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Dynamic Performance Analysis of Permanent Magnet Hybrid Stepper Motor by Transfer Function Model for Different Design Topologies

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ABSTRACT

This paper discuss about dynamic performance analysis of two phase bipolar permanent magnet hybrid (PMH) stepper motor during single stepping by transfer function model for eight topologies using PDE toolbox of Matlab. Topologies are designed with different airgaps and with different stator teeth. These results analyze motor rise time, final steady state time and final steady state position during single stepping which suggest better design topology of motor for better dynamic response of the PMH stepper motor during single stepping.

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1. INTRODUCTION

Stepper motor has a salient feature of discrete torque which makes its application in many varieties like solar array tracking system in satellites, computer peripherals and machine tools. When a large step angle is required, a permanent magnet stepper motor or a variable reluctance motor is preferred. When microstepping with good dynamic response is required permanent magnet hybrid stepper motor is preferred. Hybrid stepper motors are highly preferred in space applications as they can provide accurate positioning in open loop system [1].

The positional accuracy of the stepper motors will be high only when its step angle is very small. Hence for space applications hybrid stepper motor is the best choice as it can offer small step angles in the range of 0.50 to 1.8 mechanical degrees. The differential equations which define the response of motor are first derived. Linearized versions of these equations are used to obtain natural frequency and damping ratio at an operating point. Using these relations transfer function of stepper motor stepping angle is derived which is used for dynamic analysis during single stepping

2. PMH STEPPER MOTOR TOPOLOGIES FOR ANALYSIS

A practical 1.8^0 two phase bipolar PMH stepper motor is chosen for design. It has 4 poles in the stator and 2 sections in the rotor with $AlNiCo_5$ magnet axially magnetized. The main structural parameters of the motor are shown in Table 1.

Table 1. Structural parameters of PMH Stepper Motor

Stator poles	Tooth per stator pole	Outer diameter of stator	Inner diameter of stator	Outer diameter of stator shell
04	8	10.108 cm	5.936 cm	10.652 cm
Number of rotor tooth	Number of turns per phase	Section length of rotor	Outer diameter of rotor	Inner diameter of rotor
50	46	10.26 cm	4.2 cm	1.74 cm
No. of turns per stator pole	Rated voltage	Rated current	SWG of conductor	Torque
92	12 V	1 A	36	1.5 Nm

PDE toolbox of Matlab for FEM analysis is used to investigate inductance for different topologies which is difficult to investigate by mathematical model [3]. Different topologies considered are i)Non-uniform air-gap of 0.93 mm with extra teeth on stator ii)Non-uniform air-gap of 0.137 mm with extra teeth on stator iii)Non-uniform air-gap of 0.93 mm without extra teeth on stator iv)Non-uniform air-gap of 0.137 mm without extra teeth on stator v)Uniform air-gap of 0.93 mm with extra teeth on stator vi)Uniform air-gap of 0.137 mm with extra teeth on stator vii)Uniform air-gap of 0.93 mm without extra teeth on stator viii)Uniform air-gap of 0.137 mm without extra teeth on stator.

3. RESULTS AND ANALYSIS

3.1. Investigation of Inductance of Different Topologies

The gap permeances P_1 to P_5 are calculated using linearized tooth layer unit arrangement and flux tubes [2] as shown in Fig.1. Here, x =equivalent length of step angle mm, tooth width, t = 1.32 mm, tooth pitch, s = 1.42 mm, tooth depth, d = 1.32 mm, air-gap length, g = 0.137 mm and 0.93 mm are considered for the analysis. Suppose that the number of teeth per stator pole is Z_s.

$$\left.\begin{aligned} P_1 &= \mu_0 \frac{t-x}{g} \\ P_2 &= \mu_0 \frac{2}{\Pi} \ln\left(1+\frac{\Pi x}{2g}\right) \\ P_3 &= \mu_0 \frac{1}{\Pi} \ln\left(\frac{g+2d-\frac{1}{2}\Pi x}{g+\frac{1}{2}\Pi x}\right) \\ P_4 &= \mu_0 \frac{2}{\Pi} \ln\left(\frac{g+2d}{g+2d+\frac{1}{2}\Pi x}\right) \\ P_5 &= \mu_0 \frac{s-x-\frac{4d}{\Pi}}{g+2d} \end{aligned}\right] \quad (1)$$

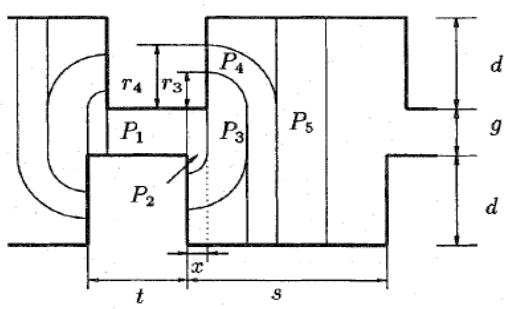

Fig.1.Linearized tooth layer unit arrangement and flux tubes

Table 2 Fourier coefficients of gap permeance for different topologies

Topology	$\rho_0 \times 10^{-6}$	$\rho_1 \times 10^{-6}$	$\rho_2 \times 10^{-6}$	$\rho_3 \times 10^{-6}$	$\rho_4 \times 10^{-6}$	$\rho_5 \times 10^{-6}$
	Fourier coefficients of gap permeance					
1	4.328	2.338	1.801	4.070	2.829	1.227
2	1.184	7.665	5.904	1.172	8.294	4.024
3	2.164	1.169	9.005	2.142	1.414	6.138
4	7.093	3.832	2.952	7.022	4.636	2.012
5	1.656	8.950	6.893	1.640	1.082	4.699
6	4.320	3.222	2.809	4.276	3.280	1.915
7	1.506	8.136	6.526	1.552	1.025	4.448
8	5.375	3.043	2.464	5.321	3.681	1.679

The total permeance per pole, P_t is given in (2), considering P_1 to P_5 are in parallel

$$P_t = Z_s\,[P_1 + 2(P_2 + P_3 + P_4) + P_5]\ \text{Wb/A} \quad (2)$$

The Permeance per pole per phase is given by (3)

$$P_{\propto} = P_t + \sum_{n=1}^{\infty} P_n \cos(n\theta) \quad (3)$$

Since the displacement is from θ = 0 to 2π radians corresponds to one-pitch of a tooth, the electric angle θ_e is related to the mechanical step angle θ_m, as in (8), where Z_r is the number of teeth of the rotor

$$\theta_e^0 = Z_r\theta_m^0 \quad (4)$$

The gap permeance Fourier coefficients (ρ) for all topologies are evaluated using (1), (2), (3) and (4) and tabulated in Table 2.

Using these Fourier coefficients self and mutual inductances of hybrid stepper motor are investigated [3], [4] for eight topologies using (5), (6).

$$L = (0.75 \times N_s^2 \times \rho_1 \times \cos(\theta)) \quad (5)$$

$$M = (0.25 \times N_s^2 \times \rho_0 - 0.25 \times N_s^2 \times \rho_1 \times \cos\left(\theta - \frac{\Pi}{2}\right)) \quad (6)$$

Here L is self inductance in H and M is mutual inductance in H. N_s is turns per phase per pole, θ is step angle in electrical degrees.

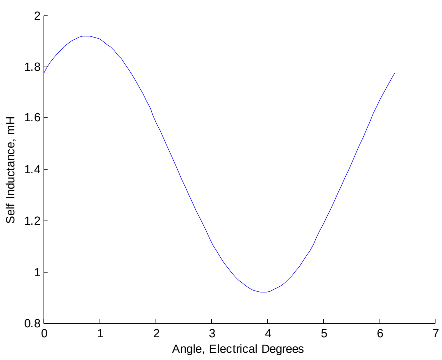

Fig.2.Self inductance of PMH for topology 6

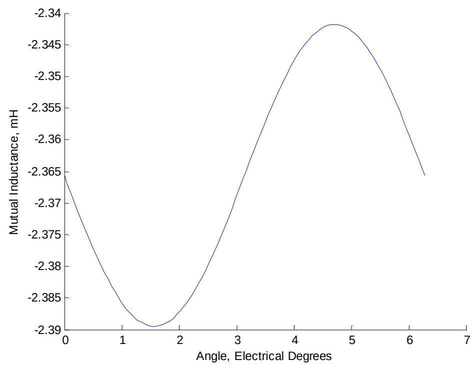

Fig.3.Mutual inductance of PMH for topology 6

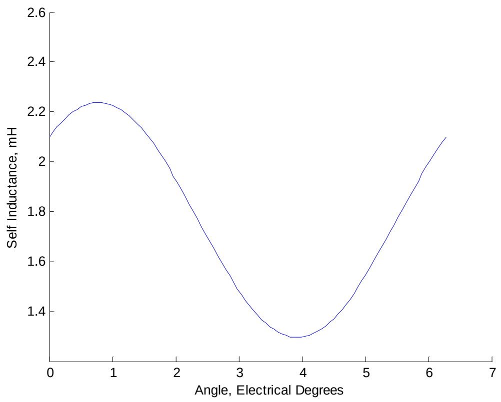

Fig.4.Self inductance of PMH for topology 8

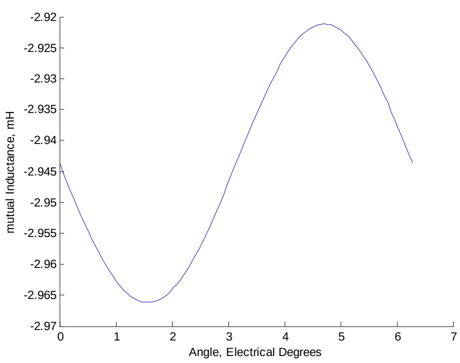

Fig.5.Mutual inductance of PMH for topology 8

Fig.2 and Fig.3 shows self and mutual inductances respectively for topology1. Fig.4 and Fig.5 shows self and mutual inductances for topology 2. Similarly for all topologies self and mutual inductances are calculated and tabulated in Table 3.

Table 3. Self and mutual inductances for different topologies

Topology	Self Inductance mH	Mutual Inductance mH
1	0.1008	0.2235
2	0.5078	0.6429
3	0.0504	0.1118
4	0.2924	0.3856
5	0.6485	0.8526
6	1.9180	2.3890
7	0.6172	0.8145
8	2.2370	2.9660

3.2. Transfer function model of PMH stepper motor

The equation of motion of the rotor with one phase excitation is given by [5]

$$J\left(\frac{d^2\theta}{dt^2}\right) + B\left(\frac{d\theta}{dt}\right) = \Phi_m N_s(I_1 \sin(p\theta) + I_2 \sin(p\theta - \lambda)) \quad (7)$$

$$V = RI_1 + L\left(\frac{dI_1}{dt}\right) + M\left(\frac{dI_2}{dt}\right) - \frac{d}{dt}(N_s \Phi_m \cos(p\theta)) \quad (8)$$

Where J is moment of inertia of the rotor, B is viscous damping coefficient, Φ_m is permanent magnetic flux in Weber, I_1, I_2 are phase currents, p is number of pole pairs, λ is stator pole pitch in electrical degrees. Neglecting hysteresis and eddy current losses, the voltage equation for the phase winding is mentioned as (8).

Using the small signal linearization method about the operating point for a stepping angle (θ),θ_0=0, I_{10}=I_{20}=0 linearized equations are written as

$$J\left(\frac{d^2}{dt^2}\right)(\delta\theta) + B\left(\frac{d}{dt}\right)(\delta\theta) = 2P^2\Phi_m NI_0 \cos\left(\frac{P\lambda}{2}\right) + P\Phi_m N_s \sin\left(\frac{P\lambda}{2}\right)(\delta I_1 - \delta I_2) \quad (9)$$

$$R(\delta I_1) + L\left(\frac{d}{dt}\right)(\delta I_1) + M\left(\frac{d}{dt}\right)(\delta I_2) - P\Phi_m N_s \sin\left(\frac{P\lambda}{2}\right)\left(\frac{d}{dt}\right)(\delta\theta) = 0 \quad (10)$$

$$R(\delta I_2) + L\left(\frac{d}{dt}\right)(\delta I_2) + M\left(\frac{d}{dt}\right)(\delta I_1) - P\Phi_m N_s \sin\left(\frac{P\lambda}{2}\right)\left(\frac{d}{dt}\right)(\delta\theta) = 0 \quad (11)$$

Where $\delta\theta = \theta - \theta_0$; $\delta I_1 = I_1 - I_{10}$; $\delta I_2 = I_2 - I_{20}$ and I_{10} and I_{20} are initial currents for phase1 and phase2 respectively before stepping. θ_0 is initial position angle of the rotor. Taking Laplace transform and eliminating all the intermediate variables, we get the hybrid motor transfer function as

$$\delta I_1(s) = -(\delta I_2) = \frac{P\Phi_m N_s \sin\left(\frac{P\lambda}{2}\right)[[s\theta(s) - \theta_0(s)]}{(R+Ls)} \quad (12)$$

$$\delta\theta(s) = \frac{\left[s^2 + \left\{\left(\frac{R}{L_p}\right) + \left(\frac{B}{J}\right)\right\}s + \left\{\left(\frac{R}{L_p}\right) + \left(\frac{B}{J}\right) + K_p \acute{\omega}_{np}^2\right\}\right]\theta(s)}{[s^3 + \left\{\left(\frac{R}{L_p}\right) + \left(\frac{B}{J}\right)\right\}s^2 + \left\{\left(\frac{R}{L_p}\right) + \left(\frac{B}{J}\right) + (1+K_p)\acute{\omega}_{np}^2\right\}s + \left(\frac{R}{L_p}\right)\acute{\omega}_{np}^2]} \quad (13)$$

Where

$$L_p = L - M \quad (14)$$

$$K_p = \frac{\Phi_m N_s \sin\left(\frac{P\lambda}{2}\right)}{L_p I_0 \cos\left(\frac{P\lambda}{2}\right)} \quad (15)$$

$$\acute{\omega}_{np}^2 = \frac{\Phi_m N_s I_0 \cos\left(\frac{P\lambda}{2}\right)}{J}; \quad (16)$$

$$I_0 = I_{10} + I_{20} \quad (17)$$

For analysis it is assumed J=1.415×10^{-3} kg-m^2, B=0.6×10^{-3} Nm/ (rad/s), R=12 Ω. Permanent magnet flux Φ_m is calculated from mmf of permanent magnet considered from Table 4 for different topologies investigated using PDE toolbox of Matlab which is found as same for different core materials and for different current densities also, L and M are considered from Table 3. Substituting these values in the above relations (13) to (17) transfer function of the hybrid stepper motor for single step is investigated. Fig.6 is step response of PMH motor for Topology 6 and Fig.7 is step response of PMH motor for Topology 8. Using this transfer function step response of the hybrid stepper motor is analyzed for eight topologies and their settling time, rise time final steady state values are tabulated in Table 5.

Table 4.Flux due to permanent magnet for different topologies

Topology	MMF due to Permanent Magnet $AT \times 10^{-4}$
1	0.649
2	1.232
3	0.568
4	1.539
5	0.970
6	4.578
7	0.811
8	6.159

Table 5.Dynamic response of PMH stepper motor for different topologies

Topology	Rise time in Seconds	Settling time in Seconds	Steady State Final value of step angle
1	0.257	0.46	0.228
2	0.907	1.57	0.747
3	0.219	0.39	0.199
4	0.740	1.32	0.612
5	0.577	1.01	0.472
6	2.240	3.98	1.800
7	0.470	0.84	0.394
8	1.880	3.34	1.600

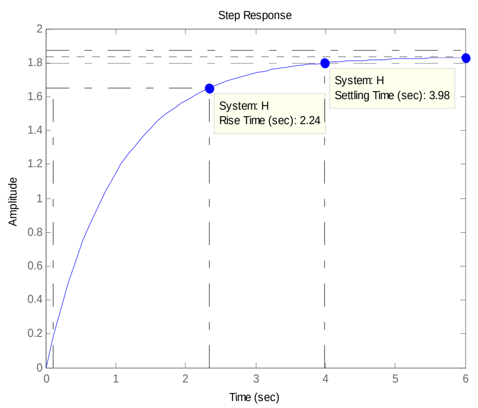

Fig.6.Step response of PMH motor for Topology 6

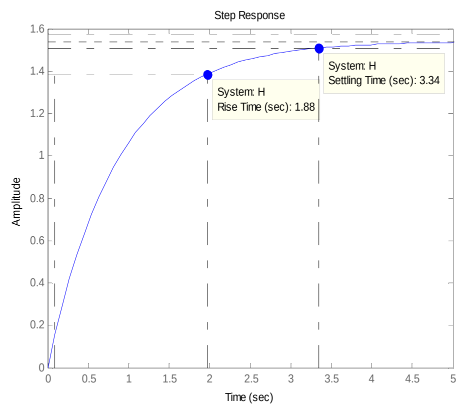

Fig.7.Step response of PMH motor for Topology 8

When airgap is non-uniform rise time and settling time are less but final step position is far from required step angle 1.8^0 mechanical as shown for topologies 1 to 4 in Table 5. For uniform airgap rise time and settling time are more comparatively but final step position is improved as shown for topologies 5 to 8 in Table 5. When there are extra teeth on stator final step position is improved when compared with topologies without extra teeth on stator as shown for topology 1, topology 2 have more final step position compared to topology 3, topology 4 respectively, similarly topology 5, topology 6 have more final step position compared to topology 7, topology 8 respectively. When airgap is decreased final step position is improved as shown for topology 2, topology 4 have more final step position compared to topology 1, topology 3 respectively, similarly topology 6, topology 8 have more final step position compared to topology 5, topology 7 respectively.

4. CONCLUSION

This paper analyzes dynamic response of permanent magnet hybrid stepper motor for different topologies. Topology 6 i.e. 0.137mm uniform airgap between stator and rotor with extra teeth on stator is only getting the required step angle of 1.8^0 mechanical but its rise time and settling time are more when compared with other topologies. For remaining topologies though settling and rising timings are low their final steady position in not the required step angle. Thus for accurate final steady state position topology one is to be concluded.

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