Robust-Neural Observer Design for Discrete-Time Uncertain Non-Affine Nonlinear System

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ABSTRACT

This paper proposed a new nonlinear discrete-time robust-neural observer (DTRNO) which capable to give estimation for the states of Discrete-Time Uncertain Non-affine Non-linear Systems in presence of external disturbances. The Neural network is a kind of discrete-time Multi Layered Perceptron (MLP) which Trained with an Extended Kalman-Filter (EKF) based algorithm, which this neural observer is robust in presence of external and internal uncertainties, using a parallel configuration.This work includes the stability proof of the estimation error on the basis of the Lyapunov approach, and for demonstrate observer performance an Uncertain Non-affine Nonlinear Systems have been simulated to formulations validate the theoretical. Simulation results confirm the proficiency of the DTRNO even at the different operating conditions and presence of parameters uncertainties.

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1. INTRODUCTION

During the recent decades, state estimation of dynamic systems and the state observation problem has been an active topic of research in different areas such as automatic control applications, fault detection, monitoring, modeling [7], etc. Due to cost, and technological constraints usually assume complete accessibility for the system state, which is not always possible [6]. It is noted that most practical systems are nonlinear and it is difficult to design a performance controller or observer. So far, the linearization techniques can be applied to overcome these problems. However, this linearization can limit enormously the performances of such approaches of control and observation. In this case, the use of neural networks (NNs) permits to approximate suitably the nonlinear functions and then to bypass the linearization problem [1], [2].

The state observation problem has been widely developed in the literature, and used in numerous applications. However in most cases, the state variables are rarely available for direct online measurements. Furthermore, there is a substantial requirement for reliable reconstruction of the state variables, especially when they are required in the synthesis of control and observation laws or for process monitoring purposes [4], [26], [16]. However, in most realistic cases merely input and output of the system are measurable. Therefore, estimating the state variables by observers plays a crucial role in the control of processes to achieve better performances [20], [13]. Observers design process is too complex have a good performance even in presence of model and disturbance uncertainties are called robust [5], [7], [11], [27]. Newly, other kind of observers has emerged: neural observers, for unknown plant dynamics [9], [15], [17], [18], [22] but all the approaches mentioned above need the previous knowledge of the plant model, at least partially.

There exist different training algorithms for neural networks, which, however, normally encounter some technical problems such as local minima, slow learning, and high sensitivity to initial conditions, among others [10]. As a viable alternative, new training algorithm, e.g., those based on Kalman filtering have been proposed [14], [21], [23], [24], [25]. Due to the fact that training a neural networks typically results in a nonlinear problem, an extended Kalman filter (EKF) is required to be used [3], [14]. EKF training for NNs allows the reduction of the epoch size and the number of required neurons [14]. Considering these two facts, we propose the use of the EKF training for DTNO in order to model complex Discrete Time Uncertain Nonlinear Systems (DTUNS).Parameter estimation , and state estimation are related in the sense of how the measurement from sensors can be used to obtain an accurate mode of the plant to be controlled. So, the learning algorithm for the DTUNS is implemented using an EKF. The respective stability analysis, based on the Lyapunov approach, is included for the proposed scheme. The applicability of this scheme is illustrated by discrete-time state estimation for a nonlinear systems.

2. DISCRETE TIME NONLINEAR SYSTEM

In this section, important mathematical preliminaries required in future sections are presented and then the state of a discrete-time nonlinear system, which is assumed to be observable, is provided.

2.1. Mathematical Preliminaries

Through this brief, we use k as the sampling step, $k \in 0 \cup Z^{+}$, as the absolute value and, as the Euclidian norm for vectors and as any adequate norm for matrices which close follows [8]. Consider a multiple input–multiple output (MIMO) nonlinear system;

$$x(k+1) = F(x(k), u(k)) \tag{1}$$

$$y(k) = h(x(k)) \tag{2}$$

Where $x \in \Re^{n}$, $u \in \Re^{m}$ and $F \in \Re^{n} \times \Re^{m} \to \Re^{n}$ is non-linear function.

Definition 1: System (1) is said to be forced, or to have inputs. In contrast, a system described by an equation without explicit presence of an input *u*, that is;

$$x(k+1) = F(x(k)) \tag{3}$$

is said to be unforced. It can be obtained after selecting the input *u* as a feedback function of the state

$$u(k) = \xi(x(k)) \tag{4}$$

Such substitution eliminates *u* and yields an unforced system [19];

Definition 2: The solution of (1)–(3) is semi globally uniformly ultimately bounded (SGUUB), if for any Ω , which is a compact subset of $\Re^{n}$ and all $x(k_0) \in \Omega$, there exists an $\varepsilon > 0$, and a number $N(\varepsilon, x(k_0))$ such that $\|x(k)\| < \varepsilon$ for all $k \geq k_0 + N$, [29].In other words, the solution of (1) is said to be SGUUB if, for any a priory given (arbitrarily large) bounded set Ω and any a priory given (arbitrarily small) set Ω_0, which contains $(0,0)$ as an interior point, there exists a control (3) such that every trajectory of the closed-loop system starting from Ω enters the set $\Omega_0 = \{x(k) | \|x(k)\| < \varepsilon\}$, in a finite time and remains in it thereafter, as is displayed in Figure 1.

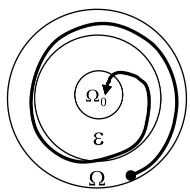

Figure 1. SGUUB, schematic representation

Theorem 1: Let $V(x(k))$ be a Lyapunov function for a discrete-time system (1), which satisfies the following properties:

$$\gamma_1(x(k)) \le V(x(k)) \le \gamma_2(\|x(k)\|)$$
$$V(x(k+1)) - V(x(k)) = \Delta V(x(k)) \le -\gamma_3(\|x(k)\|) + \gamma_3(\xi) \quad (6)$$

Where ζ is a positive constant, γ_1 and γ_2 are strictly increasing functions, and γ_3 is a continuous non-decreasing function. Thus if

$$\Delta V(x) < 0 \quad \text{For} \quad \|x(k)\| > \xi \quad (7)$$

Then $x(k)$ is uniformly ultimately bounded, i.e., there is a time instant k_T such that $\|x(k)\| < \xi, \forall k < k_T$ [8].

Definition 3: A subset $S \in \Re^n$ is bounded if there exists $r > 0$ such that $\|x\| \le r$ for all $x \in S$ [19].

2.2. Discrete-Time Nonlinear System

To estimate the state of a discrete-time nonlinear system, which is assumed to be observable, given by;

$$x(k+1) = F(x(k), u(k)) + d(k)$$
$$y(k) = h(x(k)) \quad (8)$$

where $x \in \Re^n$ is the state vector of the system, $u(k) \in \Re^m$ is the input vector, $y(k) \in \Re^p$ is the output vector, $C \in \Re^{p\times n}$ is a known output matrix, $d(k) \in \Re^n$ is a disturbance vector, G and F are smooth vectors field, G_i and F_i theirs entries. Hence, (8) can be rewritten as;

$$x(k) = [x_1(k)\dots x_i(k)\dots x_n(k)]^T, d(k) = [d_1(k)\dots d_i(k)\dots d_n(k)]^T$$
$$x_i(k+1) = F_i(x(k), u(k)) + d_i(k), \; i = 1,\dots,n \quad (9)$$
$$y(k) = Cx(k)$$

3. NEURAL STATE ESTIMATION

A Multi Layer Perceptron (MLP) is a feed-forward artificial neural network model that maps sets of input data onto a set of appropriate outputs. A MLP consists of multiple layers of nodes in a directed graph, with each layer fully connected to the next one. Except for the input nodes, each node is a neuron with a nonlinear activation function. MLP is a modification of the standard linear perceptron and can distinguish data that are not linearly separable. The structure of neural network used the proposed observer MPL neural network with four inputs, five Neurons in the hidden layer and an output is shown in Figure 2

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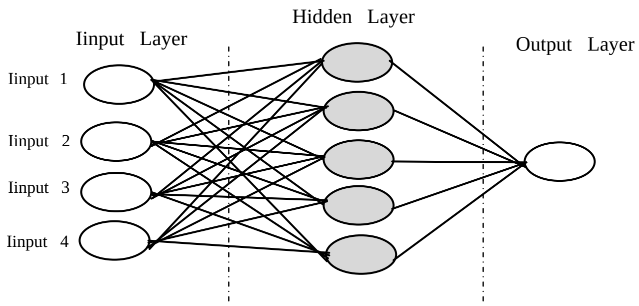

Figure 2. The structure of the neural network

By adding and subtracting Phrase $Ax(k)$ of equation (8)

$$\begin{aligned}\hat{x}(k+1) &= Ax(k) + g\,(x(k),u(k)) \\ y\,(k) &= Cx\,(k)\end{aligned} \tag{10}$$

A is Optional Horowitz matrix, (A,C) are observable, and $g\,(x(k),u(k))$ includes uncertain terms and disturbance system, where,

$$g\,(x(k),u(k)) = F\,(x\,(k),\,u\,(k)) + d(k) - Ax(k) \tag{11}$$

The key to designing a neuro-observer is to employ a neural network to identify the nonlinearity and a conventional observer to estimate the states. By invoking a Luenberger observer [28], the observer model of the system (10) can be defined as follows;

$$\begin{aligned}\hat{x}(k+1) &= A\hat{x}(k) + \hat{g}\,(\hat{x}(k),u(k)) + \Gamma(y(k) - C\hat{x}(k)) \\ \hat{y}\,(k) &= C\hat{x}\,(k)\end{aligned} \tag{12}$$

Where $\hat{x}$ denotes the state of the observer, and the observer gain $\Gamma \in \Re^{n\times m}$ is selected such that $(A-\Gamma C)$ becomes a Hurwitz matrix. It should be noted that the gain Γ is guaranteed to exist; since A can be selected such that (C,A) is observable. The structure of a neuro-observer is shown in Figure 3.

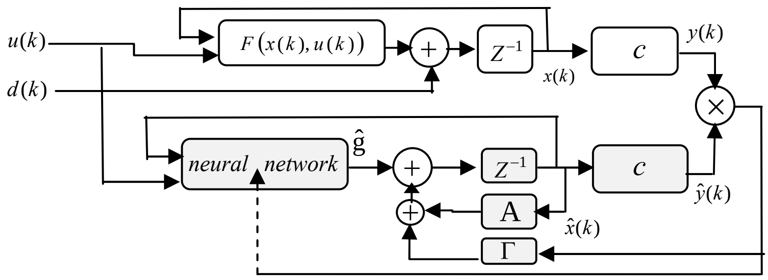

Figure 3. The structure of the proposed neural network observer

To approximate the nonlinear function $g\,(x(k),u(k))$ a multilayer NN is considered. So, a multilayer NN with sufficiently large number of hidden layer neurons can estimate the unknown function $g\,(x(k),u(k))$ as follows:

$$g = W^T\sigma\left(V^T\overline{x}\right) + \varepsilon \tag{13}$$

Where W and V are the weight matrices of the output and hidden layers, respectively, $\overline{x} = [x\ u]$, ε is the bounded neural network approximation error, and σ is the transfer function of the hidden neurons that is usually considered as a tangent hyperbolic function presenting below:

$$\sigma\left(V_i\overline{x}\right) = \frac{2}{1+\exp^{-2V_i\overline{x}}} - 1 \tag{14}$$

To obtain a linear in-parameter neural network fixing the weights is required, so the first layer as V = I. Then, the model can be expressed as

$$g = W^T\sigma\left(\overline{x}\right) + \varepsilon \tag{15}$$

The proposed observer is then given by;

$$\hat{g}\left(\hat{x}(k),\, u(k)\right) = W^{*T} \tanh\left(\hat{x}(k),\, u(k)\right) + \varepsilon \quad (16)$$

On the other hand, by defining the state estimation error $\tilde{x}(k) = x(k) - \hat{x}(k)$ and using (12), and (16), the error dynamics can be expressed as;

$$\hat{g}\left(\hat{x}(k), u(k)\right) = \hat{W}^T \tanh\left(\hat{x}(k), u(k)\right) \quad (17)$$

,

$$\hat{x}(k) = [\,\hat{x}_1(k), \hat{x}_2(k), \hat{x}_3(k) \;\; \ldots \;\; \hat{x}_i(k) \;\; \ldots. \;\; \hat{x}_n(k)]$$

$$\hat{x}(k+1) = Ax(k) + \hat{W}^T \tanh\left(\hat{x}(k), u(k)\right) + \Gamma(y(k) - C\hat{x}(k)) \quad (18)$$

Once the structure of the neural network is known, a proper learning rule should be defined to train the network. This weight-updating mechanism is usually defined in such a way that the stability of the observer is guaranteed. Therefore, the weight estimation error is defined as;

$$\tilde{W}_i(k) = w_i(k) - W_i^* \quad (19)$$

And

$$\tilde{x}_i(k) = x_i(k) - \hat{x}_i(k) \quad (20)$$

Since W_i^* is constant

$$\tilde{W}_i(k+1) - \tilde{W}_i(k) = W_i(k+1) - W_i(k), \forall k \in 0 \cup Z^+ \quad (21)$$

3.1. Extended Kalman Filter

Kalman filter, which is the set of mathematical equations, is considered as one of the important discoveries in the control theory principles. E. Kalman’s article was published in the year 1960. Its most immediate applications were in control of complex dynamic systems, such as manufacturing processes, aircrafts, ships or spaceships (it was part of the Apollo onboard guidance system). It was and still is frequently used not only in automation, but also in the graphical and economical applications, etc. However, the Extended Kalman Filter started to appear in the neural network training applications only relatively recently, which was caused by the progress of computer systems development. When the model is nonlinear, which is the case of neural networks, we have to extend Kalman filter using linearization procedure. Resulting filter is then called extended Kalman filter (EKF) [12].The weight vectors are updated online with a decoupled EKF, described by;

$$W_i(k+1) = W_i(k) + \eta_i K_i(k) e(k)$$

$$K_i(k) = P_i(k) H_i(k) M_i(k), \qquad i = 1,\ldots,n$$

$$P_i(k+1) = P_i(k) - K_i(k) H_i^T(k) P_i(k) + Q_i(k) \quad (22)$$

With

$$M_i(k) = [R_i(k) + H_i^T(k) P_i(k) H_i(k)]^{-1} \quad (23)$$

where $w_i(k)$ is a vector of all weights, η_i is a function returning a vector of actual outputs, K is the so called Kalman gain matrix, P is the error covariance matrix of the state and H is the measurement matrix (Jacobian). H_i is the partial derivative of the MLP output with respect to the MLP network parameters at the kth iteration of the Kalman recursion.

4. PROOF OF STABILITY: LYAPUNOV'S DIRECT METHOD

Theorem2*:* For uncertain Discrete Time nonlinear dynamic system (8) the Neural-Robust Observer given by equation (12) where $\hat{g}(\hat{x}(k), u(k)) = \hat{W}^T \tanh(\hat{x}(k), u(k))$ and $\hat{W}^T$ trained with the EKF-based algorithm, ensures that the estimation error and the output error are uniformly ultima-tely bounded, moreover network weights remain bounded. The output error

$$e(k) = y(k) - \hat{y}(k) \tag{24}$$

and the estimation error described by; $\tilde{x}_i(k) = x_i(k) - \hat{x}_i(k)$ then the dynamics of $x_i(k+1)$ can be expressed as

$$\tilde{x}_i(k+1) = x_i(k+1) - \hat{x}_i(k+1) \tag{25}$$

Therefore

$$\begin{aligned} &\tilde{x}_i(k+1) = x_i(k+1) - \hat{x}_i(k+1) \Rightarrow \tilde{x}_i(k+1) = Ax(k) \\ &+ W^{*T}\tanh(\hat{x}(k)) - A\hat{x}(k) - \hat{W}^T\tanh(\hat{x}(k)) - \Gamma(y(k) - C\hat{x}(k)) + \varepsilon \Rightarrow \\ &\tilde{x}_i(k+1) = A\tilde{x}(k) + \tilde{W}^T\tanh(\tilde{x}(k)) - \Gamma C\tilde{x}(k) + \varepsilon \end{aligned} \tag{26}$$

These dynamics can be considered as a linear system, where A state matrix, I input matrix and $\tilde{g} = \tilde{W}^T\tanh(\hat{x}(k)) - \Gamma_i C\hat{x}(k) + \varepsilon$ is input. If input a stable linear system is bounded, then output will be bounded, therefore if $\tilde{g}$ Remain bounded then the estimation error is bounded. $\tilde{g}$, and ε are expressed as follows:

$$\hat{g}(k) = \begin{bmatrix} \hat{g}_1(k) \\ \vdots \\ \hat{g}_n(k) \end{bmatrix} \quad , \quad \varepsilon = \begin{bmatrix} \varepsilon_1(k) \\ \vdots \\ \varepsilon_n(k) \end{bmatrix} \tag{27}$$

Where

$$\tilde{g}_i = (W_i^{*T} - \hat{W}_i^T)\tanh(\hat{x}(k)) - \Gamma_i C\hat{x}(k) + \varepsilon_i, i = 1,2,\ldots n \tag{28}$$

W^* is constant Matrix but unknown;

$$\tilde{W}_i(k) = W_i(k) - \hat{W}_i(k), \forall k \in Z \tag{29}$$

According to the EKF algorithm

$$\hat{W}_i(k+1) = \hat{W}_i(k) - \eta_i K_i(k) e(k), \tag{30}$$

$$e(k) = (y(k) - \hat{y}(k)), \tag{31}$$

$$\tilde{W}_i(k+1) = \tilde{W}_i(k) - \eta_i K_i(k) e(k) \tag{32}$$

It is evident that if $\tilde{g}$ was bounded then $\hat{g}$ and error will be bounded. In order to proof, consider the candidate Lyapunov function;

$$V_i(k) = \tilde{W}_i^T P_i(k)\tilde{W}_i(k) + \tilde{g}^T(k) P_i(k)\tilde{g}(k) \tag{33}$$

Whose first increment is defined as

$$\begin{aligned} &\Delta V_i(k) = V_i(k+1) - V_i(k) \\ &\Delta V_i(k) = \tilde{W}_i^T P_i(k+1)\tilde{W}_i(k+1) + \tilde{g}^T(k+1)P_i(k+1)\tilde{g}(k+1) - \tilde{W}_i^T P_i(k)\tilde{W}_i(k) + \tilde{g}^T(k)P_i(k)\tilde{g}(k) \end{aligned} \tag{34}$$

Using (23) and (20) in (43), then

$$\Delta V_i(k) = \left[\tilde{W}_i(k) - \eta_i K_i(k)e(k)\right]^T \sum_i(k)\left[\tilde{W}_i(k) - \eta_i K_i(k)e(k)\right] + \left[\Phi(k) - \Gamma_i C\tilde{x}(k)\right]^T \sum_i(k)\left[\Phi(k) - \Gamma_i C\tilde{x}(k)\right] - \tilde{W}_i^T P_i(k)\tilde{W}_i(k) + \tilde{g}^T(k)P_i(k)\tilde{g}(k) \quad (35)$$

With

$$\sum_i(k) = P_i(k) - \gamma_i(k) + Q_i \quad (36)$$

$$\Phi(k) = \tilde{W}_i(k)\ tanh\ \left(\hat{\tilde{x}}(k)\right) + \varepsilon_i \quad (37)$$

$$\gamma_i(k) = K_i(k)H_i^T(k)P_i(k) - Q_i \quad (38)$$

Hence, (44) can be expressed as

$$\Delta V_i(k) = 2\tilde{W}_i^T(k)P_i(k)\tilde{W}_i(k) - 2\tilde{W}_i(k)\gamma_i(k)\tilde{W}_i(k) + 2\eta^2\ \tilde{x}^T(k)C^T K^T \sum_i(k)K_i(k)C\,\tilde{x}(k) + 2\Phi^T(k)\sum_i(k)\Phi(k) + 2\tilde{x}(k)C^T\Gamma_i^T \sum_i(k)\Gamma_i C\tilde{x}(k) - \tilde{W}_i^T(k)P(k)\ \tilde{W}_i(k) - \tilde{g}^T(k)P_i(k)\tilde{g}(k) \quad (39)$$

Using the inequalities

$$X^T X + Y^T Y \geq 2X^T Y, \quad (40)$$

$$X^T X + Y^T Y \geq -2X^T Y, \quad (41)$$

$$\lambda(P)X^T X \geq -X^T PX \geq -\bar{\lambda}(P)X^T X, \forall X, Y \in \Re^n, \forall P \in \Re^{n\times n}, P = P^T > 0 \quad (42)$$

Then (48) can be rewritten as

$$\begin{aligned}\Delta V_i(k) &\leq \left\|\tilde{W}_i(k)\right\|^2\left(\bar{\lambda}(P_i(k)) - \underline{\lambda}(P_i(k))\right) + 2\|\tilde{x}(k)\|^2\|\eta_i K_i C\|^2\,\bar{\lambda}\left(\sum_i(k)\right) + 4|\varepsilon_i|^2\,\bar{\lambda}\left(\sum_i(k)\right) + \\ &4\left\|\tilde{W}_i(k)\right\|^2\|\tanh(\tilde{x}(k))\|^2\,\bar{\lambda}\left(\sum_i(k)\right) + 2\|\tilde{x}(k)\|^2\|g_i C\|^2\,\bar{\lambda}\left(\sum_i(k)\right) - \|\tilde{x}(k)\|^2\,\underline{\lambda}\left(P_i(k)\right) \Rightarrow \\ \Delta V_i(k) &\leq -\|\tilde{x}(k)\|^2\theta_i(k) - \left\|\tilde{W}_i(k)\right\|^2.\Xi_i(k) + 4|\varepsilon_i|^2\,\bar{\lambda}\left(\sum_i(k)\right) + 2\Omega_i(k)\end{aligned} \quad (43)$$

Where;

$$\begin{cases}\theta_i(k) = 2\|\eta_i K_i C\|^2\,\bar{\lambda}\left(\sum_i(k)\right) + 2\|g_i C\|^2\,\bar{\lambda}\left(\sum_i(k)\right) - \underline{\lambda}\left(P_i(k)\right), \\ \Xi_i(k) = \bar{\lambda}(P_i(k)) - \underline{\lambda}(P_i(k)) + 4\|\tanh(\tilde{x}(k))\|^2\,\bar{\lambda}\left(\sum_i(k)\right), \\ \Omega_i(k) = \left\|W_i^* - \max(W_i)\right\|.\|\tanh(\tilde{x})\|.|\varepsilon_i|.\left\|\sum_i(k)\right\|\end{cases} \quad (44)$$

As a result, $\Delta V_i(k) \leq 0$ when

$$\|\tilde{x}(k)\| = \sqrt{\frac{4|\varepsilon'_{zi}|^2\lambda_{max}\left(A_i(k)\right)}{E_i(k)}} \equiv k_1 \quad (45)$$

Or

$$\left\|\tilde{W}_i(k)\right\| = \sqrt{\frac{4|\varepsilon'_{zi}|^2\lambda_{max}\left(A_i(k)\right)}{F_i(k)}} \equiv k_2 \quad (46)$$

Therefore, the solution of (12) and (32) is stable; hence the estimation error and the DTRNO weights are DTUNNS.Considering (9) and (24), it is easy to see that the output error has an algebraic relation with $x(k)$, for that reason, $\hat{x}(k)$ is bounded, $e(k)$ is bounded too. Figure 4 illustrates the range k_1, and k_2.

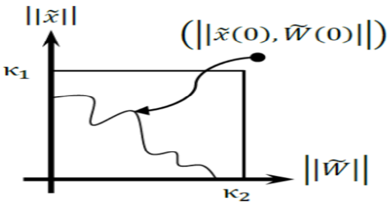

Figure 4. the range k_1 and k_2

5. SIMULATION RESULTS

The performance of the proposed observer is demonstrated through simulation results. The example is a Non-affine Nonlinear System. The simulation is performed in MATLAB software. In this section has been NN Observer by EKF learning algorithm for a second-order plant;

$$\begin{aligned} x_1(k+1) &= 0.1x_1(k) + 2\left(u(k) + x_2(k)/1 + \left(u(k) + x_2(k)\right)^2\right) + d_1(k) \\ x_2(k+1) &= 0.1x_2(k) + 2u(k) + \left(u^2(k)/1 + 1 + x_1^2(k) + x_2^2(k)\right) + d_2(k) \\ y(k) &= x_1(k) + x_2(k) + N(k) \end{aligned} \tag{47}$$

Where $x_1(k)$ and $x_2(k)$ are state variables, u is input system, $d_1(k)$ and $d_2(k)$ are disturbance and $N(k)$ is measurement noise.The numerical values of the Non-affine Non-linear System parameters and observer are described in Table 1.

Table 1. The Numerical Values

Parameter	Values	Parameter	Values	Parameter	Values
C	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$	A	$0.001\times\begin{bmatrix} 20 & 1 & 0 & 0 \\ 0 & 20 & 0 & 0 \\ 0 & 0 & 20 & 1 \\ 0 & 0 & 0 & 20 \end{bmatrix}$	Γ	$0.001\times\begin{bmatrix} 3.0 & -0.5 & -0.5 & 0.50 \\ -0.5 & 2.0 & 2.0 & 1.0 \end{bmatrix}^T$
T	0.001				

The state and error estimation obtained by our proposed neural network by EKF learning algorithm for discrete-time Non-affine Nonlinear system are shown in Figure 5.

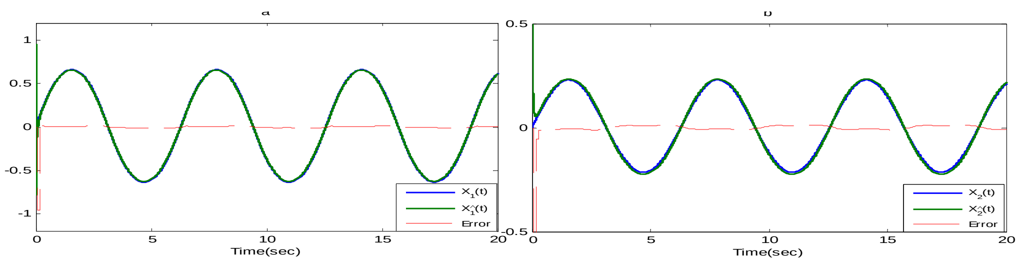

Figure 5. The state and error responses to sin (k) reference

The state and error estimation obtained by our proposed neural network by EKF learning algorithm for discrete-time Non-affine Nonlinear system with input; $u(k) = \sin(k) + 0.01\sin(50k) + 0.02\sin(100k)$, the results are given in Figure 6.

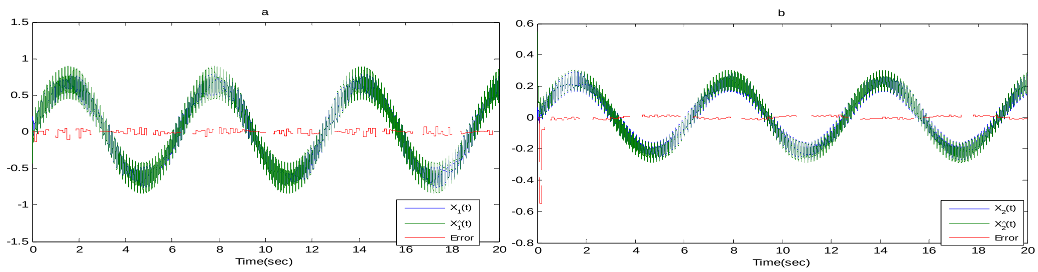

Figure 6. The state and error responses to $u(k) = \sin(k) + 0.01\sin(50k) + 0.02\sin(100k)$ reference

The state estimation and error estimation obtained by proposed neural network by EKF learning algorithm for system (56) with input $u(k) = \sin(k)$ and In the presence of measurement noise is shown in Figure 7.

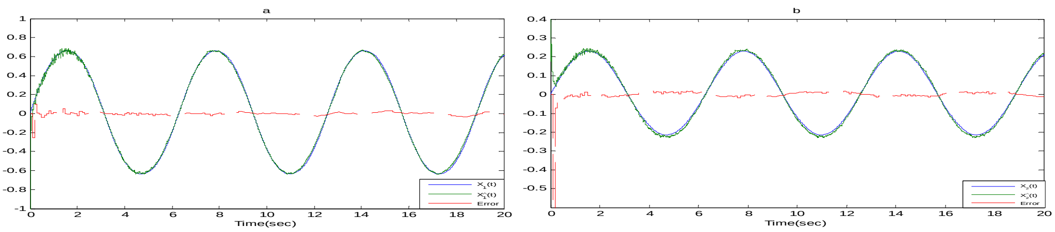

Figure 7. The state and error responses to $u(k) = \sin(k)$ reference

The state estimation and error estimation obtained by proposed neural network by EKF learning algorithm for system (47) with input $u(k) = \sin(k) + 0.01\sin(50k) + 0.02\sin(100k)$ and In the presence of measurement noise is shown in Figure 8.

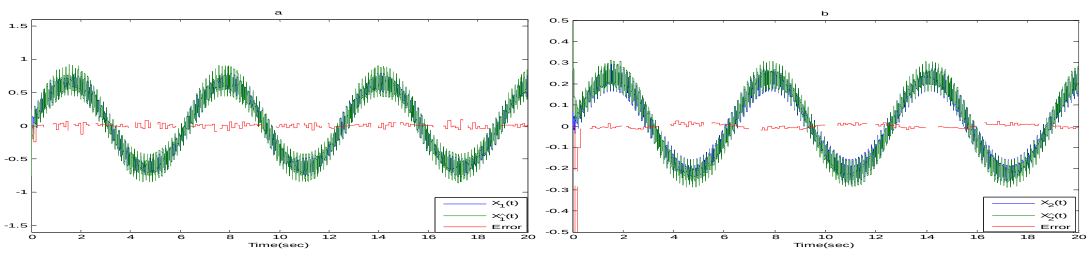

Figure 8. The state and error responses to $u(k) = \sin(k) + 0.01\sin(50k) + 0.02\sin(100k)$ reference

The state estimation and error estimation obtained by proposed neural network by EKF learning algorithm for system (47) with input $u(k) = \sin(k)$ and In the presence of disturbance are shown in Figure 9.

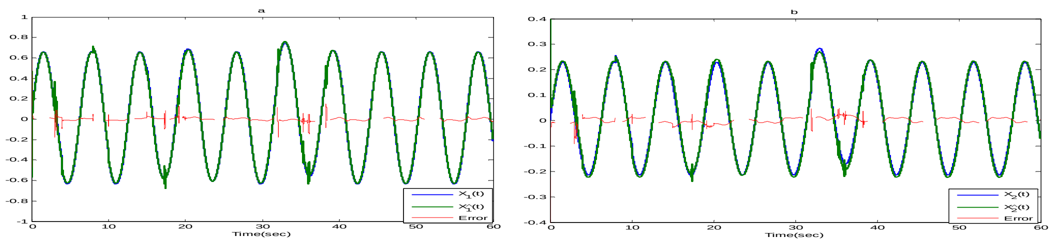

Figure 9. The state and error responses to $u(k) = \sin(k)$ reference

These results demonstrate that the NN estimation error learned by EKF algorithm is very low. The stability of the overall system was shown by Lyapunov's direct method. It is worth noting that no SPR assumption or any other constraints that restrict the applicability of the approach was imposed on the system. The proposed observer can be applied both as an online and an off-line estimator. Simulation results performed on Non-affine Nonlinear System confirm the reliable performance of the proposed observer.

6. CONCLUSION

A MLPNN structure was used to design a neural observer, named DTRNO, for a class of Discrete Time Uncertain Non-affine Nonlinear Systems (DTUNNS); the proposed observer was trained with an EKF based algorithm, which was implemented online in a parallel configuration. The boundedness of the output, state, and estimation errors was established on the basis of the Lyapunov approach. Discrete-time results show the effectiveness of the proposed observer, as applied to a Non-affine Nonlinear System in presence of time varying disturbances. However, output trajectory tracking results were included in this brief in order to show the effectiveness of the proposed observer as compared with other nonlinear observer.

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