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A Secure-Coordinated Expansion Planning of Generation and Transmission Using Game Theory and Minimum Singular Value

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ABSTRACT

A In this paper a novel method have been proposed for expansion planning of generation and transmission that considered static security of the system such as voltage security margin And loadability limit. In the same study of expansion planning Security constraints of the system are neglected. In this study at the first step minimum singular value technique is used to evaluate voltage security margin and loadabolity limit, in order to select best bus for load incrimination. After it, in order to Supply the load, coordinated expansion planning of generation and transmission is needed, therefor the strategic interaction between transmission company (TransCo) and generation company (GenCo) for Transmission expansion planning (TEP) and generation expansion planning (GEP) in a competitive electricity market is proposed using Game Theory (GT).

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1. INTRODUCTION

Power system restructuring and deregulation, introduce new definitions and methods to the power system planning. In the monopoly power market, the decision maker is just one organization which decides for the Generation Expansion Planning (GEP) and Transmission Expansion Planning (TEP) altogether. Due to emergence of competition in the power market, the decision makers of GEP and TEP become separated such that the transmission company (TransCo) decides for TEP and the generation company (GenCo) decides for GEP. In such an environment, the coordination between these two organizations becomes more crucial as capacity expansion of each organization affects the other side capacity expansion and as a consequence the profit of each company is affected. In a competitive power market with open access to the transmission system, the generation company is expected to supply the load without any congestion in the transmission lines. Transmission companies are obliged to provide a congestion-free, reliable and non-discriminative path for the generation companies to the consumers of electricity. Therefore, the transmission networks must be regulated so that optimal operation of power system is performed. In a restructured power market, Genco decides on capacity, place and time of construction of new power plants at its own discretion. The capacity expansion strategy used by GenCo involves TransCo and produces uncertainty and challenges to TEP. On the other hand, there is no absolute certainty that a transmission network can provide sufficient capacity for new generation capacity constructed by a GenCo [1-5].

This interaction between Genco and TranCo leads to a new method of GEP and TEP that considers both entities' profit. In such an environment, Game theory seems to be a useful method to predict the strategies of Genco and TransCo for generation expansion capacity (GEC) and transmission expansion capacity (TEC) and evaluate the power market equilibrium [6-10]. TEP and GEP have been studied separately in many articles but the interaction between them in a deregulated market is studied in a few papers. In [11] in order to study the relationship between generation and transmission investment, a three-stage model is presented. A study in [12] shows that in a deregulated power market, the degree of competition between different generators is dependent on the capacity of transmission lines. The interaction between transmission and generation expansion planning using game theory, is studied in a three-bus system in [13]. To study the strategic interaction between GenCo and TransCo, a single-stage deterministic model is proposed in [14]. The expansion behaviors of both GenCo and TransCo are simulated using Cournot model. The equilibrium in the power market in [14] is obtained using Mixed Complementarity Problem approach and the proposed model is applied to a three-bus system and the IEEE 14-bus system.

On the other hand, power system security which is the ability of the power system to withstand disturbances against any violation in system operating conditions should be considered in this new method of capacity expansion. The aforementioned studies in the field of generation and transmission expansion planning haven't considered power system security. However in this paper, first the load pattern of a six-bus power system is improved and then the best bus for load increment is determined using a sensitivity characteristic of ANN. Afterwards the strategic interaction between transmission company (TransCo) and generation company (GenCo) for TEP and GEP in a competitive electricity market is proposed using Game Theory. It should be taken into consideration that the load discussed in this paper is a manageable load which can be increased or decreased using reward or penalty. The paper is organized as follows. In section II, the loadability limit as a security index is introduced. The neural network used for the improvement of load pattern is presented in section III. Application of Cournot model of duopoly for TEP and GEP is discussed in section IV. Section V proposes Game Theory for solving TEP and GEP problem. A case study is presented in section VI. Finally, conclusions are presented in section VII.

2. LOADABILITY LIMIT AND VOLTAGE SECURITY MARGIN AS A SECURITY INDEX

In order to study the static voltage stability of the power system, loadability limit of system is proposed as voltage stability index for system security evaluation. Loadability limit of a power system is defined as the maximum load, which can be imposed on buses of a system without loss of voltage stability. the minimum singular value technique is used to evaluate voltage security margin for each state of loading. The minimum singular value of the load flow Jacobian matrix is obtained from solving the load flow equations that are shown in (1) and (2):

$$P_i = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}|\cos(\theta_{ij} - \delta_i + \delta_j) \tag{1}$$

$$Q_i = -\sum_{j=1}^{n} |V_i||V_j||Y_{ij}|\sin(\theta_{ij} - \delta_i + \delta_j) \tag{2}$$

Where:

P_i : Active power transfer from bus I;

Q_i : Reactive power transfer from bus i;

V_i, V_j : Voltages of bus i and bus j;

Y_{ij} : Admittances from bus i to bus j;

θ_{ij} : angle of impedances from bus i to bus j;

Y_{ij} : Admittances from bus i to bus j;

δ_i, δ_j : angle of voltage at bus i and bus j.

And:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta|V| \end{bmatrix} \tag{3}$$

ΔP : Small deviation in active power;

ΔQ : Small deviation in reactive power;

$J_1 - J_4$: Jacobian matrix elements of load flow equations;
$\Delta\delta$: Small deviations in voltage angle;
$\Delta|V|$: Small deviations in voltage magnitude.

3. MINIMUM SINGULAR VALUE

One of the effective methods of determination of voltage collapse point is minimum singular value of the load flow Jacobian matrix. In this paper, minimum singular value technique is used to selective best bus for loading. On This way load increases in desired bus when load is fixed at another bus then calculate voltage security margin with approach technique. When voltage security margin is calculated for all of the state, the state with maximum VSM is the best state for loading. Flowchart of this process has been shown in figure 1. Equation (3) can be written by equation (4). Under normal operating condition, application of singular value decomposition is applied to Jacobian matrix that has been shown in equation in (5) to (9):

$$\begin{bmatrix} \Delta P \\ \Delta\theta \end{bmatrix} = J \begin{bmatrix} \Delta\theta \\ \Delta P \end{bmatrix} \tag{4}$$

$$J = \sum_{j=1}^{2(n-1)} u_j \delta_j v_j^T \tag{5}$$

$$\begin{bmatrix} \Delta\theta \\ \Delta V \end{bmatrix} = J^{-1} \begin{bmatrix} \Delta P \\ \Delta\theta \end{bmatrix} = \sum_{j=1}^{2(n-1)} \delta_j^{-1} V_j U_j^T \begin{bmatrix} \Delta P \\ \Delta\theta \end{bmatrix} \tag{6}$$

$$\begin{bmatrix} \Delta\theta \\ \Delta V \end{bmatrix} = \delta_{2(n-1)}^{-1} V_{2(n-1)} u_{2(n-1)}^T \begin{bmatrix} \Delta P \\ \Delta\theta \end{bmatrix} \tag{7}$$

Let $\begin{bmatrix} \Delta P \\ \Delta\theta \end{bmatrix} = U_{2(n-1)}$ (8)

Then $\begin{bmatrix} \Delta\theta \\ \Delta V \end{bmatrix} = \dfrac{V_{2(n-1)}}{\delta_{2(n-1)}}$ (9)

Where:
n: number of buses in the power network,
u_j, v_j : singular vectors that u_j and v_j are the j^{th} columns of unitary matrix,
δ : Positive real singular values.

Above analysis clearly shows where the minimum singular value of the load flow Jacobian matrix is almost zero, this suitable indicator detects the closeness of power system operating condition to the voltage collapse point.

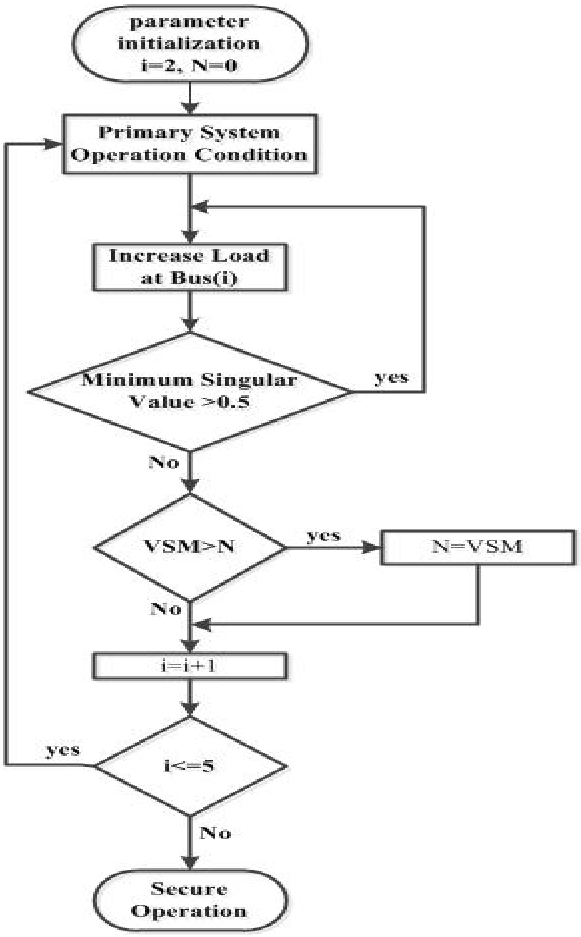

Figure 1. Flowchart of best bus Selection for loading

4. APPLICATION OF COURNOT DUOPOLY MODEL TO TEP AND GEP PROBLEM

In the Cournot duopoly model, two companies produce a homogenous product and without knowing the decision of the other one, they must decide how much product they should produce to obtain the maximum profit [18]. As the Cournot model is somehow similar to the TEP and GEP behaviors, it can be applied to the problem of TEP and GEP in the restructured power market. The first similarity between the cournot model and TEP and GEP problem is the expansion capacity of TEP and GEP, which is quantity in Cournot model. The second similarity is that in a pool market the product of each generation unit is a homogenous product which is offered by GenCo at each supply bid. The Cournot model maximizes the profit of each company and defines the amount of their outputs. In terms of mathematical formulas, the optimal quantity pair (q_1^*, q_2^*) is the Coumot equilibrium, if for firm 1, q_1^* solves (10):

$$\max \pi_1(q_1, q_2^*) \tag{10}$$

Where:

π_i : Profit for firm i; i=1, 2;

q_i : Quantities produced by firm i;

q_i^* : Optimal quantities produced by firm.

The profit function for firm 1 can be represented by (11):

$$\pi_1(q_1,q_2) = p(q_1+q_2)q_1 - c_1(q_1) \tag{11}$$

Where:

$p(.)$: Market price for aggregate quantity;

$C_i(.)$:cost function for firm i.

5. APPLYING GAME THEORY TO TEP AND GEP PROBLEM

5.1. Assumptions

In order to formulate the strategic interaction between GenCo and TransCo in the expansion planning game, some assumptions are considered.

- TransCo is owner of all transmission lines.
- Power market structure is Poolco-type in which GenCo offers the price and quantity bids to a Independent System Operator (ISO). Then, the ISO dispatches the cheapest power considering operational constraints like transfer capacity limitations and energy balance constraints.
- Demand is considered as a constant load.
- Expansion strategies of TransCo and Genco are discrete. This means that TransCo can either expand its line capacity or maintain the initial transfer capacity of the line.
- Expansion behaviors of TransCo and GenCo are based on Cournot model.

5.2. Problem Formulation

As profit maximization is the main purpose of each side of the game theory and profit is the difference between revenues and costs, the revenue of the TransCo for each line is given by a congestion charge of that line [19] which is the difference between local marginal prices (LMPs) [20-21]. The sum of congestion charges of each line is total revenue of TransCo.

In order to make the problem more simple, Transco total cost is not considered. So, profit of the TransCo can be expressed as (12):

$$\pi_T = \sum_{i \neq j} (\lambda_j - \lambda_i) P_{ij} \tag{12}$$

Where:

π_T : profit of the Transco in \$/hr;

λ_i : LMP at node i in \$/MWhr;

λ_j : LMP at node j in \$/MWhr;

P_{ij} : Active power flow from node i to node j in MW.

As GEP is performed at node i, The GenCo profit from GEP is obtained using (13):

$$\pi_G = \lambda_i P_G - c(P_G) - i(G) \tag{13}$$

Where:

π_G : profit of GenCo in \$/hr;

λ_i : LMP at node i in \$/MWhr;

P_G : active power generation in MW;

$c(P_G)$: quadratic cost function of active power generation in \$/hr;

$i(G)$: investment cost in terms of generation expansion capacity in \$/hr.

5.3. Solution Methodology

The following notations are used in the solution methodology.

S_T^i : ith expansion strategy of TEP; i= 1,2,...,n;

S_G^j : jth expansion strategy of GEP; j=1,2,…..,n;

T :transmission expansion capacity (TEC) in MW;

G : Generation expansion capacity (GEC) in MW;
T^0 : initial TEC in MW;
G^0 : initial GEC in MW;
ΔT : increment in TEC in MW;
ΔG : increment in GEC in MW;
$\bar{T}$: maximum TEC in MW;
$\bar{G}$: maximum GEC in MW.

Algorithm of finding the Nash equilibrium among all possible combinations of expansion strategies is shown in flowchart of Figure 7 and the Cournot equilibrium is found by using an iterative search procedure [22] as shown in Figure 8.

6. CASE STUDY

The proposed algorithm is applied to a six-bus system shown in Figure 9. The Data of the six-bus system is given in Tables 1 to 3. In the proposed method, first the most appropriate bus for load increment is selected using minimum singular value technique. The minimum singular value technique shows bus 2 is the best and the most suitable bus for load incremental because this state has maximum VSM, this issue is clearly shown in Figure 2. Afterwards, in order to supply the incremented load in the selected bus, Game Theory is used to study the strategic interaction between the TEP and GEP.

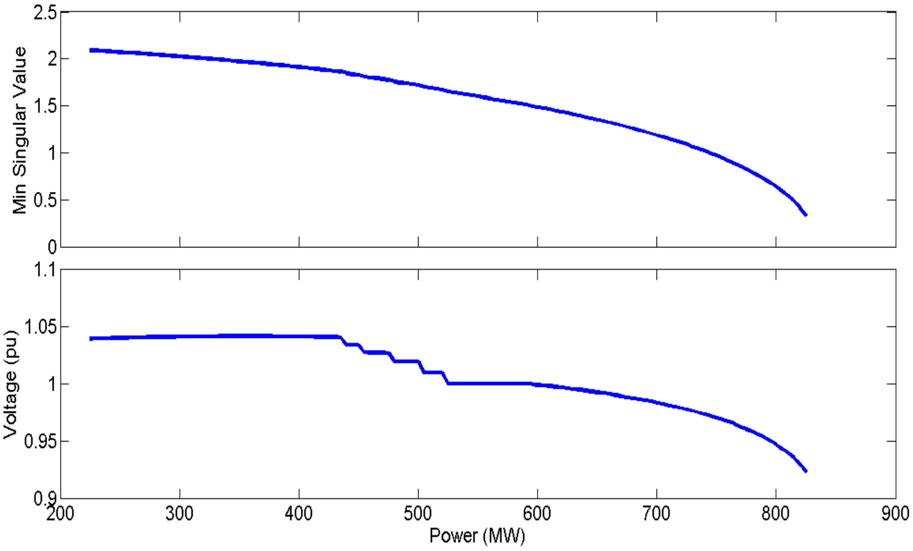

Figure 2. VSM and Min Singular Value of system when load increase at bus2

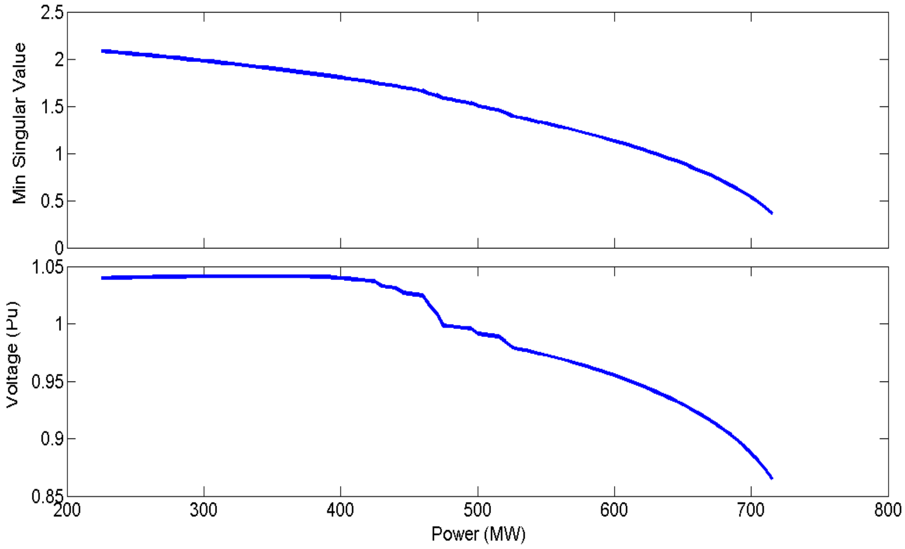

Figure 3. VSM and Min Singular Value of system when load increase at bus3

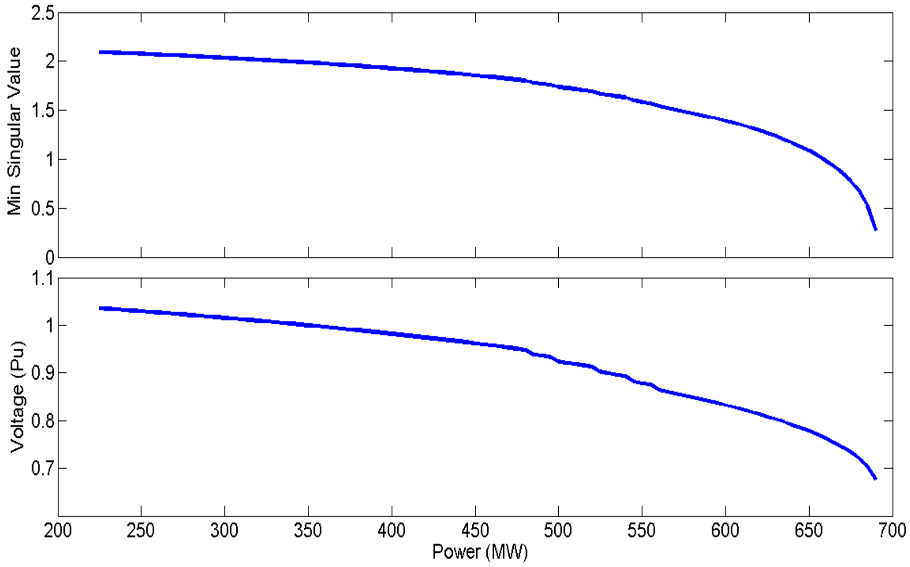

Figure 4. VSM and Min Singular Value of system when load increase at bus4

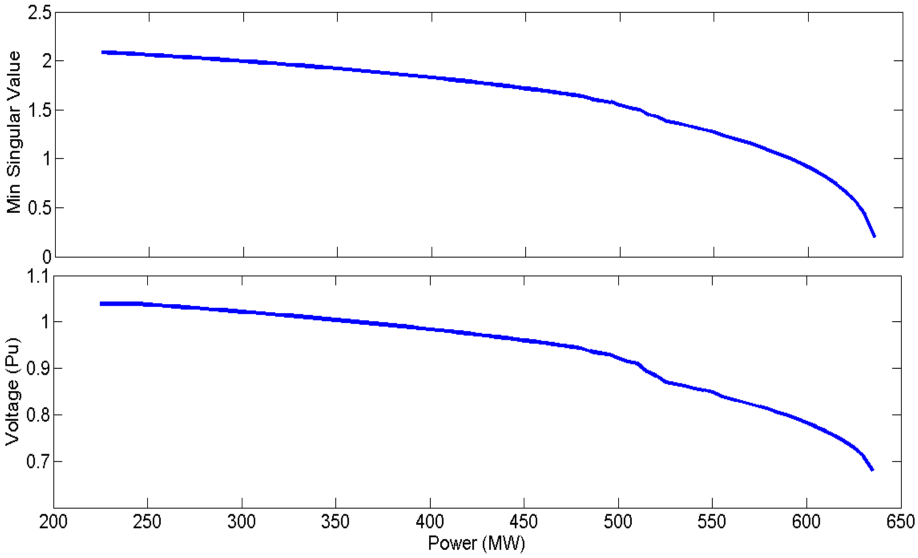

Figure 5. VSM and Min Singular Value of system when load increase at bus5

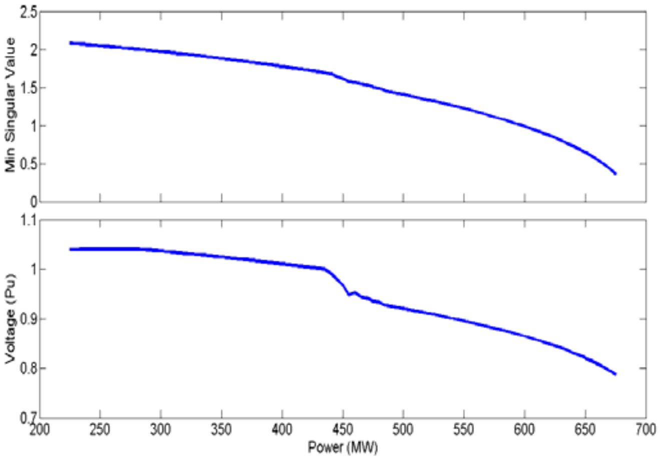

Figure 6. VSM and Min Singular Value of system when load increase at bus6

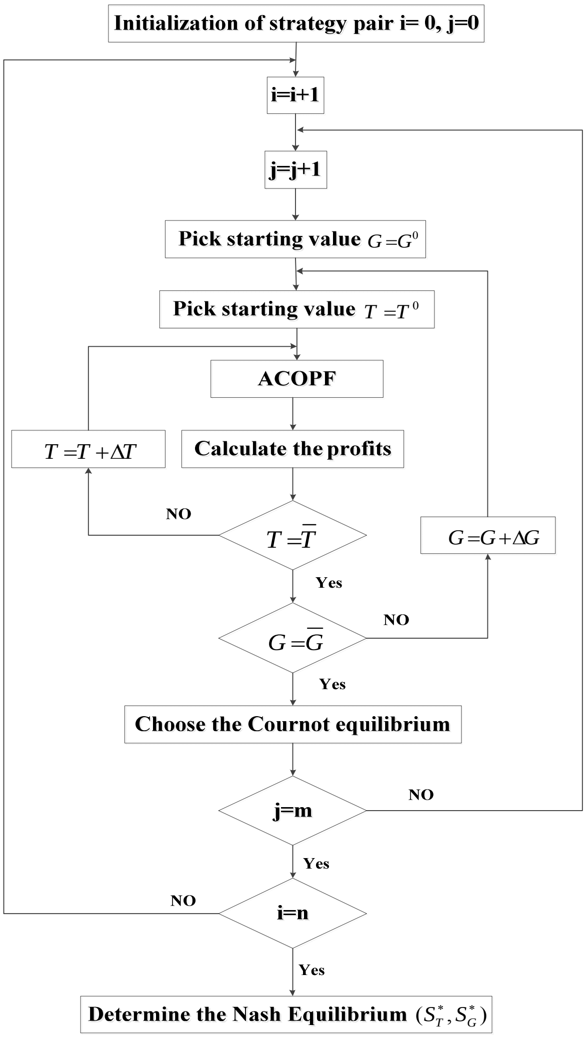

Figure 7. Flowchart of the strategic interaction between Transco and GenCo

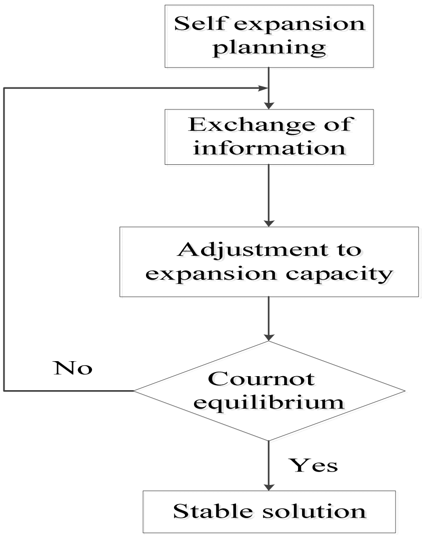

Figure 8. Flowchart of the Cournot solution algorithm

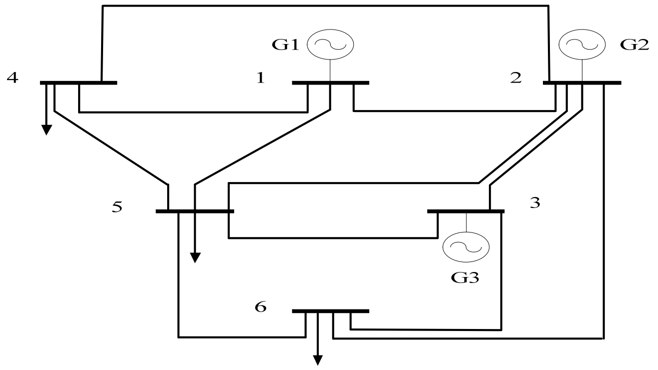

Figure 9. Six-bus system

Table 1. Generator Data

Generator	*Min Generation Power (MW)*	*Min Generation Power (MW)*
G1	50	200
G2	37.5	68
G3	45	73

Table 2. Line Data

From Bus	*To Bus*	*R (pu)*	*X (pu)*	*Transmission Capacity (MW)*
1	2	0.1	0.2	15.28
1	4	0.05	0.2	40.32
1	5	0.08	0.3	30.63
2	3	0.05	0.25	20.2
2	4	0.05	0.1	42.6
2	5	0.1	0.3	31.42
2	6	0.07	0.2	24.6
3	5	0.12	0.26	28
3	6	0.02	0.1	50.4
4	5	0.2	0.4	18.1
5	6	0.1	0.3	21.6

Table 3. Bus Data

Bus Number	*Bus Type*	*Voltage (pu V)*	*P_{gen} (pu MW)*	*P_{load} (pu MW)*	*λ ($/MWhr)*
1	Swing	1.05	-	-	12.492
2	Gen.	1.05	0.5	0	11.565
3	Gen.	1.07	0.6	0	11.877
4	Load	-	0	70	15.674
5	Load	-	0	70	12.939
6	Load	-	0	70	12.206

A. Selection of the best bus for load increment

As shown in Figure 2 the best loadability limit and the most appropriate bus for load increment, is bus 2 because in this state loadability limit ($P_{\max}$) and minimum eigenvalue of Jacobean matrix ($\lambda_{\min}$) of power flow equations that is presented in (14) are the most secure state among other states.

$$\begin{cases} P_{\max} = 850MW \\ \lambda_{\min} = 0.3 \end{cases} \tag{14}$$

B. Transmission and Generation Expansion Planning

After selection of the best bus for load incement, in order to supply the load, Game Theory studies the strategic interaction between transmission expansion planning (TEP) and generation expansion planning (GEP).

Under normal operation condition, generator 1 output is 76.5 MW and generator 2 and 3 generate their maximum allowable output power. Supposing the 40 MW load (250-210=40 MW) installed at bus 2 and as the capacity of line1-2 is 15.28MW, a line congestion occurs. Therefore, in order to supply this load, transmission capacity expansion between buses 1 and 2 or generation capacity expansion of generator at bus 2, is required. As a consequence, a competition occurs between TransCo for the capacity expansion of line 1-2 and GenCo for the generation expansion capacity of generator at bus 2. Table 4 shows the expansions strategies of the two players (TransCo & GenCo) in this case. Equation (12) represents the profit of Transco due to congestion in transmission lines, and (13) represents the profit of GenCo due to expansion of generator capacity at bus 2 and $c(P_{G2})$ for generator 2 is presented in (15).

$$c(P_G) = \alpha P_G^2 + \beta P_G + \gamma \tag{15}$$

$\alpha = 0.00889$, $\beta = 10.333$, $\gamma = 200$ and $i(G) = 200\$/hr$

In the next step, each of these strategies is applied to the system and at each step, the maximum profit of each company is obtained. Finally, the most profitable strategy for both player using Game Theory and Nash equilibrium is obtained.

Table 4. Expansion Strategies of Transco and GenCo

Symbol	*Expansion Strategy*
S_T^1	TransCo will not expand line 1-2
S_T^2	TransCo will expand line 1-2
S_G^1	GenCo will not expand line 1-2
S_G^2	GenCo will expand line 1-2

1) 1st Expansion scenario (S_T^1, S_G^1)

In this scenario, neither TransCo nor GenCo expands its capacity, so there is no revenue or cost for GenCo and TransCo gains profit from the congestion charges. profits of the two players in this strategy are given in Table 5.

2) 2nd Expansion scenario (S_T^2, S_G^1)

TransCo will expand line 1-2 capacity from 15.28 MW to 55.28 MW in steps of 5 MW. The TEC with maximum profit for TransCo is 50.28 MW with profit of 31.03 $/hr. Once again there is no profit for GenCo in this scenario. The profits of TransCo and GenCo in this scenario are given in Table 6.

Table 5.Profits of Two Players in the 1st Expansion Scenario

Line 1-2 capacity (MW)	*Total profit of TransCo ($/hr)*	*GEC in bus 2 (MW)*	*Profit of GenCo ($/hr)*
15.28	14.3	0	0

Table 6. Profits of Two Players in the 2nd Expansion Scenario

Line 1-2 TEC (MW)	*Total profit of TransCo ($/hr)*	*GEC in bus 2 (MW)*	*Profit of GenCo ($/hr)*
20.28	5.64	0	0
25.28	8.719	0	0
30.28	11.5	0	0
35.28	15.4	0	0
40.28	20.02	0	0
45.28	25.186	0	0
50.28	**31.03**	**0**	**0**
55.28	20.5	0	0

3) 3rd Expansion scenario (S_T^1, S_G^2)

GenCo will perform GEP in bus 2 and the GEC range is from 68 MW to 108 MW in steps of 5 MW. The GEC with maximum profit for the GenCo is 30 MW with profit of 290.44 $/hr. In this scenario, TransCo gains profit from congestion charges. The profits of TransCo and GenCo in this scenario are given in Table 7.

Table 7. Profits of Two Players in the 3rd Expansion Scenario

Line 1-2 Capacity (MW)	*Total profit of TransCo ($/hr)*	*GEC in bus 2 (MW)*	*Profit of GenCo ($/hr)*
15.28	13.2	5	289.4
15.28	11.84	10	279.3
15.28	10.46	15	269.7
15.28	9.1	20	260.536
15.28	7.7	25	272.33
15.28	**6.39**	**30**	**290.44**
15.28	5.03	35	275.3
15.28	3.67	40	262.1

4) 4th Expansion scenario (S_T^2, S_G^2)

Both TransCo and GenCo will perform TEC and GEC. The most profitable expansion capacity is 37 MW for TEP with profit of 32.03 $/hr and 32 MW for GEP with profit of 291.43 $/hr. So the Cournot capacity pair with maximum profit is (32 MW, 37 MW) among all other Cournot capacity pairs. The profits of TransCo and GenCo are given in Table 8.

Table 8. Profits of Two Players in the 4th Expansion Scenario

Line 1-2 TEC (MW)	*Total profit of TransCo ($/hr)*	*GEC in bus 2 (MW)*	*Profit of GenCo ($/hr)*
32	31.05	28	261.78
35	30.02	30	288.52
37	**32.03**	**32**	**291.43**

After considering all of the mentioned scenarios, a payoff matrix that comprises the payoffs of TransCo and GenCo is constructed which is shown in Table 9. In Table 9, the first entry in the box is the profit of TransCo and the second entry is the profit of GenCo. In this case, the dominant strategy which is characterized as the Nash equilibrium is the pair of strategies (S_T^2, S_G^2) which implies that the most profitable strategy for both TransCo and GenCo is to perform TEP and GEP. In this strategy, the profit of TransCo is 32.03 $/hr while GenCo profit is 291.43 $/hr.

Table 9. Payoff Matrix of the Game Theoretic TEC and GEC

		GenCo Strategy	
	Nash eq.	S_G^1	S_G^2
TransCo Strategy	S_T^1	(14.3,0)	(6.39,290.44)
	S_T^2	(31.03,0)	**(32.03,291.43)**

7. CONCLUSION

In this study it was shown a new method for expansion planning of generation and transmission that considered static security of the system such as voltage security margin And loadability limit in a six-bus system. At the first step a secure bus was selected for load incrimination that was considered voltage security margine by using minimum singular value technique. As the transmission line capacity is not sufficient for the load increment, a congestion in the transmission line occurs which can be an incentive for GenCo and TransCo to gain the maximum profit from supplying this load. Considering multiple scenarios, the Cournot model defines the most profitable TEC and GEC in each scenario. After constituting the payoff matrix which includes all of the possible strategies combination, the Nash equilibrium is obtained. The results show that the most profitable scenario for both Genco and TransCo is to perform generation expansion and transmission expansion.

REFERENCES

[1] H. He, Z. Xu and G. H. Cheng, "Impacts of transmission congestion on market power in electricity market", 2004 IEEE PES Power Systems Conference and Exposition, vol. 1, pp. 190-195, Oct. 2004.

[2] N. Yang and F. S. Wen, "A chance constrained programming approach to transmission system expansion planning", Electric Power Systems Research, vol. 75, pp. 171-177, Aug. 2005.

[3] C. W. Lee, S. K. K. Ng, J. Zhong and F. F. Wu, "Transmission expansion planning from past to future", in Proc. PSCE 2006, Oct 2006, pp. 257–265.

[4] F. F. Wu, F. L. Zheng and F. S. Wen, "Transmission investment and expansion planning in a restructured electricity market", Energy, vol. 31, pp. 954-966, May-Jun. 2006.

[5] C. W. Lee, S. K. K. Ng and J. Zhong, "Value-based transmission expansion planning in deregulated market", Proceedings of the 38th North American Power Symposium, pp. 107-114, Sept. 2006.

[6] H. Singh, "Introduction to game theory and its application in electric power markets", IEEE Comp. Applicat. in Power, vol. 12, pp. 18-22, Oct. 1999.

[7] J. Contreras and F. F. Wu, "Coalition formation in transmission expansion planning", IEEE Trans. Power Systems, vol. 14, pp. 1144-1152, 1999.

[8] H. S. Ko, T. Niimura and K. Ozawa, "A day-ahead electricity price prediction based on a fuzzy-neuro autoregressive model in a deregulated electricity market", in Proc. Int. Joint Conf on Neural Networks, May 2002, vol. 2, pp. 1362-1366.

[9] T. De la Torre, J. W. Feltes, T. G. S. Roman and H. M. Merril, "Deregulation, privatization, and competition: Transmission planning under uncertainty", IEEE Trans. Power Systems., vol. 14, pp. 460-465, May 1999.

[10] A. K. David and F. S. Wen, "Strategic bidding in competitive electricity markets: a literature survey", in Proc. IEEE Power Eng. Soc. Transmission and Distribution Conf:, 2000, vol. 4, pp. 2168-2173.

[11] E. E. Sauma and S. S. Oren, "Proactive transmission investment in competitive power systems", IEEE Power Engineering Society General Meeting, pp. 18-22, Jun. 2006.

[12] S. Borenstein, J. Bushnell and S. Stoft, "The competitive effects of transmission capacity in a deregulated electricity industry", RAND Journal of Economics, vol. 31, pp. 294-325, Jun. 2000.

[13] S. K. K. Ng, C. W. Lee and J. Zhong, "A Game-Theoretic Approach to Study Strategic Interaction Between Transmission and Generation Expansion Planning", Proceedings of the 38th North American Power Symposium, pp. 115-120, Sept. 2006.

[14] S. K. K. Ng, J. Zhong and C. W. Lee, "A Game-Theoretic Study of the Strategic Interaction between Generation and Transmission Expansion Planning", in Proc. PSCE 2009, March 2009, pp. 1-10.

[15] H.P Schmidt, R, N. Adams, "Assessment of static voltage stability using artificial neural Network", proc. Of 11th PSCC, avigon, agust 30, 1993

[16] M.R. Aghamohammadi, H. Saitoh , J, Toyada "Sensitivity characteristic of Neural Network as a tool for secure generation scheduling in power system", in proc . ICICS, 96, international Conference on Intelligent and cognitive system, Tehran, Iran, 23-26 Sept. 1996.

[17] C. Charalambous, "Conjugate gradient algorithm for efficient training of artificial neural networks", IEEE Proceedings, vol. 139, no.3, 1992, pp. 301-310.

[18] R. Gibbons, "A primer in game theory", New York, Harvester wheatsheaf, 1992.

[19] T.Krause , "Congestion management in liberalized electricity markets - theoretical concepts and international application", 2005. [Online]. Available: http://www.eeh.ee.ethz.ch

[20] California Independent System Operator, “Locational Marginal Pricing (LMP): Basics of Nodal Price Calculation”, 2004. [Online]. Available: http://www.caiso.com

[21] D. Kirschen and G. Strbac, “Fundamentals of power system economics”, Hoboken, NJ: John Wiley & Sons, 2004.

[22] A. S. Chuang, F. Wu, P. Varaiya, “A game-theoretic model for generation expansion planning: problem formulation and numerical comparisons”, IEEE Trans. Power Syst., vol. 16, pp. 885-891, Nov. 2001.

Appendix

ACOPF Equations:

$$\min \quad f(P_{ig}) = B_{ig} \times P_{ig}$$

$$s.t: \quad P_{ig} = |Y_{ij}||V_i|(V_i \cos\varphi_{ij} - V_j \cos(\varphi_{ij} + \theta_j - \theta_i))$$

$$Q_{ig} = |Y_{ij}||V_i|(V_i \sin\varphi_{ij} - V_j \sin(\varphi_{ij} + \theta_j - \theta_i))$$

$$0 \le P_{ig} \le P_{ig\,\max}$$

$$P_{ij\,\min} \le P_{ij} \le P_{ij\,\max}$$

$$Q_{ij\,\min} \le Q_{ij} \le Q_{ij\,\max}$$

$$V_{i\,\min} \le V_i \le V_{i\,\max}$$

$$\theta_{i\,\min} \le \theta_i \le \theta_{i\,\max}$$

$$\sum_{g=a}^{b} P_{ig} = \sum_{j=m}^{n} P_{ij} + P_{Load\,i}$$

Where:

$f(.)$: total operation cost in \$/hr

B_{ig} : generation offering price for each generation offering energy in \$/MWhr

P_{ig} : active power generation for each generation offering energy in MW

$P_{i\,\max}$: maximum active power generation for each generation offering energy in MW

P_{ij} : active power flow from bus i to bus j in MW

Y_{ij} : admittance of branch connecting bus i and bus j in Ω

φ_{ij} : admittance angle of branch connecting bus i and bus j in radian

V_i : voltage at bus i in volt

θ_i : voltage angle at bus i in radian

$P_{ij\,\min}$: minimum active power transmission capacity at branch ij

$P_{ij\,\max}$: maximum active power transmission capacity at branch ij

$Q_{ij\,\min}$: minimum reactive power transmission capacity at branch ij

$Q_{ij\,\max}$: maximum reactive power transmission capacity at branch ij

$V_{i\,\min}$: minimum voltage at bus i in volt

$V_{i\,\max}$: maximum voltage at bus i in volt

$\theta_{i\,\min}$: minimum voltage angle at bus i in radian

$\theta_{i\,\max}$: maximum voltage angle at bus i in radian

$P_{Load\,i}$: load at bus i