

## Curvelet Transform based Retinal Image Analysis

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### ABSTRACT

Edge detection is an important assignment in image processing, as it is used as a primary tool for pattern recognition, image segmentation and scene analysis. An edge detector is a high-pass filter that can be applied for extracting the edge points within an image. Edge detection in the spatial domain is accomplished through convolution with a set of directional derivative masks in this domain. On the other hand, working in the frequency domain has many advantages, starting from introducing an alternative description to the spatial representation and providing more efficient and faster computational schemes with less sensitivity to noise through high filtering, de-noising and compression algorithms. Fourier transforms, wavelet and curvelet transform are among the most widely used frequency-domain edge detection from satellite images. However, the Fourier transform is global and poorly adapted to local singularities. Some of these drawbacks are solved by the wavelet transforms especially for singularities detection and computation. In this paper, the relatively new multi-resolution technique, curvelet transform, is assessed and introduced to overcome the wavelet transform limitation in directionality and scaling. In this research paper, the assessment of second generation curvelet transforms as an edge detection tool will be introduced and compared with first generation curvelet transform.

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## 1. INTRODUCTION

In many important imaging applications, images exhibit edges discontinuities across curves. In traditional photographic imaging, for example, this occurs whenever one object occludes another, causing the luminance to undergo step discontinuities at boundaries. In biological imagery, this occurs whenever two different organs or tissue structures meet. When working with real rather than synthetic data, one of course doesn't 'know' where these edges are; one only has a digitized pixel array, with potential imperfections caused by noise, by blurring, and of course by the unnatural pixelization of the underlying continuous scene. Hence the typical image analyst only has recourse to representations which don't "know" about the existence and geometry of the discontinuities in the image.

Retinal vessel detection is an important step in diagnosing and treatment of many diseases affecting retina. Many diseases such as diabetic retinopathy and hypertension can be detected by retinal vessel map or scanning conjunctival vessels. There are a lot of techniques for vessel extraction from retinal images but most of them have failed to face with some patterns like haemorrhages and micro aneurysms. It was with the advent of multiscale transforms like curvelet transform that tried to solve these problems in vessel detection, accuracy and results.

In view of the fact that analysis of single scale information of an image cannot be so useful and practical in filtering and feature extraction areas, recently many studies and works have been done on multi resolution approaches to detect vessels [1], [2]. These methods try to segment objects using different scales. High processing speed and resistance to noise are the main merits of multi resolution algorithms. These methods detect the main vessel structures in lower resolutions where as the small vessels can be extracted using high resolution information. Wavelet transform is one of the efficient tools for this purpose. In [3] an automatic retinal vessel detection method is presented. This method employs a classifier to group the pixels into two vessels and non-vessels classes based on the directional features that are extracted using multi resolution wavelet transform. Contourlet transform and curvelet transform are the other tools that are used in multi scale methods to detect vessels or enhance retinal images.

Section II describes about the comparison of first generation curvelet transform & second generation curvelet transform used for analysis purpose. Section III is about the performance evaluation. Section IV gives the results and discussion of the work.

## 2. RESEARCH METHOD

### a) Retinal analysis using curvelet transform (Generation 1)

In 1999, an anisotropic geometric wavelet transform, named ridgelet transform, was proposed by Cand'es and Donoho [4], [5]. The ridgelet transform is optimal at representing straightline singularities. Unfortunately, global straight-line singularities are rarely observed in real applications. In order to analyze local line or curve singularities, a natural idea is to consider a partition of the image, and then to apply the ridgelet transform to the obtained sub-images. This block ridgelet based transform, which is named curvelet transform, was first proposed by Cand'es and Donoho, see [4]. Apart from the blocking effects, however, the application of this so-called first-generation curvelet transform is limited because the geometry of ridgelets is itself unclear, as they are not true ridge functions in digital images.

The idea is to first decompose the image into a set of wavelet bands, and to analyze each band by a local ridgelet transform as illustrated on Figure 9. The block size can be changed at each scale level. Roughly speaking, different levels of the multiscale ridgelet pyramid are used to represent different sub-bands of a filter bank output. At the same time, this sub-band decomposition imposes a relationship between the width and length of the important frame elements so that they are anisotropic and obey approximately the parabolic scaling law  $width = length^2$ .

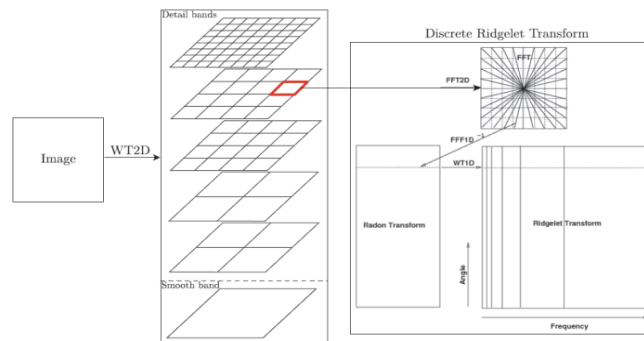


Figure 1. First Generation Discrete Curvelet Transform Flowchart

The figure illustrates the decomposition of the original image into sub-bands followed by the spatial partitioning of each sub-band. The ridgelet transform is then applied to each block. The M-term approximations in the CurveletG1 are almost rate optimal, much better than M-term Fourier or wavelet approximations for such images.

### b) Retinal analysis using curvelet transform (Generation 2)

Retinal and conjunctival vessels are linear dark objects with different width. Therefore, a filter with the constant scale cannot extract all vessels properly. Multi resolution approaches are one way to solve this problem. Wide vessels appear in coarser resolution scales and thinner ones can be extracted from finer resolution scales. Another merit of multi resolution methods is less computation time in comparison.

Candes and Donoho [6] in 1999 developed a new multiscale transform which they called the *curvelet transform*. This is the First generation curvelet transform. Motivated by the needs of image analysis,

it was nevertheless first proposed in the context of objects  $f(x_1, x_2)$  defined on the continuum plane  $(x_1, x_2) \in \mathbf{R}^2$ . The transform was designed to represent edges and other singularities along curves much more efficiently than traditional transforms, i.e. using many fewer coefficients for a given accuracy of reconstruction. Roughly speaking, to represent an edge to squared error  $1/N$  requires  $1/N$  wavelets and only about  $1/\sqrt{N}$  curvelets. The curvelet transform, like the wavelet transform, is a multiscale transform, with frame elements indexed by scale and location parameters. The transform that is a two-dimensional anisotropic extension of wavelet, originally designed to represent edges and other singularities along curves much more efficiently than traditional wavelet transforms. Although curvelets is an extension of wavelets but there exists a correspondence between curvelet and wavelet subbands. The general rule that represents correspondence between curvelet subband ( $C_s$ ) and wavelet subband ( $W_s$ ) is:

$$C_s \leftrightarrow W_s \in \{2 * C_s, 2 * C_s + 1\} \quad (1)$$

Unlike the wavelet transform, it has directional parameters, and the curvelet pyramid contains elements with a very high degree of directional specificity. In addition, the curvelet transform is based on a certain *anisotropic scaling* principle which is quite different from the *isotropic scaling* of wavelets.

The elements obey a special scaling law, where the length of the support of a frame elements and the width of the support are linked by the relation  $width \approx length$ . Moreover, frame elements in curvelets indexed by scale, location and orientation parameters in contrast to wavelets where elements have only scale and location parameters.

Discrete Curvelet Transform is obtained by dividing the Fourier space into concentric circles and further dividing them into wedges. As these radial wedges capture the structural activity in the frequency domain, high anisotropy and directional sensitivity are the inherent characteristics of the curvelet transform. The discrete curvelet transform is taken on a 2-D Cartesian grid  $f[m, n]$ .

$$c_{j,l,k} = \langle f, \varphi_{j,l,k}^D \rangle = \int_{\mathbf{R}^2} \hat{f}(\nu) \hat{\varphi}_j^D(S_{\theta}^{-1}\nu) e^{iS_{\theta}^T m\nu} d\nu \quad (2)$$

The Fast Discrete Curvelet Transform (FDCT) can be implemented via either the Unequispaced Fast Fourier Transform (USFFT) or the Wrapping method. The wrapping is done by periodic tiling of the spectrum using the wedge and then collecting the rectangular coefficient area in the centre. The Wrapping method is faster and easier to implement than the USFFT method, while producing the same results.

Since the curvelet transform is well adapted to represent the images containing edges, it is a good candidate for edge enhancement. Curvelet coefficients can be modified to enhance the edges in an image, which improves the image contrast. A nonlinear function is used to modify the representation coefficients in such a way that details of the small amplitude are enlarged at the expense of the larger ones and perform this uniformly over all scales. Definition of the function parameters based on some statistical features of curvelet coefficients of the input image is very beneficial to adapt the function better with every input image. Therefore, there is a need for a nonlinear function, such as  $y$ , to multiply against the transform coefficients. The function is defined as follows:

$$y(x) = \begin{cases} k_1 \left(\frac{m}{c}\right)^p, & \text{if } |x| < ac \\ k_2 \left(\frac{m}{|x|}\right)^p, & \text{if } ac \leq |x| < m \\ k_3, & \text{if } |x| \geq m \end{cases} \quad (3)$$

Where  $x$  is the curvelet coefficient,  $0 < p < 1$  determines the degree of nonlinearity.  $k_1, k_2$  and  $k_3$  are assigned weights to each function part allow to control the modification of coefficients with a higher severity and makes the modification more appropriate. The adjustment parameter  $a$  makes it possible to determine and regulate the coefficients modification interval. Parameters  $c$  and  $m$  are involved in determining the coefficients modification interval as well as the amplitude of corresponding multiplying  $y$ . These parameters are defined according to two statistical features of coefficients. The first one is the noise standard deviation, with the aim of preventing the noise amplification, and the second one is the maximum value of coefficients in each band. Choose  $c = \sigma_{ji}$ , where  $\sigma_{ji}$  is the noise standard deviation of coefficients being in the same direction and same scale. The parameter  $m$  can be derived from maximum curvelet

coefficients of the relative band and can be determined with regard to  $\sigma_{ji}$  ( $m = k \sigma_{ji}$ ) where  $k$  is an additional and independent parameter from the curvelet coefficient values, and therefore, much easier for a user to set.

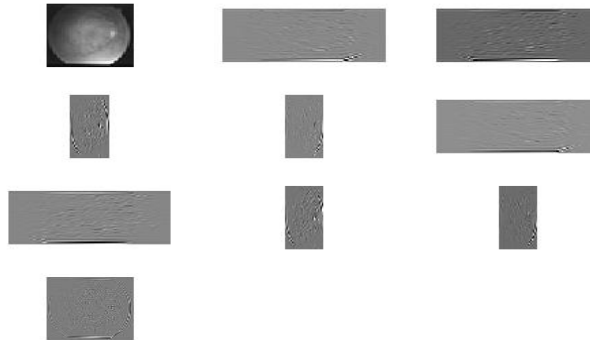


Figure 2. Retinal image and its curvelet coefficients (Generation 2)

### 3. RESULTS AND ANALYSIS

#### 3.1. Performance Evaluation

For quantitative analysis of performance, well-accepted metrics such as accuracy, sensitivity and specificity are used, which are given in Equations (4), (5) and (6). These require representing the error rate using the terms true and false positive (TP & FP) and true and false negative (TN & FN). Accuracy reflects the overall correctness of the classifier.

$$\text{Accuracy} = \frac{TP+FN}{TP+TN+FP+FN} \quad (4)$$

Sensitivity measures the accuracy among positive instances.

$$\text{Sensitivity} = \frac{TP}{TP+FN} \quad (5)$$

Specificity measures the accuracy among negative instances.

$$\text{Specificity} = \frac{TN}{TN+FP} \quad (6)$$

#### 3.2. Database Used

The methodology in this work was tested using the publicly available DRIVE database in which colored retinal images and their corresponding manual segmentations are available.

The DRIVE database [13] contains 40 color images of the retina, with 565 x 584 pixels and 8 bits per color channel, represented in LZW compressed TIFF format. These images were originally captured from Canon CR5 nonmydiatic 3 charge-coupled-device (CCD) cameras at 45° field of view (FOV), and were initially saved in JPEG format. Besides the color images, the database includes masks with the delimitation of a FOV of approximately 540 pixels in diameter for each image, and binary images with the results of manual segmentation. The 40 images were divided into a training set and a testing set. For the images of the testing set, a second independent manual segmentation exists as well.

#### 3.3. Simulation results and Discussion

We believe that the multistructure elements morphology results are suitable enough to find thin and small vessels that may be missed in our final results. The deficiency of missing some thin vessels is because of our utilizing a simple Thresholding method. There is a trade-off between removing more false edges and preserving more pixels of small vessels. The quantitative performance results of both segmentation and enhancement steps show that method effectively detects the blood vessels with accuracy of above 94% in less than 1 min. However, there is a need for a proper Thresholding algorithm to find the small vessels, while avoiding false-edge pixels detection. Also, in retinal images containing severe lesions, the algorithm needs to benefit from a higher level Thresholding method or a more proper scheme. Hence, by replacing the simple

threshold method with a more proper approach to increase the accuracy of the method and deal with the problem of presence of severe lesions in retinal images.

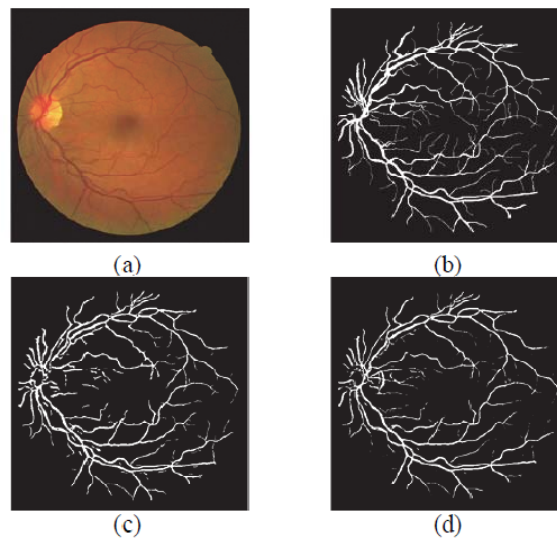


Figure 3. Vessel detection results: a) main image b) curvelet First generation based method c) curvelet second generation based method and d) proposed method in [2]

Table 1. Results of Various Vessel Detection Algorithm

Vessel detection method	Accuracy
Curvelet second generation based method	0.9840
Curvelet first generation based method	0.9411
Chaudhuri et al. [7]	0.8850
Li and Chutatape [8]	0.8939

#### 4. CONCLUSION

A new approach of Curvelet based Retinal Image analysis is proposed in this paper. The generated coefficients are compared for the first generation and second generation Curvelet Transformand with other state of the art methods. The quantitative performance results of both segmentation and enhancement steps show that method effectively detects the blood vessels with accuracy of above 94% in less than 1 min.

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#### REFERENCES

- [1] M Do, M Vetterli. The contourlet transform: An efficient directional multiresolution image representation. *IEEE Trans. Image Processing*. 2005; 14(12): 2091–2106.
- [2] DJ Cornforth, HJ Jelinek, JIG Leandro, JVB Soares, RM Cesar Jr, MJ Cree, P Mitchell, T Bossomaier. *Development of retinal blood vessel segmentation methodology using wavelet transforms for assessment of diabetic retinopathy*. Complexity International. 2005
- [3] R Ghaderi, H Hassanpour, M Shahiri. *Retinal vessel segmentation using the 2-D Morlet wavelet and neural network*. International Conference on Intelligent and Advanced Systems, ICIAS. 2007; 1251-1255.
- [4] E Candes, L Demanet, D Donoho, L Ying. Fast Discrete Curvelet Transforms. *Multiscale Model Simul.*, 2006; 5(3).
- [5] EJ Candes, DL Donoho. Ridgelets: A key to higher-dimensional intermittency?. *Phil. Trans. R. Soc. Lond. A.*, 1999; 2495 -2509.
- [6] DL Donoho, MR Duncan. *Digital curvelet transform: Strategy, implementation and experiments*. Proc. SPIE. 2000; 4056: 12 -29.

- [7] S Chaudhuri, S Chatterjee, N Katz, M Nelson, M Goldbaum. Detection of blood vessels in retinal images using two-dimensional matched filters. *IEEE Transactions on Medical Imaging*. 1989; 8: 263-269.
- [8] T Yedidya, R Hartley. Tracking of Blood Vessels in Retinal Images Using Kalman Filter. *Computing: Techniques and Applications*. 2008: 52-58.
- [9] H Li, O Chutatape. *Fundus image feature extraction*. Proceeding of the 22nd IEEE Conference on Engineering in Medicine and Biology Society. 2000; 1: 3071-3073.
- [10] Joao VB Soares, Jorge JG Leandro, Roberto M Cesar. Retinal vessel Segmentation Using the 2-D Gabor Wavelet and Supervised classification. *IEEE Trans. on Medical Imaging*. 2006; 25(9): 1214-1222.
- [11] Y Yorozu, M Hirano, K Oka, Y Tagawa. Electron spectroscopy studies on magneto-optical media and plastic substrate interface. *IEEE Transl. J. Magn. Japan*. 1987; 2: 740-741.
- [12] R Pourreza, T Banaee, H Pourreza, RD Kakhki. A Radon Transform Based Approach for Extraction of Blood Vessels in Conjunctival Images. *Springer*. 2008; 948-956.
- [13] R Derakhshani, A Ross, S Crihalmeanu. A new biometric modality based on conjunctival vasculature. *Artificial Neural Networks in Engineering (ANNIE)*, St. Louis, USA. 2006.
- [14] AL Cunha, J Zhou, MN Do. *The nonsubsampling contourlet transform: theory, design and applications*. IEEE Trans. Image Proc., submitted, 2005
- [15] [http://www.isi.uu.nl/Research/ Database/DRIVE](http://www.isi.uu.nl/Research/Database/DRIVE).
- [16] CG Owen, RSB Newsom, AR Rudnicka, TJ Ellis, EG Woodward. Vascular Response of the Bulbar Conjunctiva to Diabetes and Elevated Blood Pressure. *Ophthalmology*. 2005; 112: 1801-1808.
- [17] Zhong Hua, Jiao Li, Cheng HOU Peng. retinal vessel segmentation using nonsubsampling Contourlet Transform. *Chinese Journal of Computers*. 2011; 3: 574-582.
- [18] Z Yin Hai, et al. A Novel SVPWM Modulation Scheme. *Applied Power Electronics Conference and Exposition, 2009 Twenty-Fourth Annual IEEE*. 2009; 128-131.

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