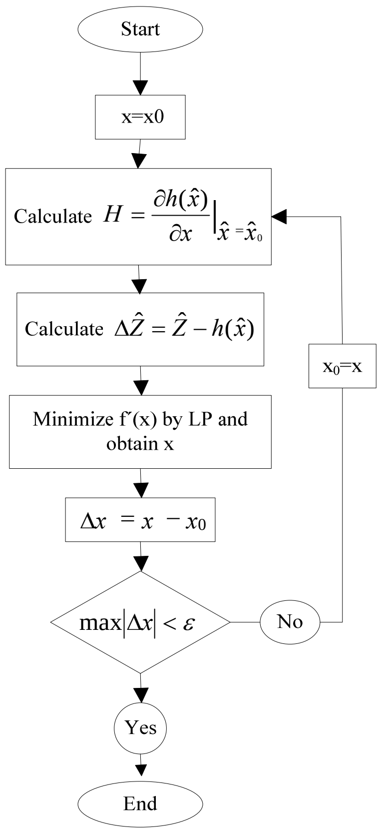


Figure 2. WLAV state estimation algorithm

Optimizing the $f'(x)$ by linear programming, it is necessary the absolute is eliminated from objective function. To eliminate absolute in WLAV method the new variables u and v was added to the objective function.

## 4. STATE ESTIMATION WLLS

Although the WLAV It is optimized by linear programming but adding the new auxiliary variables to objective function, the number of variables increases. It will increase the optimizing time so that among WLAV and WLS, WLS has better speed [16-17]. On the other hands due to the non-linearity of the objective function, WLS in some systems such as systems with diffused measurements can not find reverse of gain matrix. In other words, there is not inverse of H matrix in some systems and it is lead to WLS is disrupted.

Accordingly, this paper proposes WLLS method that it combines WLAV and WLS methods for optimization. In WLLS, according equation (28) the objective function is same WLS objective function which has applied some changes.

$$f(\hat{x}) = \sum_{i=1}^{m} w_i \left| Z_i - h_i(\hat{x}) \right|^2 \tag{28}$$

With placement $e_i$ instead measurement error we have:

$$f(\hat{x}) = \sum_{i=1}^{m} w_i \left| e_i \right|^2 \tag{29}$$

Whit linearing $h(x)$ in around the point $x_0$, we can write:

$$\begin{aligned} e_i &= \Delta Z_i - H \Delta x_i \\ &= Z_i - h_i(x_0) - H(x_0)x + H(x_0)\, x_0 \end{aligned} \tag{30}$$

The equation (24) is replaced in equation (29):

$$\begin{aligned} g(\hat{x}) &= \sum_{i=1}^{m} w_i \left| \Delta Z_i - H \Delta x_i \right|^2 \\ &= \sum_{i=1}^{m} w_i \left| Z_i - h_i(x_0) - H(x_0)x + H(x_0)x_0 \right|^2 \end{aligned} \tag{31}$$

Accordingly, C and d are as follows:

$$C = -H(x_0) \tag{32}$$

$$d = -H(x_0)\, x_0 + h_i(x_0) - Z_i \tag{33}$$

Therefore the final objective function will be as equation (34):

$$f(\hat{x}) = \sum_{i=1}^{m} w_i \left| C\hat{x} - d \right|^2 \tag{34}$$

The above function was obtained by linearing H matrix so it should be optimized by iteration method. State estimation algorithm based on WLLS can be considered as figure (3).

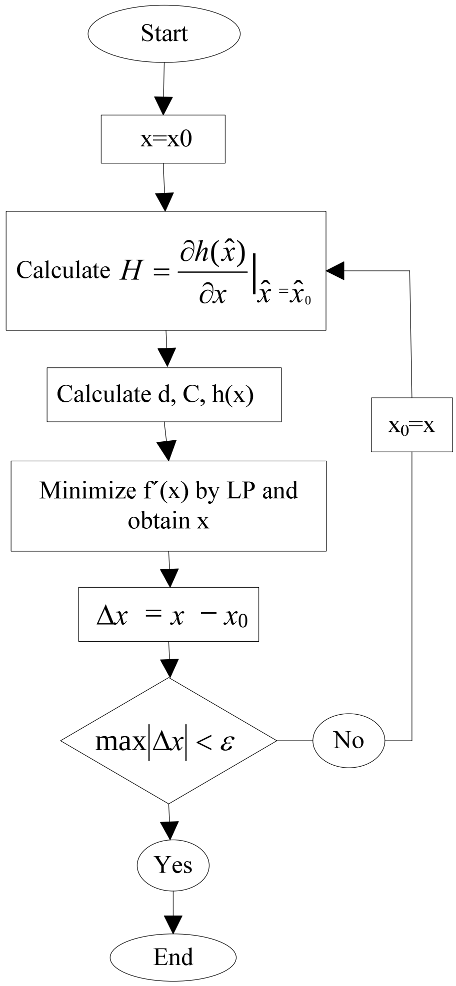


Figure 3. WLLS state estimation algorithm

## 5. CASE STUDY

According to references [16-17] WLS method is faster than the WLAV method. So in this case study WLS and WLLS is compared. is selected. The IEEE 14 bus system data for state estimation has presented in [5].

The objective function in WLS is optimized after 109 iterations ($\varepsilon$=$10^{-7}$). The resulted estimated values have presented in table (1).

Table 1. Estimated values by WLS on IEEE 14 bus system

| Measurements | Measured values | Estimated values | Actual values |
|---|---|---|---|
| | pu | Pu | pu |
| $V_1$ | 1.05 | 1.0613 | 1.06 |
| $V_3$ | 1 | 1.0136 | 1.01 |
| $V_5$ | 1.04 | 1.0237 | 1.02 |
| $V_8$ | 1.09 | 1.09 | 1.09 |
| $V_9$ | 1.05 | 1.0569 | 1.056 |
| $V_{11}$ | 1.057 | 1.0588 | 1.057 |
| $V_{13}$ | 1.03 | 1.0414 | 1.05 |
| $P_{1,2}$ | 1.4276 | 1.4441 | 1.5688 |
| $P_{2,5}$ | 0.4072 | 0.3809 | 0.4152 |
| $P_{5,6}$ | 0.4076 | 0.4153 | 0.4409 |
| $P_{6,13}$ | 0.1714 | 0.1665 | 0.1775 |
| $P_{7,8}$ | 0 | 0 | 0 |
| $P_{12,13}$ | 0.0158 | 0.0303 | 0.0161 |
| $P_{13,14}$ | 0.0518 | 0.0517 | 0.0564 |
| $P_{3,2}$ | -0.6648 | -0.6632 | -0.7091 |

| Measurements | Measured values | Estimated values | Actual values |
|---|---|---|---|
| | pu | Pu | pu |
| $P_{4,2}$ | -0.5043 | -0.5081 | -0.5445 |
| $P_{7,4}$ | -0.2581 | -0.2544 | -0.2807 |
| $P_{9,4}$ | -0.1597 | -0.1505 | -0.1608 |
| $P_{11,6}$ | -0.0661 | -0.0629 | -0.073 |
| $P_{10,9}$ | -0.0495 | -0.0463 | -0.0521 |
| $Q_{2,5}$ | 0.0108 | 0.0131 | 0.0117 |
| $Q_{10,11}$ | -0.015 | -0.0147 | -0.0162 |
| $Q_{13,14}$ | 0.016 | -0.0004 | 0.0175 |
| $Q_{5,1}$ | 0.022 | 0.0118 | 0.0223 |
| $Q_{4,2}$ | 0.0288 | 0.0185 | 0.0302 |
| $Q_{5,4}$ | -0.1332 | -0.1251 | -0.142 |
| $Q_{7,4}$ | 0.1032 | 0.0956 | 0.1138 |
| $Q_{9,4}$ | 0.0159 | 0.0112 | 0.0173 |
| $Q_{11,6}$ | -0.0313 | -0.025 | -0.0344 |
| $Q_{12,6}$ | -0.0221 | 0.0117 | -0.0235 |
| $Q_{14,9}$ | -0.0316 | -0.0491 | -0.0336 |
| $P_2$ | 0.1674 | 0.1792 | 0.183 |
| $P_4$ | -0.4763 | -0.4737 | -0.478 |
| $P_6$ | -0.1021 | -0.0991 | -0.112 |
| $P_9$ | -0.2939 | -0.2933 | -0.295 |
| $P_{10}$ | -0.0833 | -0.084 | -0.09 |
| $P_{12}$ | -0.0601 | -0.0554 | -0.061 |
| $Q_2$ | 0.3042 | 0.2938 | 0.3086 |
| $Q_3$ | 0.0571 | 0.0393 | 0.0608 |
| $Q_6$ | 0.051 | 0.0554 | 0.0523 |
| $Q_9$ | -0.1606 | -0.1648 | 0.1762 |
| $Q_{14}$ | -0.048 | -0.0478 | -0.05 |

IEEE 14 bus system is assumed with the previous parameters for comparing the WLS and WLLS estimators. The resulted estimated values with the same previous error ($\varepsilon$=$10^{-7}$) by WLLS are presented in table (2).

Table 2. Estimated values by WLLS on IEEE 14 bus system

| Measurements | Measured values | Estimated values | Actual values |
|---|---|---|---|
| | pu | Pu | pu |
| $V_1$ | 1.05 | 1.0519 | 1.06 |
| $V_3$ | 1 | 1.0036 | 1.01 |
| $V_5$ | 1.04 | 1.0281 | 1.02 |
| $V_8$ | 1.09 | 1.09 | 1.09 |
| $V_9$ | 1.05 | 1.0534 | 1.056 |
| $V_{11}$ | 1.057 | 1.057 | 1.057 |
| $V_{13}$ | 1.03 | 1.0319 | 1.05 |
| $P_{1,2}$ | 1.4276 | 1.4351 | 1.5688 |
| $P_{2,5}$ | 0.4072 | 0.3394 | 0.4152 |
| $P_{5,6}$ | 0.4076 | 0.4192 | 0.4409 |
| $P_{6,13}$ | 0.1714 | 0.1626 | 0.1775 |
| $P_{7,8}$ | 0 | 0 | 0 |
| $P_{12,13}$ | 0.0158 | 0.0428 | 0.0161 |
| $P_{13,14}$ | 0.0518 | 0.0527 | 0.0564 |
| $P_{3,2}$ | -0.6648 | -0.7068 | -0.7091 |
| $P_{4,2}$ | -0.5043 | -0.4904 | -0.5445 |
| $P_{7,4}$ | -0.2581 | -0.2564 | -0.2807 |
| $P_{9,4}$ | -0.1597 | -0.1486 | -0.1608 |
| $P_{11,6}$ | -0.0661 | -0.0627 | -0.073 |
| $P_{10,9}$ | -0.0495 | -0.0393 | -0.0521 |
| $Q_{2,5}$ | 0.0108 | -0.0345 | 0.0117 |
| $Q_{10,11}$ | -0.015 | -0.0196 | -0.0162 |
| $Q_{13,14}$ | 0.016 | -0.0109 | 0.0175 |
| $Q_{5,1}$ | 0.022 | 0.0663 | 0.0223 |
| $Q_{4,2}$ | 0.0288 | 0.0573 | 0.0302 |
| $Q_{5,4}$ | -0.1332 | -0.1044 | -0.142 |
| $Q_{7,4}$ | 0.1032 | 0.0744 | 0.1138 |
| $Q_{9,4}$ | 0.0159 | -0.0003 | 0.0173 |
| $Q_{11,6}$ | -0.0313 | -0.0233 | -0.0344 |
| $Q_{12,6}$ | -0.0221 | 0.0397 | -0.0235 |

| Measurements | Measured values | Estimated values | Actual values |
|---|---|---|---|
| | pu | Pu | pu |
| $Q_{14,9}$ | -0.0316 | -0.063 | -0.0336 |
| $P_2$ | 0.1674 | 0.1756 | 0.183 |
| $P_4$ | -0.4763 | -0.4751 | -0.478 |
| $P_6$ | -0.1021 | -0.0978 | -0.112 |
| $P_9$ | -0.2939 | -0.2933 | -0.295 |
| $P_{10}$ | -0.0833 | -0.0866 | -0.09 |
| $P_{12}$ | -0.0601 | -0.0519 | -0.061 |
| $Q_2$ | 0.3042 | 0.2939 | 0.3086 |
| $Q_3$ | 0.0571 | -0.0043 | 0.0608 |
| $Q_6$ | 0.051 | 0.0586 | 0.0523 |
| $Q_9$ | -0.1606 | -0.1761 | 0.1762 |
| $Q_{14}$ | -0.048 | -0.0511 | -0.05 |

The optimum results of objective function and iteration number have presented in table (3).

Table 3. The optimum *J(x)* and iteration number

| Estimator | $J(x)_{final}$ | Iteration number |
|---|---|---|
| WLS | 11.7290 | 111 |
| WLLS | 26.2767 | 114 |

The optimum *J(x)* in WLLS is more than the optimum *J(x)* in WLS but it should be noted that:

- The optimum *J(x)* in WLLS is acceptable by considering the number of measurements.
- In some situations, WLS estimator can not find reverse of gain matrix. One such situation is when the state variables are more than the number of measurements or when the measurements are diffused. Testing the first situation on IEEE 14 bus system shoes WLS is unable in estimation of states while WLLS can be estimate the states in this situation.

## 6. CONCLUSION

Power system state estimation is done in two ways. The first method involves the estimation of the state by mathematical methods and the methods based on intelligent techniques such as fuzzy logic or neural networks. Intelligent methods need less time than mathematical methods for estimating but they are not accurate sufficiently. The intelligent methods need many data sets to train which data collection and model learning would be nearly impossible in real systems. Among mathematical methods, WLS has suitable accuracy and speed than others.

In this paper, a new estimator, Weighted Linear Least Square (WLLS), has been proposed. The proposed estimator is better than WLS estimator so that WLS can not find the reverse of gain matrix in some situations therefore it is disturbed in some situations but WLLS don't use gain matrix and therefore it can estimate the states in all situations.

IEEE 14 bus system was selected for the case study. State estimation is done by WLS and WLLS. The results show that the proposed WLLS estimator has acceptable accuracy and is able to estimate the states in all conditions.

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