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Artificial Tune of Fuel Ratio: Design a Novel SISO Fuzzy Backstepping Adaptive Variable Structure Control

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ABSTRACT

This paper examines single input single output (SISO) chattering free variable structure control (VSC) which controller coefficient is on-line tuned by fuzzy backstepping algorithm. VSC methodology is selected as a framework to construct the control law and address the stability and robustness of the close loop system based on Lyapunove formulation. The main goal is to guarantee acceptable fuel ratio result and adjust. The proposed approach effectively combines the design technique from variable structure controller is based on Lyapunov and fuzzy estimator to estimate the nonlinearity of undefined system dynamic in backstepping controller. The input represents the function between variable structure function, error and the rate of error. The outputs represent fuel ratio, respectively. The fuzzy backstepping methodology is on-line tune the variable structure function based on adaptive methodology. The performance of the SISO VSC which controller coefficient is on-line tuned by fuzzy backstepping algorithm (FBSAVSC) is validated through comparison with VSC and proposed method. Simulation results signify good performance of trajectory in presence of uncertainty torque load.

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1. INTRODUCTION

Modeling of an entire internal combustion (IC) engine is a very important and complicated process because internal combustion engines are nonlinear, multi inputs-multi outputs (MIMO) and time variant. There have been several engine controller designs over the previous years in which the main goal is to improve the efficiency and exhaust emissions of the automotive engine [1-4]. Specific applications of air to fuel (A/F) ratio control based on observer measurements in the intake manifold were developed by Benninger in 1991 [5]. Another approach was to base the observer on measurements of exhaust gases measured by the oxygen sensor and on the throttle position, which was researched by Onder [6]. These observer ideas used linear observer theory. Hedrick also used the measurements of the oxygen sensor to develop a nonlinear, sliding mode approach to control the A/F ratio [7]. All of the previous control strategies were applied to engines that used only port fuel injections, where fuel was injected in the intake manifold. Current production A/F ratio controllers use closed loop feedback and feed forward control to achieve the desired stoichio metric mixture. These controllers use measurements from the oxygen sensor to control the desired amount of fuel that should be injected over the next engine cycle and have been able to control the A/F very well [8-9]. Classical and non-classical methods are two main categories of nonlinear plant control, where the conventional (classical) control theory uses the classical method and the non-classical control theory (e.g., fuzzy logic, neural network, and neuro fuzzy) uses the artificial intelligence methods. However both of conventional and artificial intelligence theories have applied effectively in many areas, but these methods also have some limitations [1-2]. Modeling of an entire IC engine is a very important and complicated process because these systems are nonlinear, MIMO and time variant. One purpose of accurate modeling is to save development costs of real engines and minimizing the risks of damaging an engine when validating controller designs [1-10]. Dynamic modeling of IC engine is used to describe the behavior of this system, design of model based controller, and for simulation. The dynamic modeling describes the relationship between nonlinear output formulations to electrical or mechanical source and also it can be used to describe the particular dynamic effects to behavior of system [1], [11-22].

Controller (control system) is a device which can sense information from linear or nonlinear system (e.g., IC engine) to improve the systems performance [3-20]. In feedback control system considering that there are many disturbances and also variable dynamic parameters something that is really necessary is keeping plant variables close to the desired value. Feedback control system development is the most important thing in many different fields of engineering. The main targets in designing control systems are stability, good disturbance rejection, and small tracking error [5-29]. At present, in some applications engines are used in unknown and unstructured environment, therefore strong mathematical tools used in new control methodologies to design nonlinear robust controller with an acceptable performance (e.g., minimum error, good fuel ratio, torque load rejection). One of the best nonlinear robust controllers is variable structure control which is used in nonlinear uncertain systems. One of the nonlinear robust controllers is variable structure controller, although this controller has been analyzed by many researchers but the first proposed was in the 1950 [41-62].This controller is used in wide range areas such as in control process, in aerospace applications and in IC engines because this methodology can solve some main challenging topics in control such as resistivity to the external disturbance and stability. Even though, this controller is used in wide range areas but, pure variable structure controller has two drawbacks: Firstly, output oscillation (chattering); which caused the heating in the mechanical parameters. Secondly, nonlinear dynamic formulation of nonlinear systems which applied in nonlinear dynamic nonlinear controller; calculate this control formulation is absolutely difficult because it depends on the dynamic nonlinear system's equation [20-23]. Neural network, fuzzy logic, and neuro-fuzzy are synergically combined with nonlinear classical controller and used in nonlinear, time variant, and uncertainty plant. Fuzzy logic controller (FLC) is one of the most important applications of fuzzy logic theory [30-41]. This controller can be used to control nonlinear, uncertain, and noisy systems. This method is free of some model-based techniques as in classical controllers. As mentioned that fuzzy logic application is not only limited to the modelling of nonlinear systems [31-36]but also this method can help engineers to design easier controller.Control engine using classical controllers are based on system's dynamic modelling. These controllers often have many problems for modelling. Conventional controllers require accurate information of dynamic model of engine. When the system model is unknown or when it is known but complicated, it is difficult or impossible to use classical mathematics to process this model [32]. The main reasons to use fuzzy logic technology are able to give approximate recommended solution for unclear and complicated systems to easy understanding and flexible. Fuzzy logic provides a method which is able to model a controller for nonlinear plant with a set of IF-THEN rules, or it can identify the control actions and describe them by using fuzzy rules [32]. In various dynamic parameters systems that need to be training on-line adaptive control methodology is used. Fuzzy adaptive method is used in systems which want to training parameters by expert knowledge. Traditional adaptive method is used in systems which some dynamic parameters are known. In this research in order to solve disturbance rejection and uncertainty dynamic parameter, adaptive method is applied to variable structure controller.

Even though, variable structure controller is used in wide range areas but, pure it also has chattering problem and nonlinear dynamic part challenges [8]. The boundary layer method is used to reduce or eliminate the chattering. To reduce the effect of uncertainty in proposed method, SISO novel fuzzy backstepping adaptive method is applied in variable structure controller in engine.

The main goal in this paper is to design a SISO fuzzy backstepping adaptive variable structure methodology which applied to IC engine with easy to design and implement. IC engine has nonlinear dynamic and uncertain parameters consequently; following objectives have been pursuit in the mentioned research: To develop a chattering in a position pure variable structure controller against uncertainties and to develop a position fuzzy backstepping adaptive variable structure controller in order to solve the disturbance rejection.

2. THEORY

Mathematical Modeling of IC Engine Using Euler Lagrange: In developing a valid engine model, the concept of the combustion process, abnormal combustion and cylinder pressure must be understood. The combustion process is relatively simple and it begins with fuel and air being mixed together in the intake manifold and cylinder. This air-fuel mixture is trapped inside cylinder after the intake valve(s) is closed and then gets compressed [6-9]. When the air-fuel mixture is compressed it causes the pressure and temperature to increase inside the cylinder. In abnormal combustion, the cylinder pressure and temperature can rise so rapidly that it can spontaneously ignite the air-fuel mixture causing high frequency cylinder pressure oscillations. These oscillations cause the metal cylinders to produce sharp noises called knock, which it caused to abnormal combustion. The pressure in the cylinder is a very important physical parameter that can be analyzed from the combustion process. Since cylinder pressure is very important to the combustion event and the engine cycle in spark ignition engines, the development of a model that produces the cylinder pressure for each crank angle degree is necessary.

The dynamic equations of IC engine can be written as:

$$\begin{bmatrix} \boldsymbol{PFI} \\ \boldsymbol{DI} \end{bmatrix} = \begin{bmatrix} \dot{\boldsymbol{M}}_{\boldsymbol{air}_{11}} & \dot{\boldsymbol{M}}_{\boldsymbol{air}_{12}} \\ \dot{\boldsymbol{M}}_{\boldsymbol{air}_{21}} & \dot{\boldsymbol{M}}_{\boldsymbol{air}_{22}} \end{bmatrix} \begin{bmatrix} \boldsymbol{F\ddot{R}} \\ \ddot{\boldsymbol{\alpha}}_{\boldsymbol{I}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{P}_{\boldsymbol{motor}_1} \\ \boldsymbol{P}_{\boldsymbol{motor}_2} \end{bmatrix} \left[\boldsymbol{F\dot{R}\dot{\alpha}_{I}} \right] + \begin{bmatrix} \boldsymbol{N}_{11} & \boldsymbol{N}_{12} \\ \boldsymbol{N}_{21} & \boldsymbol{N}_{22} \end{bmatrix} \times \begin{bmatrix} \boldsymbol{F\dot{R}} \\ \dot{\boldsymbol{\alpha}}_{\boldsymbol{I}} \end{bmatrix}^2 + \begin{bmatrix} \boldsymbol{M}_{\boldsymbol{a}_1} \\ \boldsymbol{M}_{\boldsymbol{a}_2} \end{bmatrix} \tag{1}$$

There for to calculate the fuel ratio and equivalence ratio we can write:

$$\begin{bmatrix} \boldsymbol{F\ddot{R}_a} \\ \ddot{\boldsymbol{\alpha}}_{\boldsymbol{I_a}} \end{bmatrix} = \begin{bmatrix} \dot{\boldsymbol{M}}_{\boldsymbol{air}_{11}} & \dot{\boldsymbol{M}}_{\boldsymbol{air}_{12}} \\ \dot{\boldsymbol{M}}_{\boldsymbol{air}_{21}} & \dot{\boldsymbol{M}}_{\boldsymbol{air}_{22}} \end{bmatrix}^{-1} \left\{ \begin{bmatrix} \boldsymbol{PFI} \\ \boldsymbol{DI} \end{bmatrix} - \left\{ \begin{bmatrix} \boldsymbol{P}_{\boldsymbol{motor}_1} \\ \boldsymbol{P}_{\boldsymbol{motor}_2} \end{bmatrix} \left[\boldsymbol{F\dot{R}\dot{\alpha}_{I_a}} \right] + \begin{bmatrix} \boldsymbol{N}_{11} & \boldsymbol{N}_{12} \\ \boldsymbol{N}_{21} & \boldsymbol{N}_{22} \end{bmatrix} \times \begin{bmatrix} \boldsymbol{F\dot{R}_a} \\ \dot{\boldsymbol{\alpha}}_{\boldsymbol{I_a}} \end{bmatrix}^2 + \begin{bmatrix} \boldsymbol{M}_{\boldsymbol{a}_1} \\ \boldsymbol{M}_{\boldsymbol{a}_2} \end{bmatrix} \right\} \right\} \tag{2}$$

To solve $\dot{\boldsymbol{M}}_{\boldsymbol{air}}$ **,** we can write;

$$\dot{\boldsymbol{M}}_{\boldsymbol{air}} = \begin{bmatrix} \dot{\boldsymbol{M}}_{\boldsymbol{air}_{11}} & \dot{\boldsymbol{M}}_{\boldsymbol{air}_{12}} \\ \dot{\boldsymbol{M}}_{\boldsymbol{air}_{21}} & \dot{\boldsymbol{M}}_{\boldsymbol{air}_{22}} \end{bmatrix} \tag{3}$$

Where $\dot{\boldsymbol{M}}_{\boldsymbol{air}_{12}} = \dot{\boldsymbol{M}}_{\boldsymbol{air}_{21}}$

Where $\dot{\boldsymbol{M}}_{\boldsymbol{air}}$ is the ratio of the mass of air.
Matrix P_{motor} is a 1×2 matrix:

$$P_{motor} = \begin{bmatrix} \boldsymbol{P}_1 \\ \boldsymbol{P}_2 \end{bmatrix} \tag{4}$$

Matrix engine angular speed matrix(N) is a 2×2 matrix.

$$N = \begin{bmatrix} \boldsymbol{N}_{11} & \boldsymbol{N}_{12} \\ \boldsymbol{N}_{21} & \boldsymbol{N}_{22} \end{bmatrix} \tag{5}$$

Where,
Matrix mass of air in cylinder for combustion matrix (M_a) is a 1×2 matrix.

$$M_a = \begin{bmatrix} \boldsymbol{M}_{\boldsymbol{a}_1} \\ \boldsymbol{M}_{\boldsymbol{a}_2} \end{bmatrix} \tag{6}$$

The above target equivalence ratio calculation will be combined with fuel ratio calculation that will be used for controller design purpose.

Sliding Mode methodology: Consider a nonlinear single input dynamic system is defined by [10-17]:

$$\boldsymbol{x^{(n)} = f(\vec{x}) + b(\vec{x})u} \tag{7}$$

Where u is the vector of control input, $x^{(n)}$ is the n^{th} derivation of$x$, $x = [x, \dot{x}, \ddot{x}, \dots, x^{(n-1)}]^T$ is the state vector, $f(x)$ is unknown or uncertainty, and $b(x)$ is of known sign function. The main goal to design this controller is train to the desired state; $x_d = [x_d, \dot{x}_d, \ddot{x}_d, \dots, x_d{}^{(n-1)}]^T$, and trucking error vector is defined by [18]-[30]:

$$\boldsymbol{\widetilde{x} = x - x_d = [\widetilde{x}, \dots, \widetilde{x}^{(n-1)}]^T} \tag{8}$$

A time-varying sliding surface $s(x,t)$ in the state space $R^n$is given by [31]-[45]:

$$\boldsymbol{s(x,t) = (\frac{d}{dt} + \lambda)^{n-1}\widetilde{x} = 0} \tag{9}$$

Where λ is the positive constant. To further penalize tracking error, integral part can be used in sliding surface part as follows [10]:

$$\boldsymbol{s(x,t)} = (\frac{d}{dt} + \boldsymbol{\lambda})^{\boldsymbol{n-1}}\left(\int_{\boldsymbol{0}}^{\boldsymbol{t}} \widetilde{x}\, \boldsymbol{dt}\right) = \boldsymbol{0} \tag{10}$$

The main target in this methodology is kept the sliding surface slope $s(x,t)$ near to the zero. Therefore, one of the common strategies is to find input U outside of $s(x,t)$ [10].

$$\boldsymbol{\frac{1}{2}\frac{d}{dt}s^2(x,t) \le -\zeta|s(x,t)|} \tag{11}$$

Where ζ is positive constant.

$$\text{If } \mathbf{S(0)>0} \rightarrow \frac{\mathbf{d}}{\mathbf{dt}}\mathbf{S(t)} \le -\boldsymbol{\zeta} \tag{12}$$

To eliminate the derivative term, it is used an integral term from t=0 to t=t_{reach}.

$$\boldsymbol{\int_{t=0}^{t=t_{reach}} \frac{d}{dt}S(t) \le -\int_{t=0}^{t=t_{reach}} \eta \rightarrow S\,(t_{reach}) - S(0) \le -\zeta(t_{reach} - 0)} \tag{13}$$

Where t_{reach} is the time that trajectories reach to the sliding surface so, suppose S $(t_{reach} = 0)$ defined as:

$$\boldsymbol{0 - S(0) \le -\eta(t_{reach}) \rightarrow t_{reach} \le \frac{S(0)}{\zeta}} \tag{14}$$

And,

$$\boldsymbol{if\ S(0)} < 0 \rightarrow 0 - S(\boldsymbol{0}) \le -\boldsymbol{\eta}(\boldsymbol{t_{reach}}) \rightarrow \boldsymbol{S(0) \le -\zeta(t_{reach})} \rightarrow \boldsymbol{t_{reach} \le \frac{|S(0)|}{\eta}} \tag{15}$$

Equation (15) guarantees time to reach the sliding surface is smaller than $\frac{|S(0)|}{\zeta}$since the trajectories are outside of $S(t)$.

$$\boldsymbol{if\ S_{t_{reach}} = S(0) \rightarrow error(x - x_d) = 0} \tag{16}$$

Suppose S is defined as:

$$\begin{aligned} \boldsymbol{s(x,t)} &= (\frac{\boldsymbol{d}}{\boldsymbol{dt}} + \boldsymbol{\lambda})\ \ \tilde{\boldsymbol{x}} \\ &= (\dot{\mathbf{x}} - \dot{\mathbf{x}}_{\mathbf{d}}) + \boldsymbol{\lambda}(\mathbf{x} - \mathbf{x}_{\mathbf{d}}) \end{aligned} \tag{17}$$

The derivation of S, namely, $\dot{S}$ can be calculated as the following;

$$\dot{\boldsymbol{S}} = (\ddot{\mathbf{x}} - \ddot{\mathbf{x}}_{\mathbf{d}}) + \boldsymbol{\lambda}(\dot{\mathbf{x}} - \dot{\mathbf{x}}_{\mathbf{d}}) \tag{18}$$

Suppose the second order system is defined as;

$$\ddot{\boldsymbol{x}} = \boldsymbol{f} + \boldsymbol{u}\ \rightarrow \dot{\boldsymbol{S}} = \boldsymbol{f} + \boldsymbol{U} - \ddot{\boldsymbol{x}}_{\boldsymbol{d}} + \boldsymbol{\lambda}(\dot{\mathbf{x}} - \dot{\mathbf{x}}_{\mathbf{d}}) \tag{19}$$

Where $\boldsymbol{f}$ is the dynamic uncertain, and also since $S = 0\ and\ \dot{S} = 0$, to have the best approximation ,$\widehat{\boldsymbol{U}}$ is defined as:

$$\widehat{\boldsymbol{U}} = -\widehat{\boldsymbol{f}} + \ddot{\boldsymbol{x}}_{\boldsymbol{d}} - \boldsymbol{\lambda}(\dot{\mathbf{x}} - \dot{\mathbf{x}}_{\mathbf{d}}) \tag{20}$$

A simple solution to get the sliding condition when the dynamic parameters have uncertainty is the switching control law:

$$\boldsymbol{U_{dis}} = \widehat{\boldsymbol{U}} - \boldsymbol{K}(\vec{\boldsymbol{x}}, \boldsymbol{t}) \cdot \mathbf{sgn}(\boldsymbol{s}) \tag{21}$$

Where the switching function $\mathbf{sgn(S)}$ is defined as [11], [16]

$$\boldsymbol{sgn(s)} = \begin{cases} \mathbf{1} & \boldsymbol{s} > 0 \\ \mathbf{-1} & \boldsymbol{s} < 0 \\ \mathbf{0} & \boldsymbol{s} = \mathbf{0} \end{cases} \tag{22}$$

And the $\boldsymbol{K}(\vec{\boldsymbol{x}}, \boldsymbol{t})$ is the positive constant. Suppose by (11) the following equation can be written as,

$$\begin{aligned} \frac{\mathbf{1}}{\mathbf{2}}\frac{\boldsymbol{d}}{\boldsymbol{dt}}\boldsymbol{s}^2(\boldsymbol{x},\boldsymbol{t}) &= \dot{\mathbf{S}} \cdot \mathbf{S} \\ &= \left[\boldsymbol{f} - \widehat{\boldsymbol{f}} - \boldsymbol{K}\mathbf{sgn}(\boldsymbol{s})\right] \cdot \boldsymbol{S} \\ &= \left(\boldsymbol{f} - \widehat{\boldsymbol{f}}\right) \cdot \boldsymbol{S} - \boldsymbol{K}|\boldsymbol{S}| \end{aligned} \tag{23}$$

And if the Equation (15) instead of (14) the sliding surface can be calculated as:

$$\begin{aligned} \boldsymbol{s(x,t)} &= (\frac{\boldsymbol{d}}{\boldsymbol{dt}} + \boldsymbol{\lambda})^2 \left(\int_{\mathbf{0}}^{\boldsymbol{t}} \tilde{\boldsymbol{x}}\, \boldsymbol{dt}\right) \\ &= (\dot{\mathbf{x}} - \dot{\mathbf{x}}_{\mathbf{d}}) + \mathbf{2}\boldsymbol{\lambda}(\dot{\mathbf{x}} - \dot{\mathbf{x}}_{\mathbf{d}}) \\ &\quad - \boldsymbol{\lambda}^2(\mathbf{x} - \mathbf{x}_{\mathbf{d}}) \end{aligned} \tag{24}$$

In this method the approximation of $\boldsymbol{U}$ is computed as [26].

$$\widehat{\boldsymbol{U}} = -\widehat{\boldsymbol{f}} + \ddot{\boldsymbol{x}}_{\boldsymbol{d}} - \mathbf{2}\boldsymbol{\lambda}(\dot{\mathbf{x}} - \dot{\mathbf{x}}_{\mathbf{d}}) + \boldsymbol{\lambda}^2(\mathbf{x} - \mathbf{x}_{\mathbf{d}}) \tag{25}$$

3. METHODOLOGY: DESIGN A NOVEL MIMO FUZZY BACKSTEPPING ADAPTIVE FUZZY ESTIMATION VARIABLE STRUCTURE CONTROL

First part is focused on eliminate the oscillation (chattering) in pure variable structure controller based on linear boundary layer method. To reduce or eliminate the chattering it is used the boundary layer method; in boundary layer method the basic idea is replace the discontinuous method by saturation (linear) method with small neighborhood of the switching surface. This replace is caused to increasethe error performance [20]-[24], [63]-[74].

$$\boldsymbol{B(t)} = \{\boldsymbol{x}, |\boldsymbol{S(t)}| \le \emptyset\}; \emptyset > 0 \tag{26}$$

Where $\emptyset$ is the boundary layer thickness.Therefore, to have a smote control law, the saturation function $\boldsymbol{Sat}(\boldsymbol{S}/_{\emptyset})$ added to the control law:

$$\boldsymbol{U} = \boldsymbol{K}(\vec{x}, \boldsymbol{t}).\boldsymbol{Sat}\left(\boldsymbol{S}/_{\emptyset}\right) \tag{27}$$

Where $\boldsymbol{Sat}\left(\boldsymbol{S}/_{\emptyset}\right)$can be defined as:

$$\boldsymbol{sat}\left(\boldsymbol{S}/_{\emptyset}\right) = \begin{cases} \mathbf{1} & (\boldsymbol{S}/_{\emptyset} > 1) \\ -\mathbf{1} & \left(\boldsymbol{S}/_{\emptyset} < 1\right) \\ \boldsymbol{S}/_{\emptyset} & (-\mathbf{1} < \boldsymbol{S}/_{\emptyset} < 1) \end{cases} \tag{28}$$

Based on above discussion, the control law for an engine is written as [18]-[24]:

$$\boldsymbol{U} = \boldsymbol{U_{eq}} + \boldsymbol{U_r} \tag{29}$$

Second step is focused on design SISO fuzzy estimation backstepping adaptive variable structure based on Lyapunov formulation. The firs type of fuzzy systems is given by:

$$\boldsymbol{f(x)} = \sum_{\boldsymbol{l=1}}^{\boldsymbol{M}} \boldsymbol{\theta^l \mathcal{E}^l(x)} = \boldsymbol{\theta^T \mathcal{E}(x)} \tag{30}$$

Where

$\theta = (\theta^1, \dots, \theta^M)^T, \mathcal{E}(x) = (\mathcal{E}^1(x), \dots, \mathcal{E}^M(x))^T, and\ \mathcal{E}^l(x) =: \prod_{i=1}^{n} \frac{\mu_{A_i^l}(x_i)}{\sum_{l=1}^{M}(\prod_{i=1}^{n}\mu_{A_i^l}(x_i))} . \theta^1, \dots, \theta^M$

are adjustable parameters in (23) .$\mu_{A_1^1}(x_1), \dots, \mu_{A_n^m}(x_n)$are given membership functions whose parameters will not change over time.

The second type of fuzzy systems is given by:

$$\boldsymbol{f(x)} = \frac{\sum_{\boldsymbol{l=1}}^{\boldsymbol{M}} \boldsymbol{\theta^l}\left[\prod_{\boldsymbol{i=1}}^{\boldsymbol{n}} \mathbf{exp}\left(-\left(\frac{x_i-\alpha_i^l}{\delta_i^l}\right)^2\right)\right]}{\sum_{\boldsymbol{l=1}}^{\boldsymbol{M}}\left[\prod_{\boldsymbol{i=1}}^{\boldsymbol{n}} \mathbf{exp}\left(-\left(\frac{x_i-\alpha_i^l}{\delta_i^l}\right)^2\right)\right]} \tag{31}$$

Where $\theta^l, \alpha_i^l\ and\ \delta_i^l$ are all adjustable parameters.From the universal approximation theorem, we know that we can find a fuzzy system to estimate any continuous function. For the first type of fuzzy systems, we can only adjust θ^l in (23). We define $f^{\wedge}(x|\theta)$ as the approximator of the real function$f(x)$.

$$\boldsymbol{f^{\wedge}(x|\theta)} = \boldsymbol{\theta^T \varepsilon(x)} \tag{32}$$

We define θ^* as the values for the minimum error:

$$\theta^{*} = \arg\min_{\theta\in\Omega}\left[\sup_{x\in U} | f^{\wedge}(x|\theta) - g(x)|\right] \quad (33)$$

Where Ω is a constraint set for θ. For specific x , $sup_{x\in U}|f^{\wedge}(x|\theta^{*}) - f(x)|$ is the minimum approximation error we can get.

We used the first type of fuzzy systems (23) to estimate the nonlinear system (11) the fuzzy formulation can be write as below;

$$\begin{aligned} f(x|\theta) &= \theta^{T}\varepsilon(x) \\ &= \frac{\sum_{l=1}^{n}\theta^{l}\left[\mu_{A^{l}}(x)\right]}{\sum_{l=1}^{n}\left[\mu_{A^{l}}(x)\right]} \end{aligned} \quad (34)$$

Where $\theta^{1}, \dots, \theta^{n}$ are adjusted by an adaptation law. The adaptation law is designed to minimize the parameter errors of $\theta - \theta^{*}$. A MIMO (multi-input multi-output) fuzzy system is designed to compensate the uncertainties of the nonlinear system. The parameters of the fuzzy system are adjusted by adaptation laws. The tracking error and the sliding surface state are defined as:

$$e = q - q_{d} \quad (35)$$

$$s = \dot{e} + \lambda_{e} \quad (36)$$

We define the reference state as:

$$\dot{q}_{r} = \dot{q} - s = \dot{q}_{d} - \lambda e \quad (37)$$

$$\ddot{q}_{r} = \ddot{q} - \dot{s} = \ddot{q}_{d} - \lambda\dot{e} \quad (38)$$

The general MIMO if-then rules are given by:

$$\begin{aligned} &R^{l}: if\ x_{1}\ is\ A_{1}^{l}\ , x_{2}\ is\ A_{2}^{l}\ , \dots, x_{n}\ is\ A_{n}^{l}\ , \\ &then\ y_{1} is\ B_{1}^{l}\ , \dots, y_{m}\ is\ B_{m}^{l} \end{aligned} \quad (39)$$

Where $l = 1,2, \dots, M$ are fuzzy if-then rules; $x = (x_{1}, \dots, x_{n})^{T}$ and $y = (y_{1}, \dots, y_{n})^{T}$ are the input and output vectors of the fuzzy system. The MIMO fuzzy system is define as:

$$f(x) = \ominus^{T}\ \varepsilon(x) \quad (40)$$

Where:

$$\ominus^{T} = (\theta_{1}, \dots, \theta_{m})^{T} = \begin{bmatrix} \theta_{1}^{1}, \theta_{1}^{2}, \dots, \theta_{1}^{M} \\ \theta_{2}^{1}, \theta_{2}^{2}, \dots, \theta_{2}^{M} \\ \vdots \\ \theta_{m}^{1}, \theta_{m}^{2}, \dots, \theta_{m}^{M} \end{bmatrix} \quad (41)$$

$\varepsilon(x) = (\varepsilon^{1}(x), \dots, \varepsilon^{M}(x))^{T}$, $\varepsilon^{1}(x) = \prod_{i=1}^{n}\mu_{A_{i}^{l}}(x_{i}) / \sum_{l=1}^{M}(\prod_{i=1}^{n}\mu_{A_{i}^{l}}(x_{i}))$, and$\mu_{A_{i}^{l}}(x_{i})$ is defined in (32). To reduce the number of fuzzy rules, we divide the fuzzy system in to three parts:

$$\begin{aligned} F^{1}(q,\dot{q}) &= \ominus^{1^{T}}\ \varepsilon\,(q,\dot{q}) \\ &= \left[\theta_{1}^{1^{T}}\varepsilon\,(q,\dot{q})\,, \dots, \theta_{m}^{1^{T}}\varepsilon\,(q,\dot{q})\right]^{T} \end{aligned} \quad (42)$$

$$\begin{aligned} F^{2}(q,\ddot{q}_{r}) &= \ominus^{2^{T}}\ \varepsilon\,(q,\ddot{q}_{r}) \\ &= \left[\theta_{1}^{2^{T}}\varepsilon\,(q,\ddot{q}_{r})\,, \dots, \theta_{m}^{2^{T}}\varepsilon\,(q,\ddot{q}_{r})\right]^{T} \end{aligned} \quad (43)$$

$$\begin{aligned} F^{3}(q,\ddot{q}) &= \ominus^{3^{T}}\ \varepsilon\,(q,\ddot{q}) \\ &= \left[\theta_{1}^{3^{T}}\varepsilon\,(q,\dot{q})\,, \dots, \theta_{m}^{3^{T}}\varepsilon\,(q,\ddot{q})\right]^{T} \end{aligned} \quad (44)$$

The control input is given by:

$$\boldsymbol{\tau} = \boldsymbol{M}^{\wedge}\ddot{\boldsymbol{q}}_r + \boldsymbol{P_m}(\boldsymbol{\theta}) + \boldsymbol{P_{net}}(\boldsymbol{\theta}) + \boldsymbol{F}^1(\boldsymbol{q},\dot{\boldsymbol{q}}) + \boldsymbol{F}^2(\boldsymbol{q},\ddot{\boldsymbol{q}}_r) + \boldsymbol{F}^3(\boldsymbol{q},\ddot{\boldsymbol{q}}) - \boldsymbol{K_D s} - \boldsymbol{W sgn}(\boldsymbol{s}) \tag{45}$$

Where $M^{\wedge}$, $\boldsymbol{P_m}(\boldsymbol{\theta}) + \boldsymbol{P_{net}}(\boldsymbol{\theta})$ are the estimations of $M(q)$ and and are positive constants; $W = diag\ [W_1, \dots, W_m]$ and $W_1, \dots, W_m$ are positive constants. The adaptation law is given by:

$$\begin{aligned}\dot{\boldsymbol{\theta}}_j^1 &= -\boldsymbol{\Gamma}_{1j}\boldsymbol{s}_j\boldsymbol{\varepsilon}(\boldsymbol{q},\dot{\boldsymbol{q}})\\ \dot{\boldsymbol{\theta}}_j^2 &= -\boldsymbol{\Gamma}_{2j}\boldsymbol{s}_j\boldsymbol{\varepsilon}(\boldsymbol{q},\ddot{\boldsymbol{q}}_r)\\ \dot{\boldsymbol{\theta}}_j^3 &= -\boldsymbol{\Gamma}_{3j}\boldsymbol{s}_j\boldsymbol{\varepsilon}(\boldsymbol{q},\ddot{\boldsymbol{q}})\end{aligned} \tag{46}$$

Where $j = 1, \dots, m$ and $\Gamma_{1j} - \Gamma_{3j}$ are positive diagonal matrices.
The Lyapunov function candidate is presented as:

$$\boldsymbol{V} = \frac{1}{2}\boldsymbol{s}^T\boldsymbol{M s} + \frac{1}{2}\sum_{j=1}^{m}\frac{1}{\boldsymbol{\Gamma}_{1j}}\emptyset_j^{1^T}\emptyset_j^1 + \frac{1}{2}\sum_{j=1}^{m}\frac{1}{\boldsymbol{\Gamma}_{2j}}\emptyset_j^{2^T}\emptyset_j^2 + \frac{1}{2}\sum_{j=1}^{m}\frac{1}{\boldsymbol{\Gamma}_{3j}}\emptyset_j^{13^T}\emptyset_j^3 \tag{47}$$

Where $\emptyset_j^1 = \emptyset_j^{1^*} - \emptyset_j^1, \emptyset_j^2 = \emptyset_j^{2^*} - \emptyset_j^2$ and $\emptyset_j^3 = \emptyset_j^{3^*} - \emptyset_j^3$ we define:

$$\boldsymbol{F}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}}_r,\ddot{\boldsymbol{q}}) = \boldsymbol{F}^1(\boldsymbol{q},\dot{\boldsymbol{q}}) + \boldsymbol{F}^2(\boldsymbol{q},\ddot{\boldsymbol{q}}_r) + \boldsymbol{F}^3(\boldsymbol{q},\ddot{\boldsymbol{q}}) \tag{48}$$

From (26) and (27), we get:

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{P_m}(\boldsymbol{\theta}) + \boldsymbol{P_{net}}(\boldsymbol{\theta}) = \boldsymbol{M}^{\wedge}\ddot{\boldsymbol{q}}_r + \boldsymbol{P_m}(\boldsymbol{\theta})\widehat{+\boldsymbol{P}}_{\boldsymbol{net}}(\boldsymbol{\theta}) + \boldsymbol{F}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}}_r,\ddot{\boldsymbol{q}}) - \boldsymbol{K_D s} - \boldsymbol{W sgn}(\boldsymbol{s}) \tag{49}$$

Since $\dot{q}_r = \dot{q} - s$ and $\ddot{q}_r = \ddot{q} - \dot{s}$, we get:

$$\boldsymbol{M\dot{s}} + (\boldsymbol{P_m}(\boldsymbol{\theta}) + \boldsymbol{P_{net}}(\boldsymbol{\theta}) + \boldsymbol{K_D})\boldsymbol{s} + \boldsymbol{W sgn}(\boldsymbol{s}) = -\Delta\boldsymbol{F} + \boldsymbol{F}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}}_r,\ddot{\boldsymbol{q}}) \tag{50}$$

Then $M\dot{s} + \boldsymbol{P_m}(\boldsymbol{\theta}) + \boldsymbol{P_{net}}(\boldsymbol{\theta})s$ can be written as:

$$\boldsymbol{M\dot{s}} + \boldsymbol{P_m}(\boldsymbol{\theta}) + \boldsymbol{P_{net}}(\boldsymbol{\theta})\boldsymbol{s} = -\Delta\boldsymbol{F} + \boldsymbol{F}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}}_r,\ddot{\boldsymbol{q}}) - \boldsymbol{K_D s} - \boldsymbol{W sgn}(\boldsymbol{s}) \tag{51}$$

Where,
$\Delta F = \widetilde{M}\ddot{q}_r + \boldsymbol{P_m}(\boldsymbol{\theta}) + \boldsymbol{P_{net}}(\boldsymbol{\theta})$, $\widetilde{M} = M - M^{\wedge}$, $\tilde{C}_1 = \boldsymbol{P_m}(\boldsymbol{\theta}) + \boldsymbol{P_{net}}(\boldsymbol{\theta}) - \boldsymbol{P_m}(\boldsymbol{\theta})\widetilde{+\boldsymbol{P}}_{\boldsymbol{net}}(\boldsymbol{\theta})$The derivative of V is:

$$\dot{\boldsymbol{V}} = \boldsymbol{s}^T\boldsymbol{M\dot{s}} + \frac{1}{2}\boldsymbol{s}^T\dot{\boldsymbol{M}}\boldsymbol{s} + \sum_{j=1}^{m}\frac{1}{\boldsymbol{\Gamma}_{1j}}\emptyset_j^{1^T}\dot{\emptyset}_j^1 + \sum_{j=1}^{m}\frac{1}{\boldsymbol{\Gamma}_{2j}}\emptyset_j^{2^T}\dot{\emptyset}_j^2 + \sum_{j=1}^{m}\frac{1}{\boldsymbol{\Gamma}_{3j}}\emptyset_j^{13^T}\dot{\emptyset}_j^3 \tag{52}$$

We know that $s^T M\dot{s} + \frac{1}{2}s^T\dot{M}s = s^T(M\dot{s} + \boldsymbol{P_m}(\boldsymbol{\theta}) + \boldsymbol{P_{net}}(\boldsymbol{\theta})s)$ from (50). Then:

$$\dot{V} = -s^T[-K_D s + Wsgn(s) + \Delta F - F(q,\dot{q},\ddot{q}_r,\ddot{q})] + \sum_{j=1}^{m} \frac{1}{\Gamma_{1j}} \phi_j^{1^T} \dot{\phi}_j^1 + \sum_{j=1}^{m} \frac{1}{\Gamma_{2j}} \phi_j^{2^T} \dot{\phi}_j^2 + \sum_{j=1}^{m} \frac{1}{\Gamma_{3j}} \phi_j^{13^T} \dot{\phi}_j^3 \tag{53}$$

We define the minimum approximation error as:

$$\omega = \Delta F - \left[F^1\left(q,\dot{q} \mid \ominus^{1^*}\right) + F^2\left(q,\ddot{q}_r \mid \ominus^{2^*}\right) + F^3\left(q,\ddot{q} \mid \ominus^{3^*}\right)\right] \tag{54}$$

We plug (47) into (48).

$$\begin{aligned}
\dot{V} &= -s^T[-K_D s + Wsgn(s) + \Delta F - F(q,\dot{q},\ddot{q}_r,\ddot{q})] + \sum_{j=1}^{m} \frac{1}{\Gamma_{1j}} \phi_j^{1^T} \dot{\phi}_j^1 + \sum_{j=1}^{m} \frac{1}{\Gamma_{2j}} \phi_j^{2^T} \dot{\phi}_j^2 + \sum_{j=1}^{m} \frac{1}{\Gamma_{3j}} \phi_j^{13^T} \dot{\phi}_j^3 \\
&= -s^T\left[-K_D s + Wsgn(s) + \omega + F^1\left(q,\dot{q} \mid \ominus^{1^*}\right) + F^2\left(q,\ddot{q}_r \mid \ominus^{2^*}\right) + F^3\left(q,\ddot{q} \mid \ominus^{3^*}\right) - F^1(q,\dot{q}) + F^2(q,\ddot{q}_r) + F^3(q,\ddot{q})\right] + \sum_{j=1}^{m} \frac{1}{\Gamma_{1j}} \phi_j^{1^T} \dot{\phi}_j^1 + \sum_{j=1}^{m} \frac{1}{\Gamma_{2j}} \phi_j^{2^T} \dot{\phi}_j^2 + \sum_{j=1}^{m} \frac{1}{\Gamma_{3j}} \phi_j^{13^T} \dot{\phi}_j^3 \\
&= -s^T K_D s - s^T Wsgn(s) - s^T \omega - \sum_{j=1}^{m} s_j \phi_j^{1^T} \varepsilon(q,\dot{q}) - \sum_{j=1}^{m} s_j \phi_j^{2^T} \varepsilon(q,\ddot{q}_r) - \sum_{j=1}^{m} s_j \phi_j^{3^T} \varepsilon(q,\ddot{q}) + \sum_{j=1}^{m} \frac{1}{\Gamma_{1j}} \phi_j^{1^T} \dot{\phi}_j^1 + \sum_{j=1}^{m} \frac{1}{\Gamma_{2j}} \phi_j^{2^T} \dot{\phi}_j^2 + \sum_{j=1}^{m} \frac{1}{\Gamma_{3j}} \phi_j^{3^T} \dot{\phi}_j^3 \\
&= -s^T K_D s - s^T Wsgn(s) - s^T \omega - \sum_{j=1}^{m} \phi_j^{1^T}\left(s_j \varepsilon(q,\dot{q}) - \frac{1}{\Gamma_{1j}} \dot{\phi}_j^1\right) - \sum_{j=1}^{m} \phi_j^{2^T}\left(s_j \varepsilon(q,\ddot{q}_r) - \frac{1}{\Gamma_{2j}} \dot{\phi}_j^2\right) - \sum_{j=1}^{m} \phi_j^{3^T}\left(s_j \varepsilon(q,\ddot{q}_r) - \frac{1}{\Gamma_{3j}} \dot{\phi}_j^3\right) \\
&= -s^T K_D s - s^T Wsgn(s) - s^T \omega - \sum_{j=1}^{m} \phi_j^{1^T}\left(s_j \varepsilon(q,\dot{q}) + \frac{1}{\Gamma_{1j}} \dot{\phi}_j^1\right) - \sum_{j=1}^{m} \phi_j^{2^T}\left(s_j \varepsilon(q,\ddot{q}_r) + \frac{1}{\Gamma_{2j}} \dot{\phi}_j^2\right) - \sum_{j=1}^{m} \phi_j^{3^T}\left(s_j \varepsilon(q,\ddot{q}_r) + \frac{1}{\Gamma_{3j}} \dot{\phi}_j^3\right)
\end{aligned}$$

Then $\dot{V}$ becomes:

$$\begin{aligned}
\dot{V} &= -s^T K_D s - s^T Wsgn(s) - s^T \omega \\
&= -\sum_{j=1}^{m} \left(s_j^2 K_{Dj} + W_j |s_j| + s_j \omega_j\right) \\
&= -\textstyle\sum_{j=1}^{m} \left[s_j\left(s_j K_{Dj} + \omega_j\right) + W_j |s_j| \right]
\end{aligned} \tag{55}$$

Since ω_j can be as small as possible, we can find K_{Dj} that $|s_j^2 K_{Dj}| > |\omega_j|(s_j \neq 0)$. Therefore, we can get $s_j(s_j K_{Dj} + \omega_j) > 0$ for $s_j \neq 0$ and $\dot{V} < 0$ $(s \neq 0)$.

Third step is focused on design Mamdani's fuzzy [30]-[40] backstepping adaptive fuzzy estimator variable structure. As mentioned above pure variable structure controller has nonlinear dynamic equivalent limitations in presence of uncertainty and external disturbances in order to solve these challenges this work applied Mamdani's fuzzy inference engine estimator in variable structure controller. However proposed MIMO fuzzy estimator variable structure has satisfactory performance but calculate the variable structure surface slope by try and error or experience knowledge is very difficult, particularly when system has structure or unstructured uncertainties; SISO Mamdani's fuzzy backstepping variable structure function fuzzy estimator variable structure controller is recommended. The backstepping method is based on mathematical formulation which this method is introduced new variables into it in form depending on the dynamic equation of robot arm. This method is used as feedback linearization in order to solve nonlinearities in the system. To use of nonlinear fuzzy filter this method in this research makes it possible to create dynamic nonlinear backstepping estimator into the adaptive fuzzy estimator variable structure process to eliminate or reduce the challenge of uncertainty in this part. The backstepping controller is calculated by;

$$\boldsymbol{U_{B.S}} = \boldsymbol{U_{eq_{B.S}}} + \boldsymbol{M.I} \tag{56}$$

Where $\boldsymbol{U_{B.S}}$is backstepping output function, $\boldsymbol{U_{eq_{B.S}}}$is backstepping nonlinear equivalent function which can be written as (51) and $\boldsymbol{I}$ is backstepping control law which calculated by (52).

$$\boldsymbol{U_{eq_{B.S}}} = \left[\left(\boldsymbol{P_m(\theta)} + \boldsymbol{P_{net}(\theta)}\right)\right] \tag{57}$$

$$\boldsymbol{I} = \left[\boldsymbol{\ddot{\theta}} + \boldsymbol{K_1(K_1 - 1)\cdot e} + \boldsymbol{(K_1 + K_2)\cdot \dot{e}}\right] \tag{58}$$

Based on (11) and (51) the fuzzy backstepping filter is considered as:

$$\left(\boldsymbol{P_m(\theta)} + \boldsymbol{P_{net}(\theta)}\right) = \sum_{l=1}^{M} \boldsymbol{\theta^T\, \zeta(x)} - \boldsymbol{\lambda S} - \boldsymbol{K} \tag{59}$$

Based on (50) the formulation of fuzzy backstepping filter can be written as;

$$\boldsymbol{U} = \boldsymbol{U_{eqB.S_{fuzzy}}} + \boldsymbol{MI} \tag{60}$$

Where $\boldsymbol{U_{eq_{B.Sfuzzy}}} = \left[\left(\boldsymbol{P_m(\theta)} + \boldsymbol{P_{net}(\theta)}\right)\right] + \sum_{l=1}^{M} \boldsymbol{\theta^T\, \zeta(x)} + \boldsymbol{K}$

The adaption low is defined as:

$$\boldsymbol{\dot{\theta}_j} = \boldsymbol{\gamma_{sj} S_j \zeta_j(S_j)} \tag{61}$$

Where the $\gamma_{sj}$is the positive constant and $\zeta_j(S_j) = [\zeta_j^1(S_j), \zeta_j^2(S_j), \zeta_j^3(S_j), \ldots\ldots, \zeta_j^M(S_j)]^T$

$$\boldsymbol{\zeta_j^1(S_j)} = \frac{\boldsymbol{\mu_{(A)}{}_j^l(S_j)}}{\sum_i \boldsymbol{\mu_{(A)}{}_j^l(S_j)}} \tag{62}$$

The dynamic equation of IC engine can be written based on the variable structure surface as;

$$\boldsymbol{M\dot{S}} = -\boldsymbol{VS} + \boldsymbol{M\dot{S}} + \boldsymbol{VS} \tag{63}$$

It is supposed that:

$$S^T(\dot{M} - 2V)S = 0 \tag{64}$$

The derivation of Lyapunovfunction ($\dot{V}$) is written as:

$$\dot{V} = \frac{1}{2}S^T\dot{M}S - S^TVS + \sum \frac{1}{\gamma_{sj}}\phi_j^T\dot{\phi}_J$$
$$= S^T(-\lambda S + \Delta f - K) + \sum \frac{1}{\gamma_{sj}}\phi_j^T\dot{\phi}_J$$
$$= \sum_{j=1}^{m}[S_j\,(\Delta f_j - K_j)] - S^T\lambda S + \sum \frac{1}{\gamma_{sj}}\phi_j^T\dot{\phi}_J$$
$$= \sum_{j=1}^{m}[S_j\,(\Delta f_j - \theta_j^T\zeta_j(S_j))] - S^T\lambda S + \sum \frac{1}{\gamma_{sj}}\phi_j^T\dot{\phi}_J$$
$$= \sum_{j=1}^{m}[S_j\,(\Delta f_j - (\theta_j^*)^T\zeta_j(S_j) + \phi_j^T\zeta_j(S_j))] - S^T\lambda S + \sum \frac{1}{\gamma_{sj}}\phi_j^T\dot{\phi}_J$$
$$= \sum_{j=1}^{m}[S_j\,(\Delta f_j - \left((\theta_j^*)^T\zeta_j(S_j)\right)] - S^T\lambda S)] + \sum (\frac{1}{\gamma_{sj}}\phi_j^T[\gamma_{sj}\zeta_j(S_j)S_j + \dot{\phi}_J])$$

Where $\dot{\theta}_j = \gamma_{sj}S_j\zeta_j(S_j)$ is adaption law and $\dot{\phi}_J = -\dot{\theta}_j = -\gamma_{sj}S_j\zeta_j(S_j)$, consequently $\dot{V}$ can be considered by:

$$\dot{V} = \sum_{j=1}^{m}[S_j\,\Delta f_j - \left((\theta_j^*)^T\zeta_j(S_j)\right)] - S^T\lambda S \tag{65}$$

The minimum error can be defined by:

$$e_{mj} = \Delta f_j - \left((\theta_j^*)^T\zeta_j(S_j)\right) \tag{66}$$

$\dot{V}$ is intended as follows:

$$\dot{V} = \sum_{j=1}^{m}[S_j\,e_{mj}] - S^T\lambda S$$
$$\le \sum_{j=1}^{m}|S_j||e_{mj}| - S^T\lambda S$$
$$= \sum_{j=1}^{m}|S_j||e_{mj}| - \lambda_j {S_j}^2$$
$$= \sum_{j=1}^{m}|S_j|\left(|e_{mj}| - \lambda_j S_j\right) \tag{67}$$

For continuous function $U_{eqB.S_{fuzzy}}$ and suppose $\varepsilon > 0$ it is defined the fuzzy backstepping controller in form of (59) such that.

$$Sup_{x\in U}\left|U_{eqB.S_{fuzzy}} + MI\right| < \epsilon \tag{68}$$

As a result MIMO fuzzy backstepping adaptive fuzzy estimation variable structure is very stable which it is one of the most important challenges to design a controller with suitable response.

4. RESULTS

Variable structure controller (VSC) and proposed SISO fuzzy backstepping adaptive fuzzy estimator variable structure controller were tested to sinus response trajectory. The simulation was implemented in Matlab/Simulink environment. Links trajectory, disturbance rejection and error are compared in these controllers. It is noted that, these systems are tested by band limited white noise with a predefined 40% of relative to the input signal amplitude. This type of noise is used to external disturbance in continuous and hybrid systems.

Trajectory: Figure 1 shows the FR trajectory in VSC and proposed controller without changes in system dynamics for variable desired fuel ratio in general and zoom scaling.

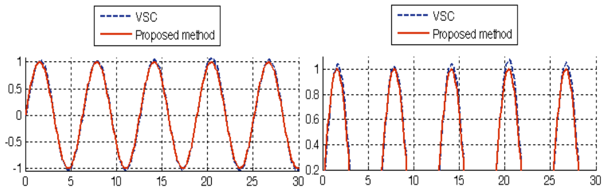

Figure 1. VSC vs. Proposed controller to adjust FR

By comparing sinus response, Figure 1, in SMC and proposed controller, proposed controller 's overshoot (0%) is lower than VSC's (3.8%) which caused to adjust FR ratio in proposed methodology.

Torque Load Rejection: Figure 2 is indicated the power torque load removal in VSC and proposed controller. Besides a band limited white torque lode with predefined of 40% the power of signal is applied to the VSC and proposed controller; it found slight oscillations in VSC trajectory responses.

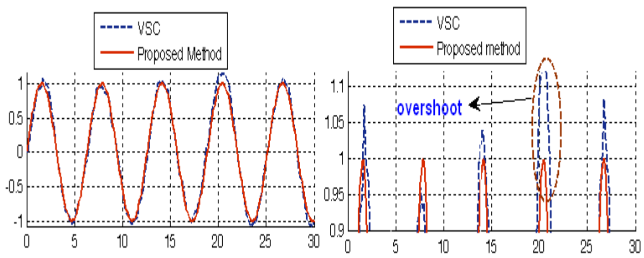

Figure 2. VSC vs. Proposed controller to adjust FR: with variant torque load

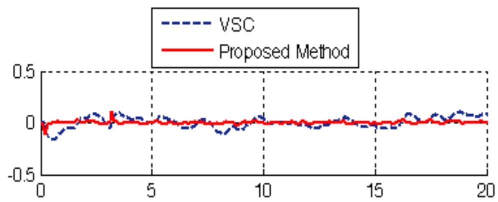

Figure 3. VSC Vs. proposed controller to adjust FR in presence of external disturbance

Among above graph, relating to variable trajectory following with external disturbance, VSC has slightly fluctuations. By comparing overshoot; proposed controller's overshoot **(0%)** is lower than VSC's **(16%).**

Errors in the model: Proposed controller has lower error rate (refer to Table.1), VSC has oscillation tracking which causes chattering phenomenon at the presence of disturbances. Figure 3 is shown steady state and RMS error in VSC and proposed controller in presence of external disturbance.

Table 1. RMS Error Rate of Presented controllers

RMS Error Rate	VSC	Proposed method
Without Noise	1e-3	0.56e-9
With Noise	0.01	0.09e-8

5. CONCLUSION

Refer to the research, a Lyapunov based SISO fuzzy backstepping adaptive fuzzy estimator variable structure controller design and application to automotive engine has proposed in order to design high performance nonlinear controller in the presence of valiant torque load. Regarding to the positive points invariable structure controller, fuzzy inference system and adaptive fuzzy backstepping methodology it is found that the adaptation laws derived in the Lyapunov sense. The stability of the closed-loop system is proved mathematically based on the Lyapunov method. To remove the chattering linear boundary layer method is used. To compensate the model uncertainty, SISO fuzzy inference system is applied to VSC. If we define k_1 membership functions for each input variable, the number of fuzzy rules that applied to each joint is K_1 which it will result in a low computational load. Finally, fuzzy backstepping methodology with minimum rule base is used to online tuning and adjusted the fuzzy variable structure method and eliminates the chattering with minimum computational load. In this case the performance is improved by using the advantages of variable structure, artificial intelligence compensate method and adaptive algorithm while the disadvantages removed by added each method to previous method.

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