International Journal of Electrical and Computer Engineering (IJECE)
Vol. 3, No. 1, February 2013, pp. 93~101
ISSN: 2088-8708

Performance Evaluation of Unscented Kalman Filter for Gaussian and Non-Gaussian Tracking Application

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Article Info

Article history:

Received Sep 30, 2012
Revised Dec 29, 2012
Accepted Jan 10, 2013

Keyword:

Kalmanfilte
Extended Kalman filter
Unscented Kalman filter
Gaussian***/Non***Gaussiandistribution

ABSTRACT

State estimation theory is one of the best mathematical approaches to analyze variants in the states of the system or process. The state of the system is defined by a set of variables that provide a complete representation of the internal condition at any given instant of time. Filtering of Random processes is referred to as Estimation, and is a well defined statistical technique. There are two types of state estimation processes, Linear and Nonlinear. Linear estimation of a system can easily be analyzed by using Kalman Filter (KF) but is optimal only when the model is linear .But Most of the state estimation problems are nonlinear, thereby limiting the practical applications of the KF and EKF. Unscented Kalman filter and Particle filter are best known for nonlinear estimates. The approach in this paper is to analyze the algorithm for maneuvering target tracking using bearing only measurements for both Gaussian /Nongaussian distributions where UKF provides better probability of state estimation. Montecarlo computer simulations are used to analyse the performance .The simulations results showed that UKF provides better performance for Gaussian distributed models compared to the nongaussian models.

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1. INTRODUCTION

Control of any process modeling, obtained from a priori knowledge of certain observable parameters is standard practice for Engineers. For many of the applications simple models with linear approximations around a design point suffice the requirement. Since all the natural phenomena are non-linear, it is very important to study the nonlinear models and their control for the following reasons:

1) Some systems have a linear approximation that is non controllable near interesting working points. Linearization is ineffective even locally for such cases.
2) Even if the linearized model is controllable one may wish to extend the operational domain beyond the validity domain into nonlinear region for better prediction.
3) Some control problems are external to the process and cannot be answered by a linearly approached model.

The success of the linear model in identification or in control has its cause in the good understanding of it. A better mastery of invariants of nonlinear models for some transformations is a prerequisite to a true theory of nonlinear identification and control. And all nonlinear systems are supposed to have a state space of finite dimension. State Estimation techniques are handled by filtering technique models for performance.

A common approach to overcome this problem is to linearize the system before using the Kalman filter, resulting in an extended Kalman filter. This linearization does however pose some problems, e.g. it can result in nonrealistic estimates [1, 2] over a period of time. The development of better estimator algorithms for nonlinear Systems has therefore attracted a great deal of interest in the scientific community, because the improvements will undoubtedly have great impact in a wide range of engineering fields. The EKF has been considered the standard in the theory of nonlinear state estimation. This paper deals with how to estimate a nonlinear model with unscented kalman filter (UKF). The approach in this paper is to analyze **Unscented**Kalman filter where UKF provides better probability of state estimation for a bearing only passive target tracking.

2. UNSCENTED KALMAN FILTER

Instead of linearising the functions, UKF transform uses a set of points and propagates them through the actual nonlinear function, eliminating linearization altogether. The points are chosen such that their mean, covariance and higher order moments match the Gaussian random variable. Mean and covariance can be recalculated from the propagated points, to yield more accurate results compared to Taylor's series ordinary function linearization.

Selection of sample points is not arbitrary. Gaussian random variable in N dimensions uses 2N+1 sample points. Matrix square root and Covariance definitions are used to select sigma points in such a way that their covariance is same as the Gaussian random variable.

The unscented Transform approach has the advantage that noise is treated as a nonlinear function to account for non Gaussian or non additive noises. The strategy for doing so involves propagation of noise through functions by first augmenting the state vector to include noise sources. Sigma points are then selected from the augmented state, which includes noise values also. The net result is that any nonlinear effects of process and measurement noise are captured with the same accuracy as the rest of the state, which in turn increases estimation accuracy in presence of additive noise sources.

3. AMODELLING EXAMPLE FOR MANEUVERING TARGET TRACKING USING BEARING ONLY MEASUREMENTS

There are many methods available to obtain target motion parameters in sonar signal processing[3-8].Target is assumed moving at constant course and constant speed. Its motion is updated every second. The own ship is also assumed to be stationary. It is assumed that noise in one bearing measurement is uncorrelated with that of the other. Another assumption is that the mean value of the noise is zero. In the simulator, random numbers are generated using central limit theorem. The output of Gaussian random generator is used as Gaussian noise for the Bearing measurements. The raw bearings are corrupted with the Gaussian noise. The output of another Gaussian random generator with given percentage input error is used to corrupt the frequency measurements.

The obtained bearing is modified according to the quadrant in which it exists such that its range is from 0-360 deg. (clock wise positive). The bearing is considered with respect to North.

Target parameters [R, B, C and S] and Own ship parameters [ocr and ospd] are read and taken as input by the simulator. Assumed error in Bearing measurement (sigma_b) and range measurement (sigma_r) are also fed as input.

Assumptions:

Following are the assumptions made in the simulator.

1) At start, own ship is at the origin.
2) Target is moving at constant velocity and
3) All angles are considered with respect to Y-axis.

3.1. Own ship motion

The own ship motion is introduced as follows. Consider the fig 2 shown below. The own ship is moving with a velocity v_0, x_0 is the distance of the own ship from the x-coordinate, $y_0$is the distance of the own ship from the y and Ocr is the angle making with north. From Fig 1.

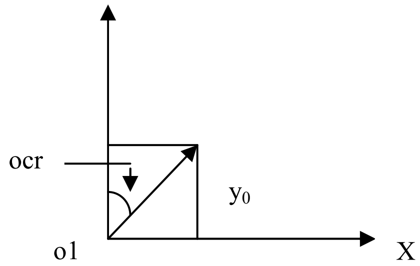

Fig 1

$$\text{Sin (ocr)} = x_0 / v_0 \quad (1)$$

$$\text{Cos (ocr)} = y_0 / v_0 \quad (2)$$

For every second change in X and Y component of own ship position is found and added to the previous X, Y components of own ship position.

For t_s=1sec

$$dX_0=v_0*\sin(Ocr)*ts \quad (3)$$

$$dY_0=v_0*\cos(Ocr)*ts \quad (4)$$

Where dX_0 is change in X-component of own ship position in 1 sec. dY_0 is change in Y-component of own ship position in 1 sec. v_0 is own ship velocity.Ocr is own ship course. (X_0 Y_0) is own ship position. Then

$$X_0 = (X_0 + dX_0) \quad \& \quad Y_0 = (Y_0 + dY_0) \quad (5)$$

3.2. *Initial target position*

From input bearing, initial position of target is known as follows.Considering Fig 2. Shown below.

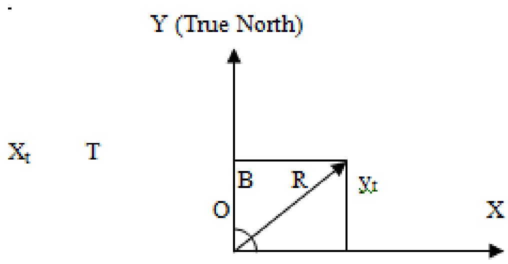

Fig 2

R-Range T-Target O-Observer

For t_s=1sec

X_t= range*sin (bearing) (6)

Y_t= range*cos (bearing) (7)

Where (X_t, Y_t) is target position with respect to own ship as the origin

3.3. *Target Motion*

The target motion is introduced as follows. Consider the Fig 3.shown below.

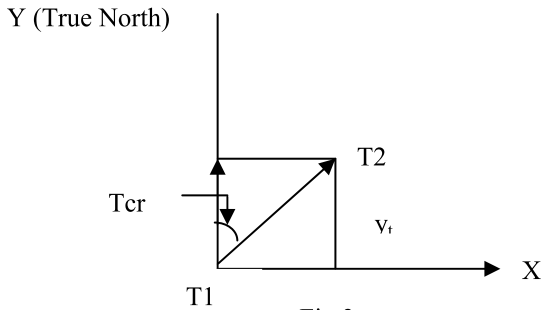

Fig 3.

From input range and Bearing initial position of target is known,

X_t=Range*sin(Bearing) (8)

Y_t=Range*cos (Bearing) (9)

($X_{t,}$ Y_t) is target position with respect to own ship as the origin.

For every 1 sec, change in $X_{t,}$ and Y_t are calculated and added to previous target position.

dX_t= v_t*sin (Tcr)*ts (10)

dY_t=v_t*cos (Tcr)*ts (11)

$X_t = (X_t + dX_t)$and $Y_t = (Y_t + dY_t)$ (12)

Where $dX_t$is change in X-component of target position in 1 sec $dY_t$is change in Y-component of target position in 1 sec.

$V_t$is target velocity.Tcr is target course with respect to true ϕ north.

$X_t = (X_t + dX_t)$ and $Y_t = (Y_t + dY_t)$.

The target is assumed to maintain fixed course and velocity through the observation duration.

3.4. Target tracking and mathematical modeling

State and measurement equations**:**The target is assumed to be moving with constant velocity as shown in the fig1. And is defined to have the state vector.

$$Xs\,(k) = [\,\dot{x}\,(k)\ \ \dot{y}\,(k)\ R_x\,(k) R_y\,(k)\ W_x(k)\ W_y(k)]^T \quad (13)$$

Where $R_x\,(k) R_y(k)$ denote the relative range components between observer and target. The observer state is similarly defined as

$$X_0 = [\,\dot{x}_0\,\dot{y}_0\,x_0\,y_0]^T \quad (14)$$

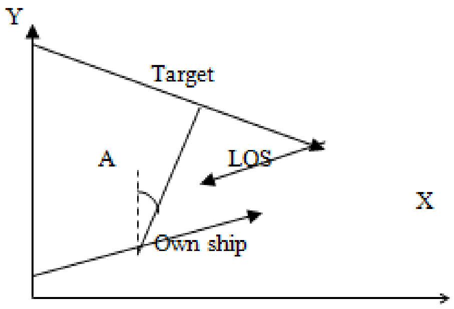

Fig 4.Target and observer encounter

The target state dynamic equation is given by

$$Xs\,(k+1) = \Phi(k+1/k)\ Xs\,(k) + b\,(k+1) + W\,(k) \quad (15)$$

Where Φ(k+1/k), b (k+1) and W (k) are transient matrix, deterministic vector and plant noise respectively. The transient matrix is given by

$$\Phi(K+1/k)= \begin{pmatrix} 1 & 0 & 0 & 0 & t_s & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & t_s & 0 \\ t_s & 0 & 1 & 0 & t_s^2/2 & 0 & 0 \\ 0 & t_s & 0 & 1 & 0 & t_s^2/2 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad (16)$$

Where $t_s$is sample time and b (k+1) is given by

b (k+1)=[0 0 -(X(k+1)-X(k)) -(y(k+1)-y(k)) 0 0 0 $]^T$ (17)

w (k) is a zero mean gaussian noise vector with E [W (k) W (k) ^{T}] =Qδ_{kj}. It is assumed that the measurement noise and plant noise are uncorrelated. The bearing measurement, modeled as

$$B_m\,(k+1) = \tan^{-1}\frac{Rx(k+1)}{R\,y(k+1)} \quad (18)$$

Where $\delta_{kj}$the Kronecker delta function, and ς(k) is error in the measurement and this error is assumed to be zero mean Gaussian with variance σ^2.The measurement and plant noises are to be uncorrelated to each other.

Kronecker Delta Function:

δ_{kj} =1 if k=j

0 if k≠j (19)

4. FILTER MODEL FORMULATION

4.1. Augmentation of State Vector

The filter starts by augmenting the state vector to N Dimensions, where N is the sum of dimensions in the original state-vector, model noise and measurement noise.

The covariance matrix is similarly augmented to a N^2 matrix. Together this forms the augmented state$\boldsymbol{\mathcal{X}^a}$ estimate vector and covariance matrix $\boldsymbol{P^a}$:

$$x^a_{k-1} = \begin{pmatrix} x_{k-1} \\ 0w \\ 0v \end{pmatrix} \quad (20)$$

$$P^a_{k-1} = E(x^a_{k-1} - \hat{x}^a_{k-1})(x^a_{k-1} - \hat{x}^a_{k-1})^T \begin{bmatrix} P^{k-1} & 0 & 0 \\ 0 & q_{k-1} & 0 \\ 0 & 0 & R_{k-1} \end{bmatrix} \quad (21)$$

4.2. Creating 2N+1 sigma-points

The $\boldsymbol{\mathcal{X}^a}$matrix is chosen to contain these points, and its columns are calculated as follows:

$$\begin{aligned} \chi^a_{0,k-1} &= X^a_{k-1} && i = 0 \\ \chi^a_{i,k-1} &= X^a_{k-1} + (\alpha\sqrt{NP^a_{k-1}})_i && i = 1\cdots, N \\ \chi^a_{i,k-1} &= X^a_{k-1} - (\alpha\sqrt{NP^a_{k-1}})_{i,})_{i-N} && i = N+1\cdots, 2N \end{aligned} \quad (22)$$

Subscript ‘i’meansith column of the square root of thecovariance matrix. The parameter α , in the interval 0 < α<1, determines sigma-point spread. This parameter is typically quite low, normally around 0.001, to avoid non-local effects. The resulting matrix$\boldsymbol{\mathcal{X}^a_{k-1}}$ can now be decomposed vertically into the$\boldsymbol{\mathcal{X}^x_{k-1}}$ rows, which contains the state;

The rows $\boldsymbol{\mathcal{X}^W_{k-1}}$, which contain sampled process noise and

The rows $\boldsymbol{\mathcal{X}^V_{k-1}}$, which contain sampled measurement noise.

4.3. Weightage in Estimation

Each sigma-point is also assigned a weight. The resulting weights for mean and covariance (C) estimates then become:

$$\begin{aligned} w_0^{(m)} &= 1 - 1/\alpha^2 i = 0 \\ w_0^{(c)} &= 4 - 1/\alpha^2 - \alpha^2 i = 0 \\ w_0^{(m)} &= w_0^{(c)} = 1/2\alpha^2 N i = 1\cdots, 2N \end{aligned} \quad (23)$$

4.4. Estimation

The filter then predicts next state by propagating the sigma-points through the state and measurement models, and then calculating weighted averages and covariance matrices of the results:

$$\chi^x_{k/k-1} = f(\chi^x_{k-1}, u_k, \chi^w_{k-1})$$
$$\hat{x}_{k/k} = \sum_{i=0}^{2N} W_i^{(m)} \chi^x_{k-1} P_{k/k-1} = \sum_{i=0}^{2N} W_i^{(c)} [\chi^x_{k-1} - \hat{x}_{k/k-1}][\chi^x_{k/k-1} - \hat{x}_{k/k-1}]^T$$
$$z_{k/k-1} = h(\chi^x_{k/k-1}, \chi^v_{k-1})$$
$$\hat{z}_{k/k-1} = \sum_{i=0}^{2N} W_i^{(m)} z_{i,k/k-1} \quad (24)$$

4.5. Mean and Covariance

The predictions are then updated with new measurements by first calculating the measurement covariance and state measurement cross correlation matrices, which are then used to determine Kalman gain - New state of the system; - Its associated covariance - Expected observation; - Cross-correlation matrix - Kalman Gain

$$P_{zz} = \sum_{i=0}^{2N} W_i^{(c)} [z_{i,k/k-1} - \hat{z}_{k/k-1}][z_{i,k/k-1} - \hat{z}_{k/k-1}]^T \quad (25)$$
$$P_{xz} = \sum_{i=0}^{2N} W_i^{(c)} [\chi^x_{i,k/k-1} - \hat{x}_{k/k-1}][z_{i,k/k-1} - \hat{z}_{k/k-1}]^T \quad (26)$$
$$K_k = P_{xz} P^{-1} \quad (27)$$
$$\hat{x}_{k/k} = \hat{x}_{k\{k-1} + K_k(z_k - \hat{z}_{k/k-1}) \quad (28)$$
$$P_{k/k} = P_{k/k-1} - K_k P_{yy} {K_k}^T \quad (29)$$

$\hat{x}_{k/k}$ - New state of the system; $P_{k/k}$ - Its associated covariance
$\hat{z}_{k/k-1}$ - Expected observation; P_{xz} - Cross-correlation matrix K_k -Kalman Gain

The properties of this algorithm:
1) Since the mean and covariance of x are captured precisely up to the second order, the calculated values of the mean and covariance of Nonlinear function (Yi = f [Xi]) are correct to the second order as well. This means that the mean is calculated to a higher order of accuracy than the EKF, whereas the covariance is calculated to the same order of accuracy. However, there are further performance benefits. Since the distribution of x is being approximated rather than the function, its series expansion is not truncated in a particular order. It can be shown that the unscented algorithm is able to partially incorporate information from the higher orders, leading to even greater accuracy.
2) The sigma points capture the same mean and covariance irrespective of the choice of matrix square root which is used.
3) The mean and covariance are calculated using standard vector and matrix operations. This means that the algorithm is suitable for any choice of process model, and implementation is extremely rapid because it is not necessary to evaluate the Jacobeans which are needed in an EKF.

5. RESULTS

The results are anlysed for UKF in the presence of Gaussian noise and Nongaussian noise for the initial conditions given below.

Parameter	Scenario1
Initial range, meters	5000
Initial bearing, deg	**0**
Target speed, meters/sec	2
Target course,deg	135
Observer speed, meters/sec	10
Observer course, deg	90
Error in the bearing, deg(one sigma)	0.33

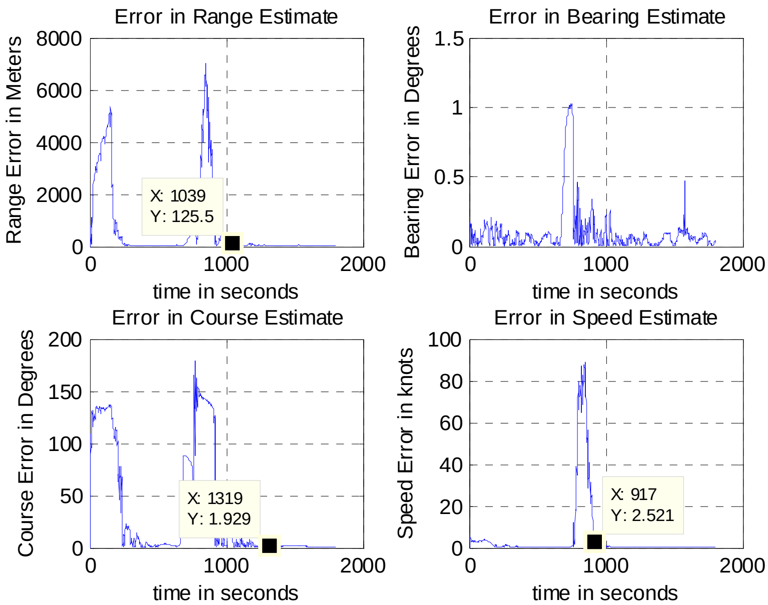

Fig 5. estimation errors of Target Motion Parameters (Range,Bearing,Course and Speed) in the precence of Gaussian noise

Analysis:
Duration of run: 1800 sec.
Time taken for convergence of Range for maneuver target is 1039 sec.
Time taken for convergence of Course for maneuver target is 1319 sec.
Time taken for convergence of Speedfor maneuver target is 917 sec.

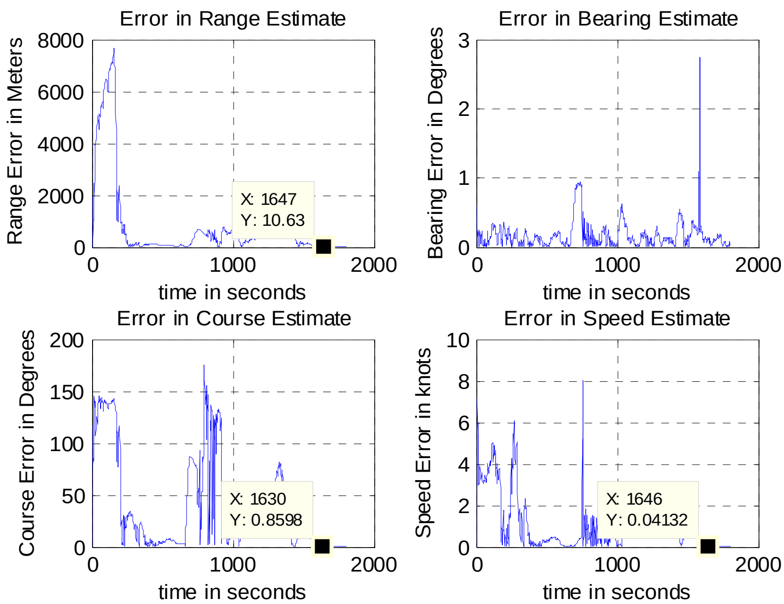

Fig 6. estimation errors of Target Motion Parameters (Range,Bearing,Course and Speed) in the precence of Nongaussian noise

Analysis:

Duration of run: 1800 sec.
Time taken for convergence of Range for maneuver target is 1647 sec.
Time taken for convergence of Course for maneuver target is 1630 sec.
Time taken for convergence of Speedfor maneuver target is 1646sec.

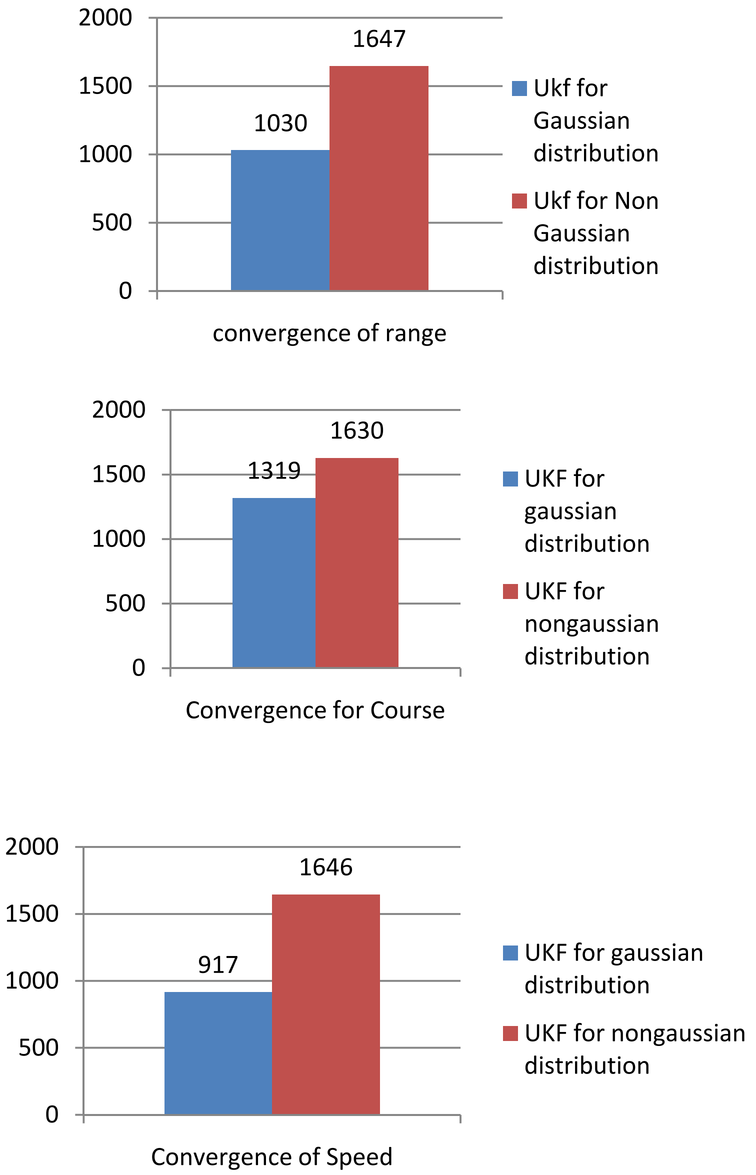

Fig 7. Comparison UKF for Gaussian and Non-Gaussian disribution

6. CONCLUSIONS

Application of KF to nonlinear systems results in highly inaccurate estimates. This paper looks into the need to consistently predict the new state and observation of the system with the presentation of UKF for nonlinear systems. We have used the nonlinear algorithm, UKF that has two great advantages over the KF. First, it is able to predict the state of the system more accurately. Second, it is much less difficult to implement. The benefits of the algorithm were demonstrated in a realistic example, bearing only passive target tracking. This paper has considered one specific form of the unscented transform for one particular set of assumptions. It is shown that the number of sigma points can be extended to yield a Filter which matches moments up to the fourth order. This higher order extension effectively de-biases almost all common nonlinear coordinate transformations.

The paper began with the simulation of the motion of the target and determining the initial target parameter namely bearing. This parameter was then corrupted with noise (The noise is assumed to be Gaussian and nongaussian and the results are compared and analyzed for two cases) to get the noisy measurements. Extended Kalman Filter can filter the noisy measurements and extend the target motion parameters but is having computational difficulties. The unscented Kalman Filter algorithm reduces this difficulty. Subsequently, maneuvering of the own ship was detected using relative Bearing algorithm and

CPA algorithm. Then the state of the target was corrected accordingly after the detection of correct own ship evasion. Monte-Carlo simulation was carried out in the end in a number of scenarios.

The results confirm that the failure rate of UKF is insignificant. For the UKF the initial errors in x position were more than 160m under the assumption of Gaussian noise, and are more in the case of nongaussian distributions, i.e thetarget motion parameters converges at an earlier time Under the assumption of Gaussian noise than Nongaussian noise. Therefore we may conclude that UKF is robust algorithm for Gaussian distributions than for Nongaussian distributions.

The performance can further be improved in the presence of nongaussian noise by taking more number of sample points and is referred to as Particle filter.

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