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A Comparative Study of Identification Techniques for Fractional Models

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ABSTRACT

A comparative study of methods for fractional system identification is presented in this paper. The fractional system is modeled by the help of a non integer integrator which is approximated by a J+1 dimensional modal system composed of an integrator and first order systems. This identification method is compared to other techniques available in the Matlab toolbox. The model parameters are estimated by an output-error technique using a non linear iterative optimization algorithm. Numerical simulations show the performance of the modal approach for modeling and identification.

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1. INTRODUCTION

The aim of any system identification technique is to establish a mathematical model able to reproduce the dynamic behaviour of a system. Many methods have been developed using continuous time models [1], [2], [3].

Studies on real systems such as thermal [4] or electrochemical [5], reveal inherent fractional differentiation behavior. The use of classical methods (based on integer order differentiation) is thus inappropriate in identifying these fractional systems. Thus, fractional models, using fractional differentiation, have been developed [6], [7], [8], [9].

A fractional model is defined by an equation or a system of differential equations characterized by real derivative orders, integer or not integer, i.e. in the monovariable case:

$$D_{m_N}(y(t))+a_{N-1}D_{m_{N-1}}(y(t))+\ldots+a_1D_{m_1}(y(t))+a_0y(t)=b_MD_{m_M}(u(t))+\ldots+D_{m_1}(u(t))+b_0u(t) \quad (1)$$

Where $u(t)$ and $y(t)$ are respectively the input and the output of the system.

The fractional derivative orders verify:

$$m_1 < m_2 < \cdots < m_N \quad (2)$$

In the context of parameter estimation, the study of Equation (1) reveals that the differentialoperators coefficients act linearly whereas the derivative orders act non-linearly. Two cases of study are then to distinguish.

The first is the case of a dynamic system where the derivative orders are fixed a priori. Only the coefficients of operators are then subject to parametric estimation. Based on the equation error method, the optimization techniques used are linear towards the parameters and allow a direct estimate.

In the second case, presented in this paper, the derivative orders have to be estimated in the same way that the coefficients. Based on the output error method, the optimization techniques used are non linear towards the parameters and algorithms involve non linear programming (NLP).

The paper is organized as follows. Definitions related to fractional integration in section II. After a reminder of principles related to state-space representation of the fractional integration operator in section III, the state space model of a fractional system is presented in section IV. An output error technique is presented in section V. Using the Matlab toolbox, the frequency domain approach and the modal approach of the non integer integrator, an application to numerical simulation on an example is presented in section VI. Finally, in section VII, we propose a comparison between the identification techniques.

2. FRACTIONAL DIFFERENTIATION AND INTEGRATION

Fractional integration is defined by the Riemann-Liouville Integral [10], [11], [12], [13]. The nth order integral (nreal positive) of the function $f(t)$ is defined by the relation:

$$I_n(f(t)) = \frac{1}{\Gamma(n)} \int_0^t (t-\tau)^{n-1} f(\tau) d\tau \tag{3}$$

Where $\Gamma(n) = \int_0^\infty x^{n-1} e^{-x} dx$ is the gamma function.

$I_n(f(t))$ is interpreted as the convolution [11] of the function $f(t)$ with the impulse response:

$$h_n(t) = \frac{t^{n-1}}{\Gamma(n)} \tag{4}$$

Of the fractional integration operator whose Laplace transform is:

$$I_n(s) = L\{ h_n(t) \} = \frac{1}{s^n} \tag{5}$$

Fractional differentiation is the dual operation of the fractional integration.

Consider the fractional integration operator $I_n(s)$ whose input/output are respectively $x(t)$ and $y(t)$. Then:

$$y(t) = I_n(x(t)) \tag{6}$$

or

$$Y(s) = \frac{1}{s^n} X(s) \tag{7}$$

Reciprocally, $x(t)$ is the nth order fractional derivative of $y(t)$ defined as:

$$x(t) = D_n(y(t)) \tag{8}$$

Or

$$X(s) = s^n Y(s) \tag{9}$$

Where s^n represents the Laplace transform of the fractional differentiation operator (with zero initial conditions).

3. SATE-SPACE REPRESENTATION OF THE FRACTIONAL INTEGRATION OPERATOR

3.1. Fractional integrator based on a frequency approach

3.1.1. Principle

Let us consider the Bode plots of a fractional integrator truncated in low and high frequencies (Figure 1) [14].

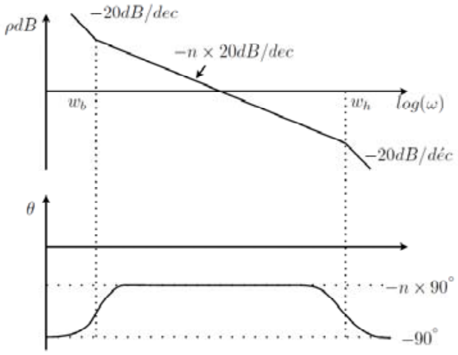

Figure 1. Bode Diagram of the Fractional Integrator

It is composed of three parts. The intermediary part corresponds to non-integer action, characterized by the order n. In the two other parts, the integrator has a conventional action, characterized by its order equal to 1. In this way, the operator $\tilde{I}_n(s)$ is defined as a conventional integrator, except in a limited band $[\omega_b, \omega_h]$ where it acts like s^{-n}. The operator $\tilde{I}_n(s)$ is defined using a fractional phase-lead filter [10] and an integrator s^{-1}.

$$\tilde{I}_n(s) = \frac{G_n}{s}\prod_{j=1}^{J}\frac{1+\frac{s}{\omega_j'}}{1+\frac{s}{\omega_j}} \tag{10}$$

The coefficient G_n is a normalized factor, such as $\tilde{I}_n(s)$ and $I_n(s)$ are identical on $[\omega_b, \omega_h]$.

This operator is completely defined by the following relations demonstrated by A. Oustaloup [10]:

$$\begin{aligned} &\omega_j = \alpha w_j' \text{ with } \alpha > 1 \\ &\omega_{j+1}' = \eta\omega_j \text{ with } \eta > 1 \\ &n = 1 - \frac{\log\alpha}{\log\alpha\eta} \end{aligned} \tag{11}$$

α and η are recursive parameters related to the non integer order n. When J is sufficiently large, the bode diagram of $\tilde{I}_n(s)$ tends towards the ideal one of Figure1.

3.1.2. State-space model $\tilde{I}_n(s)$

There is an infinite number of possibilities to represent $\tilde{I}_n(s)$ by a state space model. Practically, we have chosen the one where the state variables correspond to the outputs of the elementary cells of $A_v(s)$. Let:

$$Z_{j+1}(s) = \frac{1+\frac{s}{\omega_j'}}{1+\frac{s}{\omega_j}} Z_j(s) \tag{12}$$

or

$$-\alpha\dot{z}_{j-1}+\dot{z}_j=\omega_j(z_{j-1}-z_j)\ \text{for}\ j=1\ \text{to}\ J \quad (13)$$

with $Z_0(s)=\frac{G_n}{s}V(s)$

Where v(t) is the input of $\tilde{I}_n(s)$ and $z_J(t)=x(t)$ its output. The corresponding state space model is:

$$M_I\,\underline{\dot{z}}_I(t)=A_I\,\underline{z}_I(t)+\underline{B}_I v(t) \quad (14)$$

With:

$$M_I=\begin{bmatrix}1&0&\cdots&\cdots&0\\-\alpha&1&&&\vdots\\0&\ddots&\ddots&&\vdots\\\vdots&0&\ddots&\ddots&0\\0&\cdots&0&-\alpha&1\end{bmatrix}\quad A_I=\begin{bmatrix}0&0&\cdots&\cdots&0\\\omega_1&-\omega_1&&&\vdots\\0&\omega_2&-\omega_2&&\vdots\\\vdots&0&\ddots&\ddots&0\\0&\cdots&0&\omega_J&-\omega_J\end{bmatrix}\quad \underline{B}_I=\begin{bmatrix}G_n\\0\\\vdots\\\vdots\\0\end{bmatrix}\quad \underline{z}_I=\begin{bmatrix}z_0\\\vdots\\z_i\\\vdots\\z_J\end{bmatrix}$$

3.2. Fractional integrator based on a time approach
3.2.1. Principle

Diffusive representation, used by D. Matignon [15], [11] and G. Montseny [16] provides the theoretical basis for a time approximation of $I_n(s)$.

Consider a linear system such as:

$$x(t)=h(t)*v(t) \quad (15)$$

Where *h(t)* is its impulse response.

Let us define the function $\mu(\omega)$: it represents the diffusive representation (or the frequency weighting function) of the impulse response *h(t)*. *h(t)* and $\mu(\omega)$ verify the pseudo Laplace transform definition [16]:

$$h(t)=\int_0^\infty \mu(\omega)e^{-j\omega t}d\omega \quad (16)$$

A continuous frequency weighted state space model is associated to $\mu(\omega)$, according to:

$$\begin{cases}\dfrac{dz(\omega,t)}{dt}=-\omega z(\omega,t)+v(t)\\ x(t)=\displaystyle\int_0^\infty \mu(\omega)z(\omega,t)d\omega\end{cases} \quad (17)$$

For a fractional integration operator, it has been demonstrated [15], [16] that:

$$I_n(s)=\frac{1}{s^n} \quad (18)$$

with 0<n<1 and $h(t)=\frac{t^{n-1}}{\Gamma(n)}$

and

$$\mu(\omega)=\frac{\sin(n\pi)}{\pi}\omega^{-n} \quad (19)$$

3.2.2. Discrete frequency state model

This continuous frequency weighted model is not directly usable. A practical model is obtained by frequency discretization of $\mu(\omega)$, where the function $\mu(\omega)$ is replaced by a multiple step function (with K steps) (refer to Figure 2).

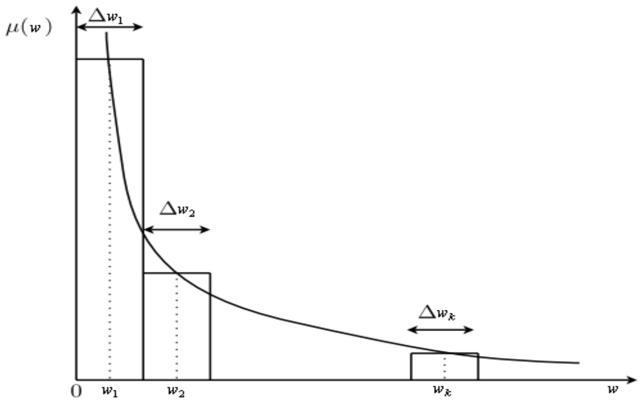

Figure 2. Frequency discritezation of $\mu(\omega)$

For an elementary step, its height is $\mu(\omega_k)$, and its width is $\Delta\omega_k$. Let c_k be the weight of the k^{th} element:

$$c_k = \mu(\omega_k)\Delta\omega_k \tag{20}$$

Thus, the continuous distributed model (17) becomes a conventional state model with dimension equal to K.

$$\begin{cases} \dfrac{dz_k(t)}{dt} = -\omega_k z_k(t) + v(t) \\ x(t) = \displaystyle\sum_{k=1}^{K} \mu(\omega_k) z_k(t)\Delta\omega_k \quad \text{for } k=1..K \\ \quad\; = \displaystyle\sum_{k=1}^{K} c_k z_k(t) \end{cases} \tag{21}$$

Or equivalently:

$$\begin{aligned} \dot{\underline{Z}}(t) &= A\underline{Z}(t) + \underline{B}v(t) \\ x(t) &= \underline{C}^T \underline{Z}(t) \end{aligned} \tag{22}$$

With,

$$\underline{Z}(t) = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_K \end{bmatrix}, \quad A = \begin{bmatrix} -\omega_1 & & 0 \\ & \ddots & \\ 0 & & -\omega_K \end{bmatrix}, \underline{B}^T = \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}, \ \underline{C}^T = \begin{bmatrix} c_1 & c_2 & \cdots & c_K \end{bmatrix}$$

With this approach, we obtain a discrete state-space model which is frequency distributed with the constraints: $\omega_1 \to 0$, $\omega_K \to \infty$ et $K >> 1$.

It is easy to transform the model (14) of $I_n(s)$ into a modal form because the ω_j are known a priori. This transformation is based on the following definition by decomposition in simple elements:

$$\tilde{I}_n(s) = \frac{c_0}{s} + \sum_{j=1}^{J} \frac{c_j}{s+\omega_j} \tag{23}$$

Where c_0 and c_j coefficients are linked to G_n, ω_j and $\omega_j^{'}$ by the relation:

$$c_0 = G_n$$

$$c_j = \frac{G_n(\omega_j - \omega_j^{'})}{\omega_j^{'}} \prod_{\substack{i=1 \\ i \neq j}}^{J} \frac{1-\dfrac{\omega_j}{\omega_i^{'}}}{1-\dfrac{\omega_j}{\omega_i}} \tag{24}$$

This second definition of $I_n(s)$ corresponds to a modal state model:

$$\begin{cases} \underline{\dot{Z}}^{'}(t) = A_I^{'}\underline{Z}^{'}(t) + \underline{B}_I^{'}(t)v(t) \\ x(t) = \underline{C}_I^{'T}\underline{Z}^{'}(t) \end{cases} \tag{25}$$

With:

$$A_I^{'} = \begin{bmatrix} 0 & & & 0 \\ & -\omega_1 & & \\ & & \ddots & \\ 0 & & & -\omega_J \end{bmatrix}; \underline{B}_I^{'} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}; \underline{C}_I^{'T} = \begin{bmatrix} c_0 & c_0 & \dots & c_J \end{bmatrix}$$

In the frequency domain approach, the modes ω_j are indirectly obtained by $I_n(s)$ in the $[\omega_b;\omega_h]$ interval, they correspond to the modes of the modal approach. The interest of this last representation is that the modes are decoupled, which allows fast computations. Moreover, the interest of $\omega_0 = 0$ is to reject static error in simulation applications.

4. STATE-SPACE OF FRACTIONAL MODEL

In the context of non integer system simulation and particularly for output error identification, the state space representation (17) of the operator is inserted in a non integer state representation describing the system to be simulated.

Consider the following transfer function with two non integer derivative orders:

$$H_{n_1,n_2}(s) = \frac{b_0 + b_1 s^{n_1}}{a_0 + a_1 s^{n_1} + s^{n_1+n_2}} \tag{26}$$

This transmittance corresponds to the fractional differential equation:

$$D_{n_1+n_2}(y(t)) + a_1 D_{n_1}(y(t)) + a_0 y(t) = b_1 D_{n_1}(u(t)) + b_0 u(t) \tag{27}$$

The pseudo state-space representation of this system is:

$$\begin{cases} \dfrac{d^{n_1}x_1(t)}{dt^{n_1}} = x_2(t) \\ \dfrac{d^{n_2}x_2(t)}{dt^{n_2}} = u(t) - a_0 x_1(t) - a_1 x_2(t) \\ y(t) = b_0 x_1(t) + b_1 x_2(t) \end{cases} \tag{28}$$

The global state-space representation:

$$\begin{cases} \underline{\dot{x}} = \begin{bmatrix} A'_{I_1} & \underline{B}'_{I_1}\underline{C}_{I_2} \\ -\underline{B}'_{I_2}a_0\underline{C}_{I_1} & A'_{I_2}-\underline{B}'_{I_2}a_1\underline{C}_{I_2} \end{bmatrix}\underline{x} + \begin{bmatrix} 0 \\ \underline{B}'_{I_2} \end{bmatrix}u \\ y = \begin{bmatrix} b_0\underline{C}_{I_1} & b_1\underline{C}_{I_2} \end{bmatrix}\underline{x} \end{cases} \tag{29}$$

Where (A'_{I_1} , $\underline{B}'_{I_1}$) and (A'_{I_2} , $\underline{B}'_{I_2}$) are matrix defining the two integrators $I_{n_1}(s)$ and $I_{n_2}(s)$.

Remark: (28) is the pseudo state space model of the system because x_1 and x_2 are not true state variables.

5. OUTPUT ERROR METHOD

Next, we present a method allowing the estimation of derivative orders as well as the coefficients.

Whereas parametric estimation can be performed by a linear optimization technique in case only the coefficients are estimated, the estimation of the derivative orders and of the coefficients requires the use of a nonlinear programming algorithm.

The method suggested by Trigeassou, Lin and Poinot, is based on the definition of non integer integration operator limited in frequency.

The model of the system is in continuous time representation, thus it is preferable to use an output error technique (OE) to estimate its parameters [17].

The state-space model of the non integer system is:

$$\begin{cases} \underline{\dot{x}} = A(\underline{\theta})\underline{x} + B(\underline{\theta})u \\ y = C^T(\underline{\theta})\underline{x} + D(\underline{\theta})u \end{cases} \tag{30}$$

For the model $H_{n_1,n_2}(s)$, the parameter vector is defined by:

$$\underline{\theta}^T = [a_0 \quad a_1 \quad b_0 \; b_1 \; n_1 \quad n_2]$$

The state-space model is simulated using a numerical integration algorithm, thus one gets:

$$\hat{y}_i = f_i(u, \underline{\hat{\theta}}_i) \tag{31}$$

Where $\underline{\hat{\theta}}_i$ is an estimation of $\underline{\theta}$ at iteration i.

The optimal value of $\underline{\hat{\theta}}(\underline{\theta}_{opt})$ is obtained by minimization of the quadratic criterion:

$$J_c = \sum_{k=1}^{K} (y_k^* - \hat{y}_k(u, \underline{\hat{\theta}}_k))^2 \tag{32}$$

We obtain:

$$\underline{\hat{\theta}}_{i+1} = \underline{\hat{\theta}}_i + \Delta\underline{\theta} \tag{33}$$

Where $\Delta\underline{\theta}$ depends on the optimization algorithm.

We can use a black box technique such as the Matlab toolbox functions in order to minimize J_c In this case we seek to obtain the optimal $\underline{\theta}_{opt}$ without worrying of how we reach that point. But this technique presents some defects such as the absence of direct informations on the criterion at the optimum, thus in particular on the precision (sensitivity of J_c in comparison with the different estimates).

To remedy this defect, we use sensitivity functions of the output. Because $\hat{y}(t)$ is non linearin $\underline{\hat{\theta}}$, a Non Linear Programming technique is used to estimate iteratively $\underline{\hat{\theta}}_i$:

$$\underline{\hat{\theta}}_{i+1} = \underline{\hat{\theta}}_i - \left\{ \left[J''_{\theta\theta} + \lambda I \right]^{-1} \underline{J}'_\theta \right\}_{\hat{\theta}=\theta_i} \quad (34)$$

With [18], [20]:

$$\begin{cases} \underline{J}'_\theta = -2\sum_{k=1}^{K} \varepsilon_k \underline{\sigma}_{k,\underline{\theta}_i} : \text{ gradient} \\ J''_{\theta\theta} = 2\sum_{k=1}^{K} \underline{\sigma}_{k,\underline{\theta}_i} : \text{ hessien} \\ \lambda : \text{Marquardt parameter} \\ \underline{\sigma}_{k,\underline{\theta}_i} = \dfrac{\partial \hat{y}_k}{\partial \underline{\theta}_i} : \quad \text{sensitivity function} \end{cases} \quad (35)$$

This algorithm, known as Marquardt's one [18], often used in non linear optimization, ensures a robust convergence in spite of a bad initialization of $\underline{\hat{\theta}}$. A good precision on the output sensibility functions $\underline{\sigma}_{k,\underline{\theta}_i}$ [17], is however necessary to ensure a good convergence and precision.

6. APPLICATION

In order to compare the identification techniques of a non integer system, an illustrative example is treated to exhibit the performances of each technique.

$$H_{n_1,n_2}(s) = \frac{b_0 + b_1 s^{n_1}}{a_0 + a_1 s^{n_1} + s^{n_1+n_2}}$$

With:

$a_0 = 0.5$, $a_1 = 1.5$, $b_0 = 1$, $b_1 = 2$, $n_1 = 0.6$, $n_2 = 0.5$

The data set is composed of K data pairs $\{u_k, y_k^*\}$ with $t = kT_e$ (T_e: sampling period) and K=500, $T_e = 10^{-4} s$.

6.1. Identification using matlabtoolbox

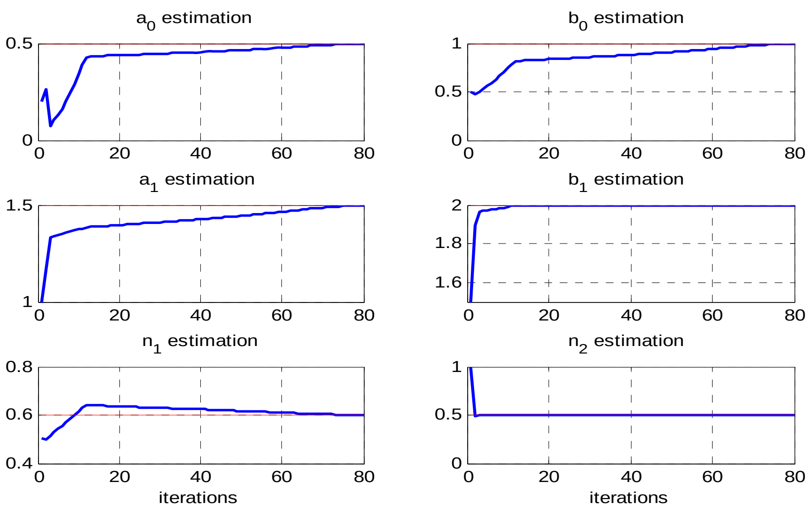

Figure 3. Identification using Matlab Toolbox

The Curve Fitting Toolbox uses the nonlinear least squares formulation to fit a nonlinear model to data. A nonlinear model is defined as an equation that is nonlinear in the coefficients.

Fitting requires a parametric model that relates the response data to the predictor data with one or more coefficients. The result of the fitting process is an estimate of the model coefficients.To obtain the coefficient estimates, the least squares method minimizes the criterion J_c.It uses a predefinedfunction “LSQNONLIN”, an implementation of the Levenberg-Marquardt algorithm, to minimize a nonlinear function of several variables. We obtain the identification results from Figure 3.

6.2. Identification using frequency domain approach

In this section, we present the identification results on Figure 4 and performed by the frequency approach.

This method is based on the simulation of the sensitivity functions. It gives better results than the direct approach, but it leads to an important calculation load.

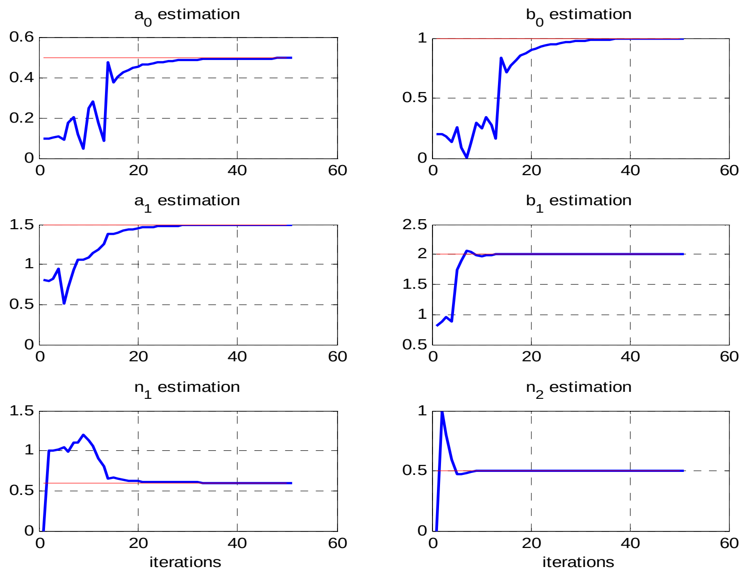

Figure 4. Identification by frequency approach

Moreover, the analytical calculation of sensitivity functions can be inextricable, even unnecessarily complex, concerning the output sensitivity of the parameter n_i (with respect to the coefficients: α_i η_i). For this reason we prefer now to use the modal model.

6.3. Identification using modal approach

The modal formulation is not adapted to the calculation of $\frac{\partial x(t)}{\partial n}$ because the ω_k and c_k are complicated functions of n. It is possible to simplify and proceed directly the calculation of the sensitivity functions [19], [20], [21], [22] by numerical differentiation, in the form:

$$\frac{\partial x(\hat{n},t)}{\partial \hat{n}} = \lim_{\Delta n \to 0} \frac{x(\hat{n}+\Delta n,t)-x(\hat{n},t)}{\Delta n} \tag{36}$$

A preliminary study is essential for the choice of Δn. In the general case, $\Delta\theta$ is difficult to choose because θ can vary from 0 to ∞. Because $0 < n < 1$, it is easy to find an optimal value of Δn, which will be always the same. Then the calculation becomes more simple.

The simulation of the modal model is simple and powerful. This modal representation guarantees precision and reduces calculation time. We have represented on Figure 5 the identification result using the modal representation.

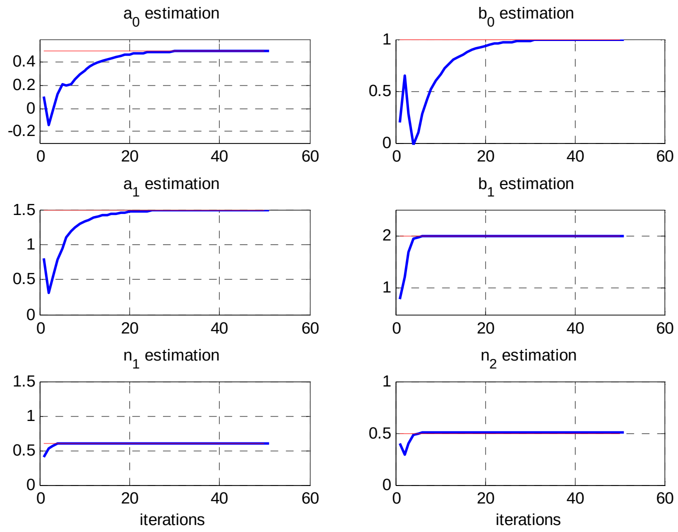

Figure 5. Identification by modal approach

7. COMPARISON OF THE METHODS

The use of the Matlab toolbox as a black box technique is simple, but this technique presents some defects such as the absence of direct informations on the criterion at the optimum and the precision. Moreover, the convergence appears to be very slow.

The method of the fractional integrator is more complex to implement. However, it relies on a state-space representation allowing to generalize the fundamental concepts related to ODEs.

Finally, the use of the modal representation of the fractional integrator reduces the convergence time compared to poles/zeros approach and its programming is much simple, which is an important feature in the context of more complex systems.

8. CONCLUSION

In this paper, we have presented and compared some techniques for the identification of fractional systems. We have presented the output error method based on the definition of a fractional state space representation. The modal model has confirmed the interest and the validity of this new approach for calculation time and simplicity compared to the frequency approach and the Matlab toolbox techniques.

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Abdelhamid JALLOUL was born in Monastir in 1979. He received the engineer diploma and master degrees from National Engineering School of Sfax and Monastir, Tunisia, respectively in 2004 and 2006. He is currently working for the PhD degree at Tunis University, Tunisia. His research interests are in the fractional modeling of rotor skin effect in induction machines with application to control and diagnosis.

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Jean-Claude TRIGEASSOU was born in Libourne (33) France on december 12, 1946. He received the Ph.D. degree in automatic control from ENSM Nantes in 1980 and the Doctoratd'EtatEs Sciences in automatic control from Poitiers University in 1987.
From 1988 to 2006, he has been professor at ESIP, an engineering school at Poitiers University. Since 2006, he is retired and Honorary Professor.
His major research interests have been in the method of moments with applications to identification and control and in the parameter estimation of continuous systems with application to the diagnosis of electrical machines.
At present, he his associated to the activities of the IMS-LAPS at Bordeaux University and his research works deal with modelling, stability, identification and control of fractional order systems.