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The Kalman Filter Performance for Dynamic Change in System Parameters

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Article Info	ABSTRACT
	This paper studies the performance of the simple kalman filter for dynamic changes of the system variables. The kalman filter was used to detect the fundamental component of power system signal contained harmonics. First the performance of the kalman filter was examined for a sudden change in the amplitude of the fundamental component, then for a frequency variation. The noise covariance matrices were assumed to be unknown, the values of these matrices were changed to study the kalman filter behavior in such circumstances.

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1.   INTRODUCTION

The kalman filter is an optimal filter under several conditions, one of them is related to the noise nature, where the noise should be independent white additive Gaussian noise with known covariance matrices; the measurement noise covariance (R) and the state noise covariance matrix (Q), for that reason several researches focus on on-line and off-line evaluating of the noise covariance matrices, where they need a lot of computation time and computation memory.

The effect of the covariance matrices were studied in several previous researches [1-9], There is strong argument about the effect of the noise covariance matrices on the kalman filter performance, some of researches concluded that it is necessary to estimate the real values of these matrices to guarantee that the kalman filter located the optimal solution, otherwise the kalman filter will converges to suboptimal solution. Some researchers concluded that the real value of these matrices are not important as the relative ratio between them [3, 4], while other researchers concluded that the knowledge of the ratio between the noise covariance matrices is not sufficient when the signal to noise ratio exceed 30dB [1], other researchers concluded the importance of resetting these matrices when a dynamical change occurs [6].

Since it is hard to estimate the noise covariance matrices in many of practical applications or it may required extra calculations and increase the system complexity, in this paper the covariance matrices were assumed to be unknown, and the value of these matrices are changed to several values (included the actual value) to investigate the kalman filter performance, then the kalman filter will be used to detect the amplitude of the fundamental signal to diagnose the voltage sag problem of a measured signal contains harmonics and measurement noise under a dynamic change in the fundamental signal amplitude and its frequency.

2. KALMAN FILTER

Kalman filter is a recursive linear optimal filter, the model of kalman filter can be described as follows:

$$X_{k/k} = A_k X_{k/k-1} + B_k U_k + W_k$$
$$Y_k = C_k X_{k/k} + D_k U_k + N_k$$
(1)

Where:

A$_k$: is the transition matrix

B$_k$: is the input control vector.

W$_k$: is the process noise, and it is assumed to be white Gaussian noise with zero mean and covariance matrix Q$_k$, i.e $N(0, Q_k)$.

Z$_k$: is observation of the state X_k.

C$_k$: is the observation matrix.

V_k : is the observation noise, and it is also assumed to be Gaussian white noise with zero mean and R_k covariance matrix, i.e $N(0, R_k)$.

The kalman filter calculations can be divided to two stages; predicted stage and updating stage, in the predicted stage, the state variables and the output are calculated based on the system model, while in updating stage, the predicted state variables and the output are modified based on the measurement data. The calculation of each stage as follows:

- Predicted stage:

$$X_{k/k-1} = A_k X_{k-1/k-1} + B_{k-1} U_{k-1}$$
$$P_{k/k-1} = A_k P_{k-1/k-1} + A_k^T + Q_k$$
(2)

- Updating stage

$$Y_k = Z_k - C_k X_{k/ k-1}$$
$$K_k = P_{k/k-1} C_k^T [C_k P_{k/k-1} C_k^T + R_k]^{-1}$$
$$X_{k/k} = X_{k/k-1} + K_k Y_k$$
$$P_{k/k} = [I - K_k C_k] P_{k/k-1}$$
(3)

Where: P$_{K/K}$ is the estimate covariance matrix.

3. POWER SYSTEM HARMONICS

The power system signals (voltage and current) have harmonic components due to the nonlinear devices such as transformer, machines and power electronics devices, the harmonics components may leads to un correct decision of power system protection devices, for that reason, it is essentially in many applications to detect precisely the fundamental component, the kalman filter will be useful in such case. One of the major problem in power system is the voltage sag problem, where the voltage is decreased due to increasing in load demand, at the beginning, the voltage decreases continuously, then at certain threshold value the voltage collapse rapidly down to zero, which causes severe problems in power system, the protection device must detect the voltage and operate before the threshold value to avoid the voltage collapse problem.

To detect the fundamental component, the system can be modeled in two ways; the first model will include only the fundamental component and treat the harmonics as noise, while the other model will include all the harmonics components in the system model.

The two models are as follows:

Model 1:

$$A_k = \begin{bmatrix} \cos(wT) & -\sin(wT) \\ \sin(wT) & \cos(wT) \end{bmatrix}$$
(4)

$$B_k = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \ C_k = \begin{bmatrix} 1 & 0 \end{bmatrix}, D_k = [0]$$
(5)

Where; w: the angular frequency.
        T: the sampling time.

Model 2:

$$A_k = \begin{bmatrix} A_{k,1} & Z & \cdots & Z \\ Z & A_{K,2} & \cdots & Z \\ \vdots & \vdots & \ddots & \vdots \\ Z & Z & \cdots & A_{k,n} \end{bmatrix} \tag{6}$$

$$A_{k,j} = \begin{bmatrix} \cos(jwT) & -\sin(jwT) \\ \sin(jwT) & \cos(jwT) \end{bmatrix} \tag{7}$$

$$Z = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{8}$$

$$B_k = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{n \times 1} \tag{9}$$

$$C_k = \begin{bmatrix} 1 & 0 & 1 & 0 & \cdots & 0 \end{bmatrix}_{1 \times n} \tag{10}$$

Where; n: is the order of harmonic to be detected.
The amplitude of the fundamental component can be calculated as follows:

$$Amp_k = \sqrt{X_{1,k}^2 + X_{2,k}^2} \tag{11}$$

In this paper, model 1 will be used to detect the fundamental components, the exact model of all the harmonics will be studied in our future works.

4.   SIMULATION RESULTS

Suppose, the electrical signal is given as follows:
$$x(t) = 1.414\cos(100\pi t + \pi/6) + 0.3\cos(300\pi t + \pi/8) + 0.1\cos(500\pi t)$$

The signal has the fundamental, first and third harmonic components, the measurement noise covariance matrix in this experiment is 0.1, the sampling time is 0.1ms, the value of R and Q in the kalman model, tare assumed to be unknown (the ral value of them will be constant) and they are changed to investigate their effects in kalman filter performance. Figure 1 shows the measured signal with harmonics and noise, Figure 2 shows the estimated of the fundamental component using kalman filter for Q =1×10-9 and R = 0.1, which is the real value of the measurement noise covariance .

The error signal, which is the difference between the amplitude of the fundamental signal and the amplitude of the fundamental signal calculated by the kalman filter, for several values of R and Q are shown in Figure 3 and Figure 4.

As it is discussed in [10], the value of Q and R matrices in kalman filter represent how much the kalman filter can trust the predicted and the measurement value to estimate the output, when R in the kalman filter model is increased this will give indication to kalman filter that the measurement signal is highly noisy, which cause the kalman filter to trust more the predicted value, where the predicted value is calculated using the model matrices and it strongly depends on the initial values of the state variables, the model in this experiment predicted a sinusoidal signal with 1.41 amplitude. When the value of Q is increased, the kalman filter will trust the measurement signal more than the predicted value, and this will lead the kalman filter to have a noisy output similar to the input, based on the results shown in Figure 2 and Figure 3, the error signal goes to zero even though that the noise covariance R is not true, which shows a good response of kalman filter, so by balancing Q and R matrices, a good response of kalman filter could be achieved.

Based in most researches, the ratio between Q and R is more important than the exact values of them, Figure 5 shows the error signal for different values of Q/R ratio.
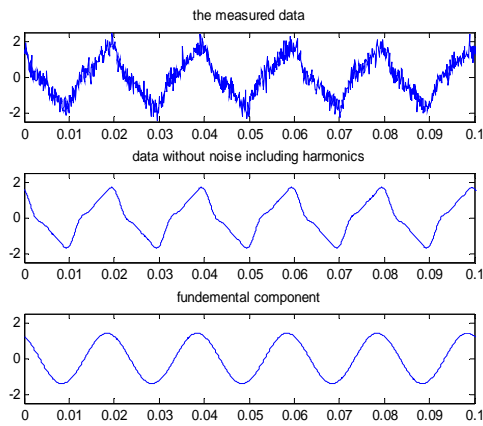
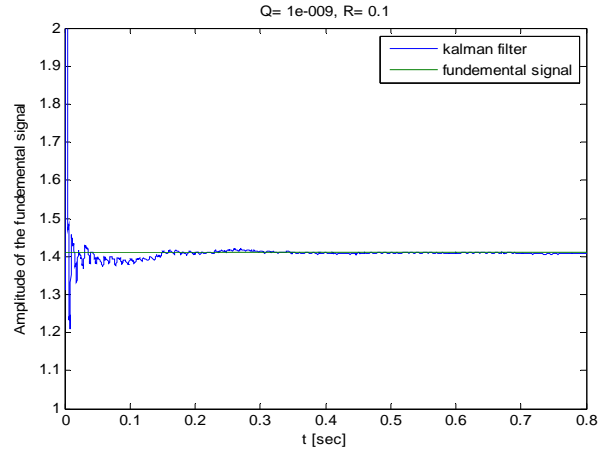Figure 1. The measured signal for R =0.1

Figure 2. The fundamental signal amplitude of Kalman filter for Q= 1×10^{-9} and R= 0.1
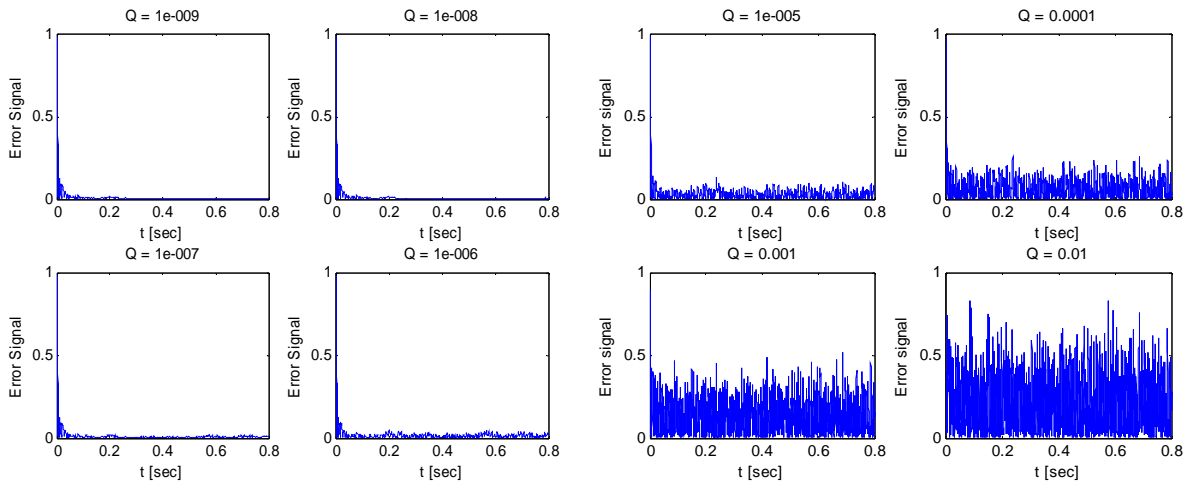

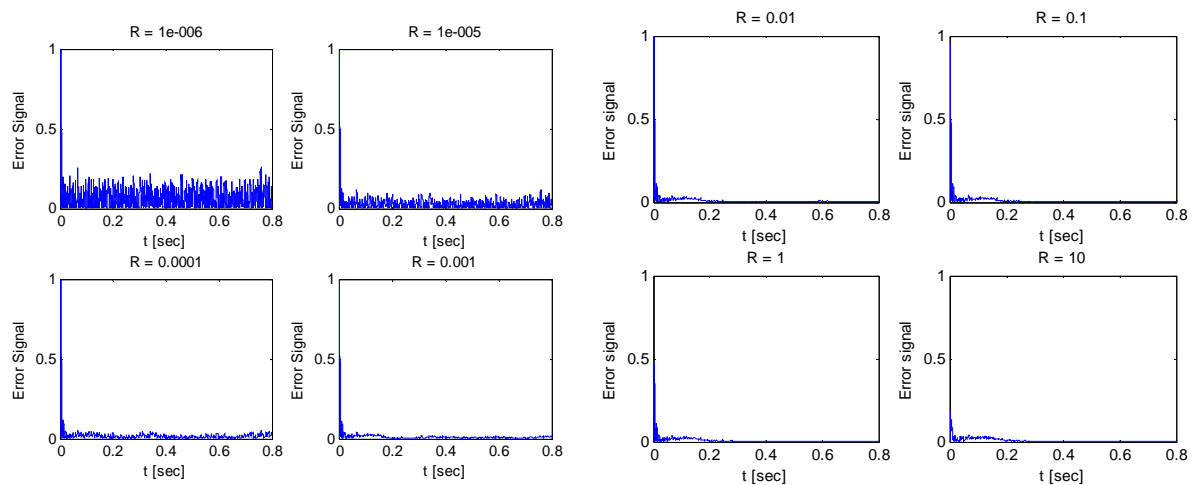Figure 3. The error signal of kalman filter with different values of Q and R= 0.1

Figure 4. The error signal of kalman filter with different values of R and Q= 1×10^{-9}
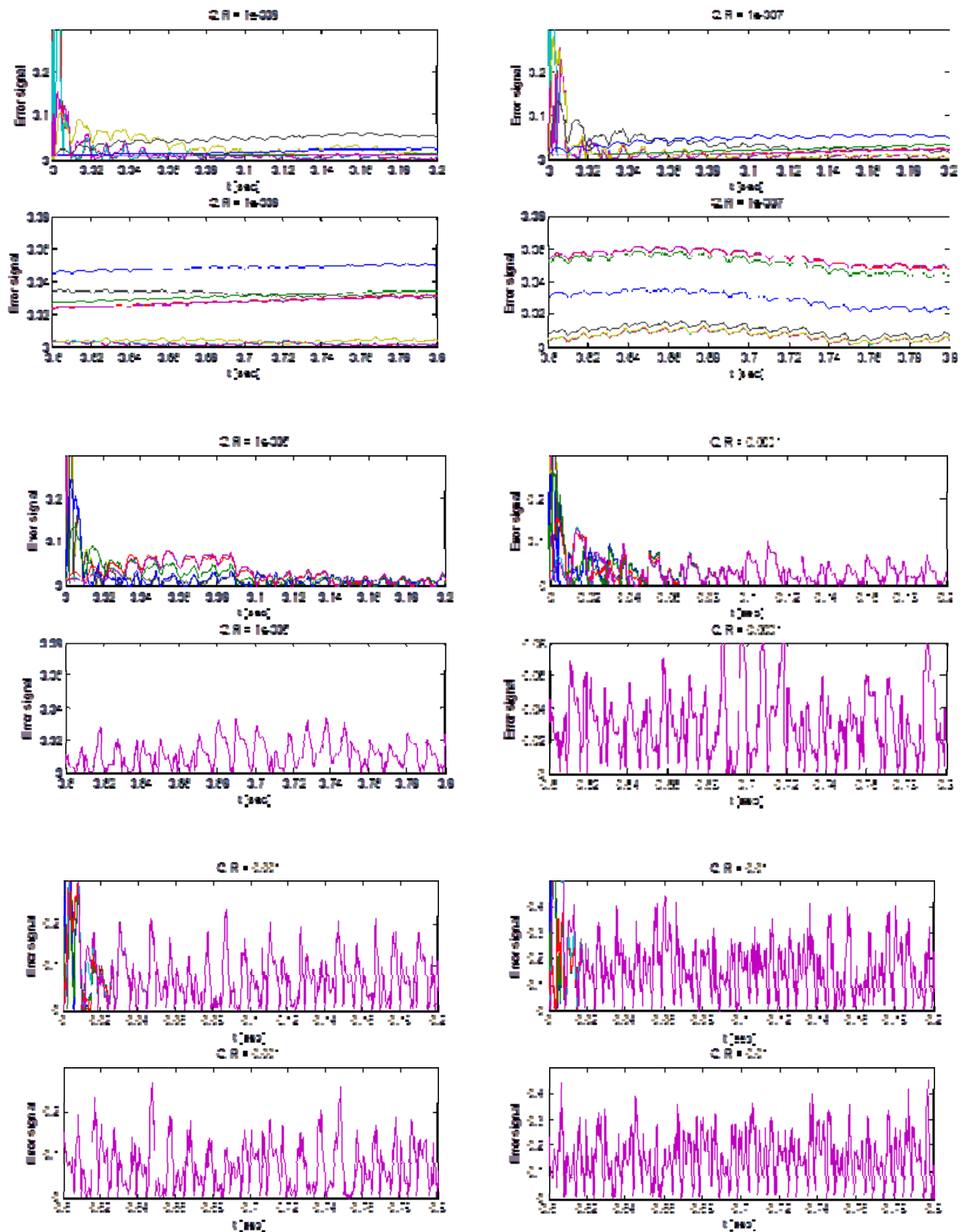
Figure 5. Error signal for several values of Q and R with different constant Q/R ratios

Figure 5 shows the error signal for different values of Q and R matrices with different Q/R ratios, there is slightly differences in the error signal for different values of Q and R even though that the ratio between them is still constant, by examining Figure 5 carefully, when the ratio between Q and R is decreased the kalman filter has the same error signal for the same Q/R ratio.

Figure 6 shows a dynamic change in the amplitude of the fundamental signal at o.4 sec, first Q and R matrices are constant, the kalman filter shows a bad performance in this case, then the value of Q and R are updated to several values listed in the Figure 6, the performance of kalman filter is strongly effected due to changing in Q and R matrices. Based on these results the best performance of kalman filter was occurred when both Q and R matrices are changed. Even though that the amplitude of the fundamental signal was changed at 0.4sec then it returns to its original value at 0.8 sec as it shown in Figure 7, the performance of kalman filter was poor, in Figure 7 (a) the value of Q and R are constant in the interval 0 sec to 0.4 sec, the kalman filter detect the fundamental component successfully, then the amplitude of the fundamental component is changed through the interval 0.4sec to 0.8 sec, Q and R remain constant, the kalman filter couldn't estimated the amplitude in this interval, then at 0.8sec the signal is returned to its original value (where the value of Q and R are sufficient to have good performance of kalman filter), the kalman filter is failed to estimate the fundamental signal correctly in this interval, which points to the importance of changing Q and R matrices for a dynamic change in the system variables, in Figure 7 (b) the value of Q and R matrices are modified for the first change and they are kept constant for the second sudden change, the kalman filter performance was good, when Q and R matrices are changed and poor when they are kept constant through a dynamical changing of the system variables.

At this point, this will lead to ask , what prevent the kalman filter to detect the amplitude, even though it did in previous interval, by examining the kalman model matrices, the difference was in the P matrix (estimate covariance matrix) on interval 0sec to 0.4 sec, the value of P is set to identity matrix, and through the calculations the kalman filter converges to the solution and the value of P matrix decreases and approach to zero, for interval 0.4 sec to 0.8 sec, the P matrix is very small at the beginning of this interval, so in order to investigate the effect of setting the P matrix for a dynamic change, the P matrix is changed in Figure 8 and Figure 9. The performance of the kalman filter is improved, when the value of the P matrix is changed, the value of P matrix approaches zero for normal case, as soon as a dynamic change occurs, the P matrix should be increased to give indication to kalman filter that it doesn't converge to the solution yet, based on the results in Figure 8 and Figure 9, the kalman filter estimated the fundamental component correctly, for different values P matrix.

Up to this point, the kalman filter performance is investigated under constant frequency condition, while the frequency in power system is usually varied due to the load changing, the control devices in power system allow the frequency to vary in narrow band limits, since the frequency is assumed to be constant in the kalman filter transition matrix, which will cause problem in both of the predicted and the estimated signal, the frequency of the fundamental signal is changed in Figure 10.

Figure 10 shows the state variables of the kalman filter when the fundamental frequency is changed to different value and the amplitude of the fundamental signal is kept constant, in Figure 10 (a) the fundamental frequency is changed slightly to 312.9, neither of the noise covariance matrices nor P matrix are changed, the kalman filter failed to track the amplitude in such case, while in Figure 10 (b) and Figure 10 (c) the frequency is also changed to 312.9 but in Figure 10 (b) the noise covariance matrices are changed and in Figure 10 (c) P matrix is changed, the kalman filter shows better performance in these figures, the result obtained from changing P matrix looks better. In Figure 10 (d) and (e) the fundamental frequency is changed to 308.7 rad/sec (i.e 49.15 Hz) neither of changing P matrix nor noise covariance matrices improves the kalman filter capability to detect the amplitude of the fundamental signal, based on the results shown in Figure 10 the variation of the fundamental frequency has a great effect of the kalman filter performance, in normal operation of power system, the frequency is usually distinct or changed slightly from the nominal value due to load changing, so updating P matrix can improve the kalman filter performance, while if the frequency is changed more, the simple kalman filter failed to detect the amplitude of the fundamental signal, in this case the frequency should be considered in the kalman filter model which leads to a nonlinear model, then extended kalman filter and unsecent kalman filter can be used.
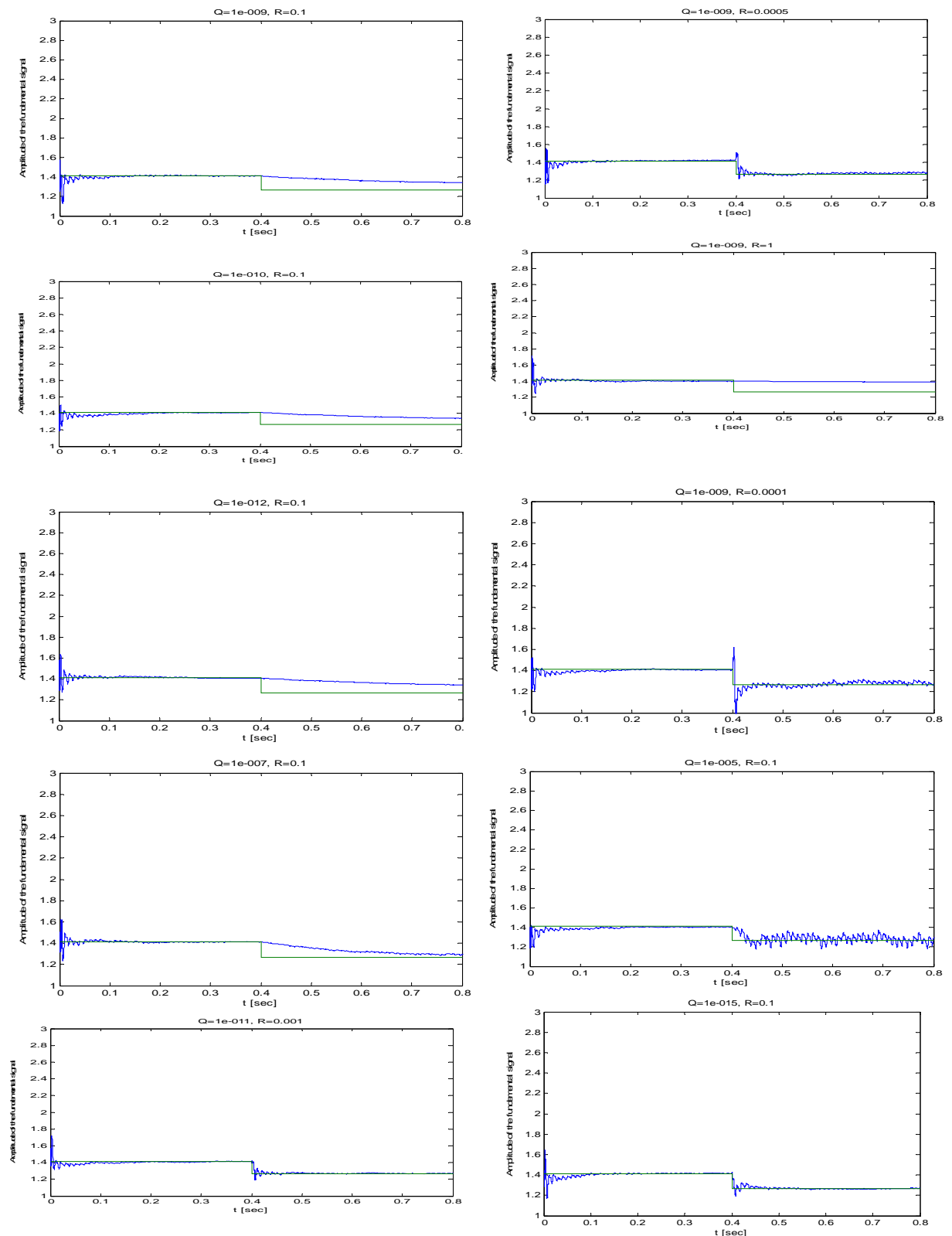
Figure 6. Amplitude of the fundamental signal estimated using kalman filter for a sudden change of the
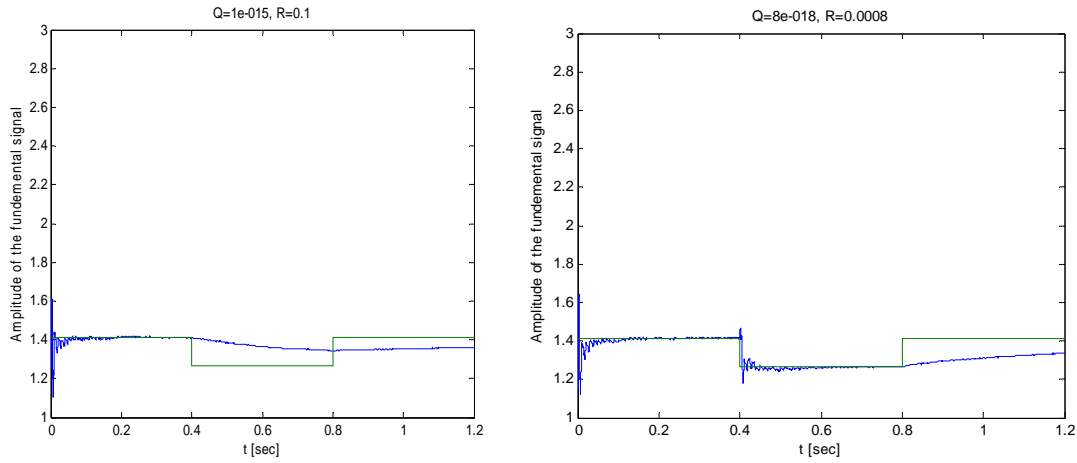fundamental amplitude for different values of Q and R matrices

Figure 7. The amplitude of the fundamental signal estimated by kalman filter for a sudden change on the fundamental signal at t=0.4 then its return to its original value, (a) Q and R kept constant, (b) Q and R are updated at t=0.4 then they are kept constant.
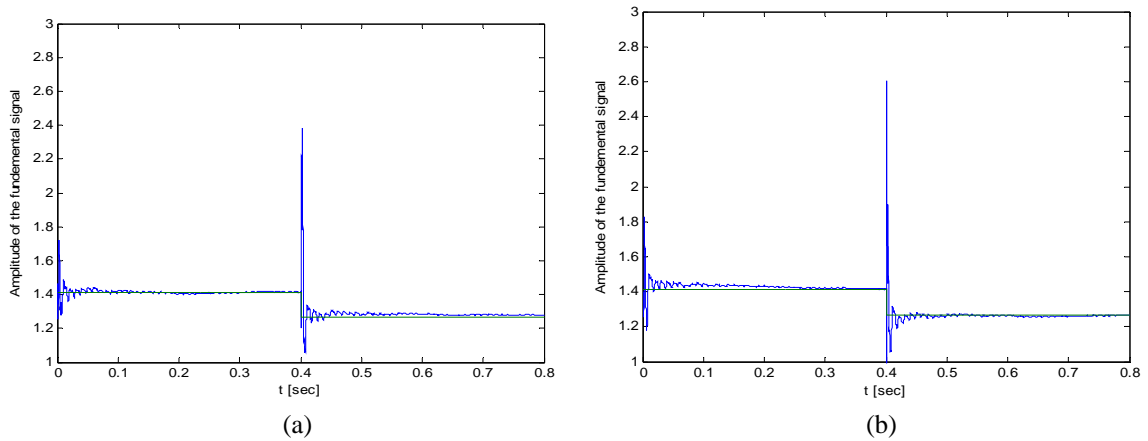

Figure 8. Amplitude of the fundamental signal using kalman filter for a sudden change of the fundamental amplitude, Q , R are constant (a) P is reset (b) P is changed to [0.5 0;0 0.5]
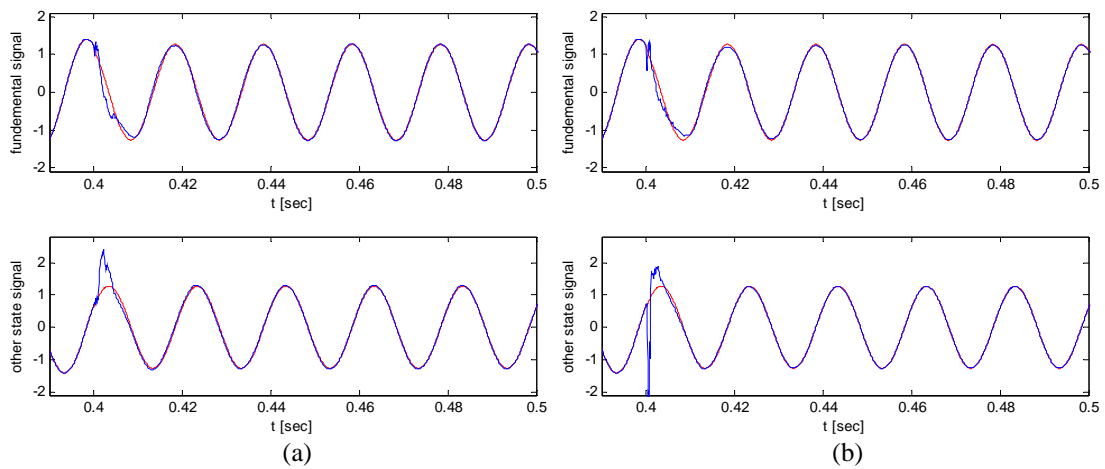

Figure 9. State signals using kalman filter for a sudden change of the fundamental amplitude, Q, R are constant and (a) P is reset (b) P is changed to [0.5 0; 0 0.5]
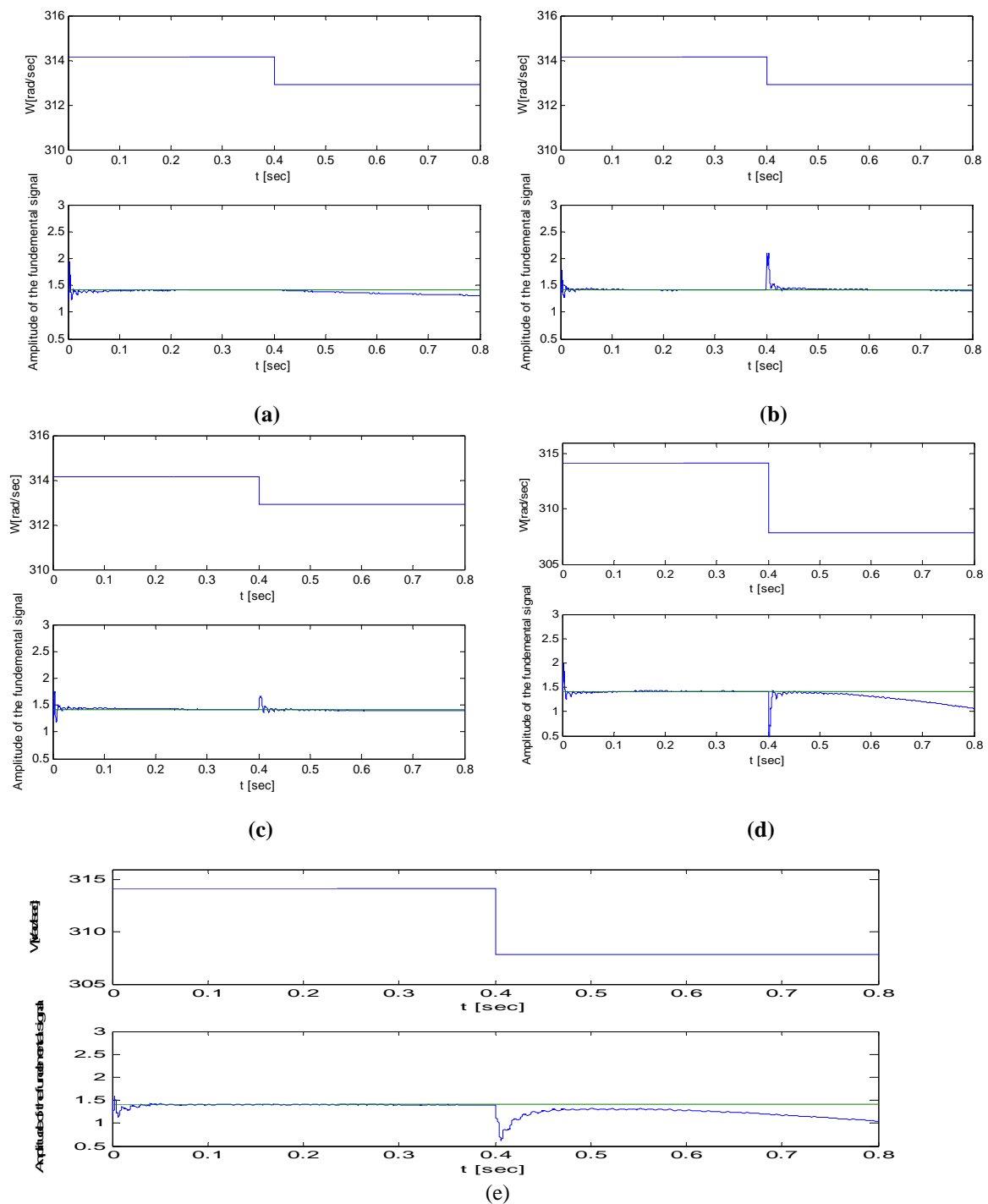
(a)

(b)

(c)

(d)

(e)

Figure 10. Amplitude of the fundamental signal using kalman filter for a sudden change in the frequency to (a) 312.9 rad/sec, Q, R and P are constant, (b) 312.9 rad/sec, Q, R are constant and P is changed, (c) to 312.9 rad/sec, Q, R are changed and P is constant, (d) to 307.8 rad/sec, Q, R are constant and P is changed and (e) to 307.8 rad/sec, Q, R are changed and P is constant

5. CONCLUSIONS AND FUTURE WORKS

the kalman filter was used to detect the fundamental component of a signal contains harmonics and noise, the measurement noise covariance matrices in the kalman filter model were assumed to be unknown and the values of these matrices were changed through the research to investigate the kalman filter performance under the uncertainty of these matrices (the real noise covariance matrix is kept constant), the

amplitude of the fundamental component is changed several times and it is concluded that, it is necessary to change the values of both Q and R matrices for the dynamic change, it is also concluded that changing P matrix in kalman model is also sufficient to improve the kalman filter in such case. The frequency of the fundamental component is also changed, the frequency is included in the kalman transition matrix, which caused error in the estimated output of the kalman filter, for a normal operation of power system, the frequency is changed slightly, changing P matrix can improve the kalman the filter performance, otherwise, the performance of the kalman filter can be improved by changing the kalman state variables to inclued the frequency, in such case the system becomes nonlinear , which leads to use extended kalman filter or unsecent kalman filter.

      The covariance noise matrices and P matrix are updated when the amplitude is changed, to investigate their effect in kalman filter performance, even though, in practical this time is unknown, this information will be helpful for design adaptive kalman filter, where combining the kalman filter with other mathematic method to figure the necessary time for updating these matrices or by investigating the kalman filter converge criteria to set the necessary conditions for updating them, and this will be one of our future works.

      The extended kalman filter could be used to detect the amplitude under a large frequency deviation, investigating the effects of the covariance matrices in such case could be done. In this paper, transition matrix includes only the fundamental component, including the harmonic frequencies in the transition matrix will leads to more accurate model and this may increase the range of the frequency deviation that the simple kalman filter could be used, even though that the size of matrices will be larger, this can be investigated in the future works.

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