Optimal Controller Design for Thermal Power System with Feedback Linearization

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ABSTRACT

The main objective of this paper is to device a method for construction of state-feedback control law for a single area thermal power system .In this method, first plant model is transformed into the controller form and constructs a state feedback controller in the new coordinates and then using the inverse transformation; represent the controller in the original coordinates. While constructing the controller in the new coordinates, a part of the controller is used to cancel nonlinearities, thus resulting in a linear system in the new coordinates. Simulation results in a single reheat thermal system are provided to illustrate the effectiveness of the proposed nonlinear control scheme.

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1. INTRODUCTION

The speed oscillation of the turbo-generator in thermal power system is caused by a load changesor sudden electrical demanded load and the frequency deviation in the power system is highly undesirable. These oscillations may also depend upon the different parameters of the thermal power plant such as governor, generation rate constraint, reheat turbine etc. In previous some investigations the different authors investigated and focused on thermal power plant only non-reheat turbine is considered. But in present paper work, thermal power plant with reheat turbine is considered and mainly focused on the designing of the optimal controller [9], [13] for the thermal power plant via feedback linearization method [4]. With the help of block-diagram of the thermal power plant state-space equations is obtained [11], [12], thus defined the different parameter's relations of the thermal power plant of single area is obtained. Again with the help of Lie-derivatives, the nonlinear coordinate system of the thermal power plant is transformed into its equivalent linear and controllable form. After transforming the coordinate system of the thermal power plant, we are able to find out the nonlinear optimal control law via feedback linearization. Also the optimal control input and its corresponding optimal gains are obtained in views of Linear Quadratic Riccati principal and Quadratic Performance Index.

2. FEEDBACK LINERIZATION METHOD

Consider the following nonlinear system:

$$\dot{x} = f(x) + g(x)u \tag{1}$$

$$y = h(x) \qquad (2)$$

Where $x \in R^n$, $u \in R^1$ are state and control vectors respectively, $y \in R^1$ is the regulation output vectors. $f(x)$, $g(x)$ and $h(x)$ are the smooth vector fields with n-dimension and supposing that the dimensions of $h(x)$ is smaller than the dimensions of the system.

Let us define the Lie derivate of the function $h(x)$ along the vector field $f(x)$ as:

$$L_f h(x) = \left[\frac{\partial h}{\partial x_1}, \frac{\partial h}{\partial x_2} \cdots \frac{\partial h}{\partial x_n}\right] * \left[f_{1(x)}, f_2(x) \dots f_n(x)\right]^T \qquad (3)$$

And Lie derivative of $h(x)$ along $g(x)$ is:

$$L_g h(x) = \left[\frac{\partial h}{\partial x_1}, \frac{\partial h}{\partial x_2} \cdots \frac{\partial h}{\partial x_n}\right] * [g_1(x), g_2(x) \dots g_3(x)]^T \qquad (4)$$

Again $L_g L_f^0 h(x) = L_g L_f^1 h(x) = \cdots = L_g L_f^{n-2} h(x) = 0$

And $L_g L_f^{n-1} h(x) \neq 0$, getting the relative degree r of $h(x)$ to the nonlinear system equation described in equation (1).

If $r < n$ and $L_g L_f^{i-1} h(x) = 0 \;\; (0 \leq i \leq r)$; then, there exists a mapping,

$$Z = \varphi(x) \qquad (5)$$

$$Z = [\varphi_1(x)\varphi_2(x) \dots \varphi_r(x) \dots \varphi_n(x)]^T \qquad (6)$$

Where,

$$z = T(x) = \begin{bmatrix} z_1(x) \\ z_2(x) \\ \vdots \\ z_n(x) \end{bmatrix} = \begin{bmatrix} y \\ \dot{y} \\ \vdots \\ y^{(n-1)} \end{bmatrix} = \begin{bmatrix} h(x) \\ L_f h(x) \\ \vdots \\ L_f^{(n-1)} h(x) \end{bmatrix} \qquad (7)$$

And Z is the new coordinate iff,

$$L_g \varphi_{r+1}(x) = L_g \varphi_{r+2}(x) = \cdots = L_g \varphi_n(x) = 0 \qquad (8)$$

The jaccobian matrix is non-singular at equilibrium point x_0 .

$$J_\varphi = \left(\frac{\partial \varphi(x)}{\partial x}\right)_{at\ x=0} = \begin{bmatrix} \frac{\partial \varphi_1}{\partial x_1} & \cdots & \frac{\partial \varphi_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial \varphi_n}{\partial x_1} & \cdots & \frac{\partial \varphi_n}{\partial x_n} \end{bmatrix} \qquad (9)$$

transformstrajectories from the original *x* coordinate system into the new *z* coordinate system. So long as this transformation is a diffeomorphism, smooth trajectories in the original coordinate system will have unique counterparts in the *z* coordinate system that are also smooth. Those *z* trajectories will be described by the new system,

$$\dot{z}_1 = L_f h(x) = z_2(x)$$

$$\dot{z}_2 = L_f^2 h(x) = z_3(x)$$

$$\vdots$$

$$\dot{z}_n = L_f^n h(x) + L_g L_f^{n-1} h(x) u \qquad (10)$$

From the above discussion, we can transform the original nonlinear coordinate system (1) into linear and controllable system,
Hence, the feedback control law,

$$u = \left\{ \left(-L_f^n h(x) + v \right) \Big/ \left(L_g L_f^{n-1} h(x) \right) \right\} \quad (11)$$

Here v is optimal input in the view of Linear Quadratic Riccatti principal for the linearized system of new Z - Coordinate system.
Again,

$$v = -K_i Z_i$$

And v may be expressed as

$$v = -K_1 Z_1 - K_2 Z_2 - K_3 Z_3 - K_4 Z_4 \ldots\ldots\ldots\ldots\ldots\ldots -k_n Z_n \quad (12)$$

Where $K_i =$ corresponding optimal gain vectors which can be deduced from the Algebraic Riccatti Equation corresponding to the linear system.The Quadratic Perfomance index is:

$$J = \lim 1/2 [\int_0^\infty (Z^T Q Z + V^T R V) dt] \quad (13)$$

Where, Q and R is the semi-positive and semi-positive definite weighting matrix.

3. NONLINEAR OPTIMAL CONTROLLER DESIGN FOR SINGLE AREA REHEAT TURBINE THERMAL POWER SYSTEM

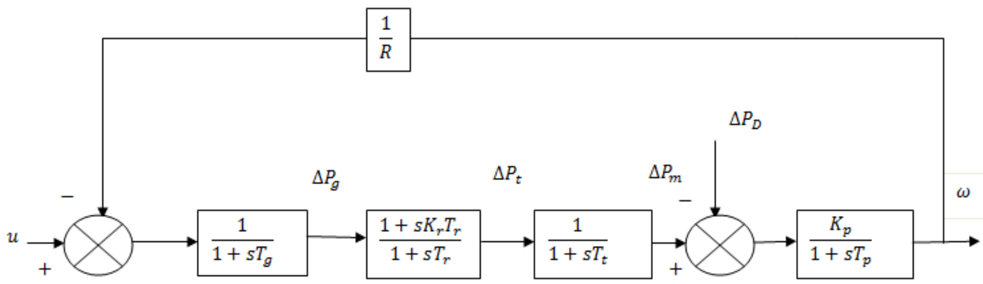

Figure 1. Block diagram of reheat thermal power system

3.1. A. Model of the Thermal reheatTurbine power System

Consider the block-diagram of the system; the thermal turbine regulation can be described as:

$$\Delta\dot{\delta} = \Delta\omega \quad (14)$$

$$\Delta\dot{\omega} = \left\{ \frac{K_p}{T_p}(\Delta P_m - \Delta P_D) - \frac{\Delta\omega}{T_p} \right\} \quad (15)$$

$$\Delta\dot{P}_m = \left\{ \frac{(\Delta P_t - \Delta P_m)}{T_t} \right\} \quad (16)$$

$$\Delta\dot{P}_t = \left\{ \frac{(\Delta P_g - \Delta P_t)}{T_r} + \frac{K_r}{T_g}\left(u - \left(\frac{1}{R}\right) + \Delta P_g\right) \right\} \quad (17)$$

$$\Delta\dot{P}_g = \left\{ \frac{u - \left(\frac{1}{R}\right) - \Delta P_g}{T_g} \right\} \quad (18)$$

The above thermal system equations could be written as the following nonlinear affine system

$$\dot{x} = f(x) + g(x)u \tag{19}$$

Where,

$$f(x) = \begin{bmatrix} \left\{\frac{K_p}{T_p}(\Delta P_m - \Delta P_D) - \frac{\Delta\omega}{T_p}\right\} \\ \left\{\frac{(\Delta P_t - \Delta P_m)}{T_t}\right\} \\ \left\{\frac{(\Delta P_g - \Delta P_t)}{T_t} - \frac{K_r}{T_g}\left(\frac{1}{R} + \Delta P_g\right)\right\} \\ \left\{-\left(\frac{\Delta P_g + \frac{1}{R}}{T_g}\right)\right\} \end{bmatrix}, \quad \dot{x} = \begin{bmatrix} \Delta\dot{\omega} \\ \Delta\dot{P}_m \\ \Delta\dot{P}_t \\ \Delta\dot{P}_g \end{bmatrix} \quad \text{and} \quad g(x) = \begin{bmatrix} 0 \\ 0 \\ \left(\frac{K_r}{T_g}\right) \\ \left(\frac{1}{T_g}\right) \end{bmatrix}$$

3.2. Design of the Nonlinear Governor Optimal control Law

To design the nonlinear thermal governor optimal law of single area, we need to transform the coordinate using feedback linearization into equivalent linear one via following steps:

Step1: Coordinate transformation,

$$z_1 = \Delta\delta \tag{20}$$

$$z_2 = \Delta\omega \tag{21}$$

$$z_3 = \left\{\frac{K_p}{T_p}(\Delta P_m - \Delta P_D) - \frac{\Delta\omega}{T_p}\right\} \tag{22}$$

$$z_4 = \Delta P_t\left(\frac{K_p}{T_p T_t}\right) - \Delta P_m\left(\frac{K_p}{T_p T_t} + \frac{K_p}{T_p^2}\right) - \Delta\dot{P}_D\left(\frac{K_p}{T_p}\right) + P_D\left(\frac{K_p}{T_p^2}\right) - \frac{\Delta\omega}{T_p^2} \tag{23}$$

$$z_5 = \frac{\Delta P_g K_p}{T_p T_t}\left(\frac{1}{T_r} + \frac{K_r}{T_g}\right) - \Delta P_m\left(\frac{K_p}{T_p^2 T_t} + \frac{K_p}{T_p^3}\right) + \frac{\Delta P_t K_p}{T_p T_t}\left(\frac{1}{T_t} - \frac{1}{T_r}\right) + \frac{\Delta\omega}{T_p^3} - \frac{K_r K_p}{T_p T_t T_g R} + \dot{P}_D\left(\frac{K_p}{T_p}\right) \quad 24)$$

Step2: The new Z - coordinate of the system can be changed into the following way,

$$\dot{z_1} = z_2 \tag{25}$$

$$\dot{z_2} = z_3 \tag{26}$$

$$\dot{z_3} = z_4 \tag{27}$$

$$\dot{z_4} = z_5 \tag{28}$$

$$\dot{z_5} = v \tag{29}$$

Hence,

$$v = \frac{\Delta P_g K_p}{T_p T_t}\left(\frac{1}{T_t} + \frac{K_r}{T_g}\right) - \Delta P_m\left(\frac{K_p}{T_p^2 T_t} + \frac{K_p}{T_p^3}\right) + \frac{\Delta P_t K_p}{T_p T_t}\left(\frac{1}{T_t} - \frac{1}{T_r}\right) + \frac{\Delta\omega}{T_p^3} - \frac{K_r K_p}{T_p T_t T_g R} + \Delta\ddot{P}\left(\frac{K_p}{T_p}\right) + \dot{P}_d\left(\frac{K_p}{T_p^3}\right) + \frac{\Delta P_D}{T_p^3} + \left(\frac{K_r K_p}{T_p T_t T_g}\right)u \tag{30}$$

Step3: Consider the above linear system described by the linearized part of the system is as,

$$\dot{z} = Az + Bv \tag{31}$$

$$y = Cz \tag{32}$$

$$\text{Where, } A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} C = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T$$

and $z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \end{bmatrix}$

Step 4: With the help of the solution of Riccatiequation and keeping in view of the linear quadratic performance index, the final governor optimal control law is,

$$\bar{P}_D T_r T \text{[illegible]} \tag{33}$$

4. RESULT AND DISCUSSION

The proposed design of the **optimal controller via feedback linearization for thermalpower system** of a single area power system with reheat turbine has been considered and applied. The system is simulated& tested in MATLAB and SIMULATION TOOLBOX. The input data used here is given in Table.1 which is given in the appendix and nomenclature section. The performance of the controller is demonstrated for 0.01pu to 0.05pu step load changes forthermal power simulated system of a single area and frequency deviation graph against time as well as mechanical output power deviation graph is given below in the simulation graphs respectively. For finding out the corresponding optimal gains of the controller, **Quadratic Performance Index** and **Linear Quadratic Riccati Principle** theory and its application has been consider and applied. So the optimal input control via feedback linearization for the thermal power system brings back the frequency deviation and mechanical output power deviation graph to desired value or zero deviation value for all the load changes. The figures from figure (2) to figure (11) shows the frequency deviations and total mechanical output power of the thermal power system with time when the load change varies from 0.01pu to the 0.05pu respectively and hence demonstrate the performance of the controller using feedback linearization methods for reheat turbine thermal power system.

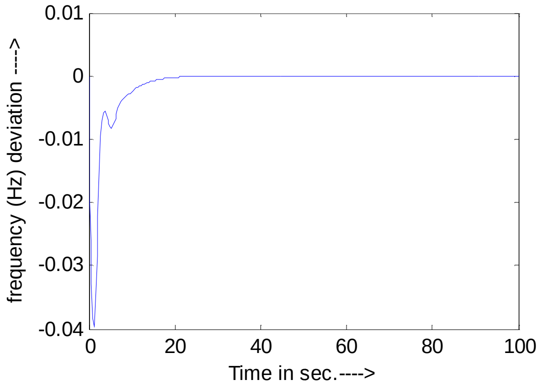

Figure 2. Frequency deviation when the load change is 0.01pu.

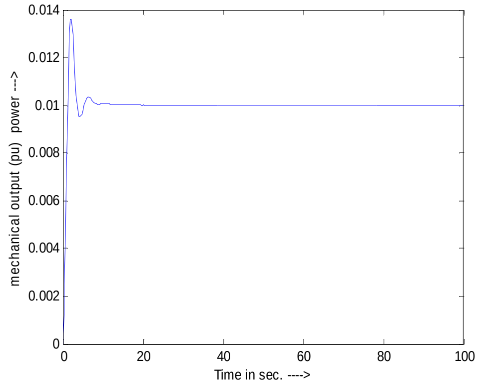

Figure 3 .Mechanical output power when the load change is 0.01pu.

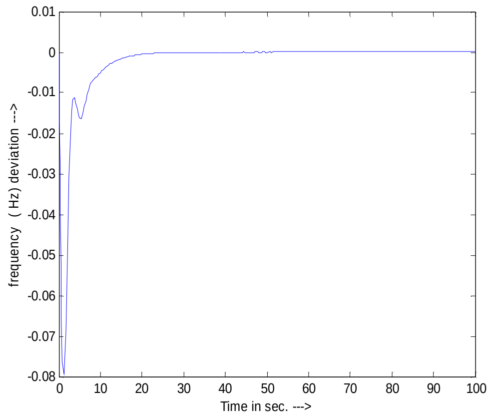

Figure 4. Frequency deviation when the load change is 0.02pu

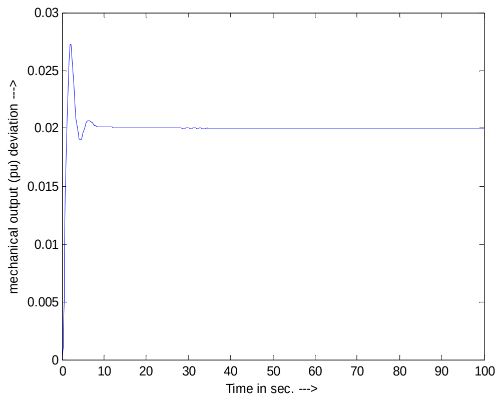

Figure 5. Mechanical output power when the load change is 0.02pu

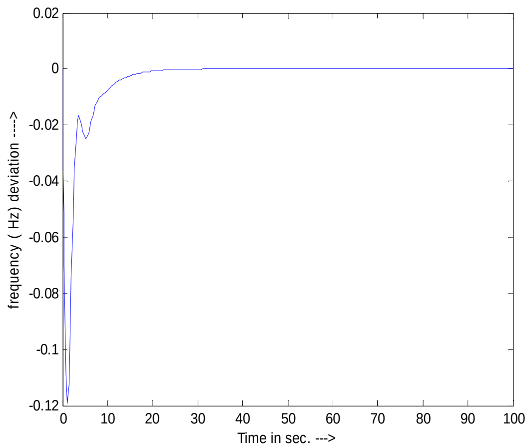

Figure 6 .Frequency deviations when the load change is 0.03pu

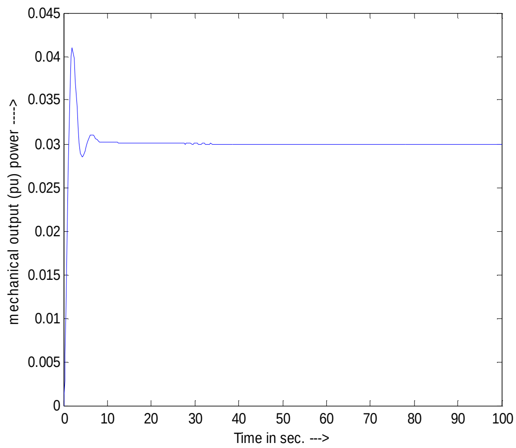

Figure 7. Mechanical output power when the load change is 0.03pu

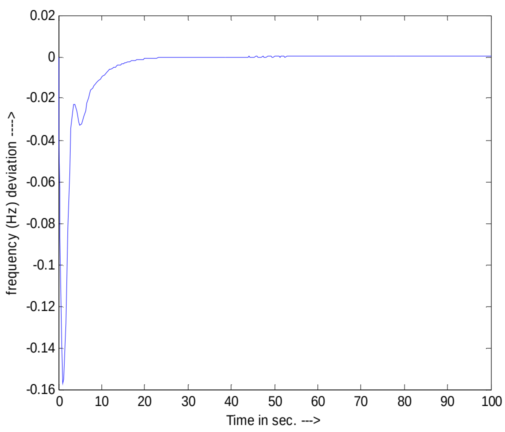

Figure 8 .Frequency deviations when load change is 0.04pu

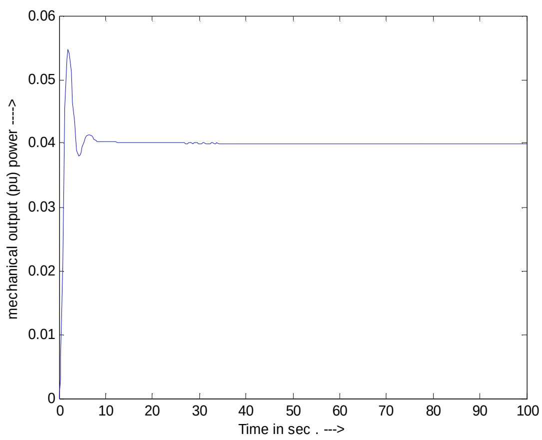

Figure 9. Mechanical output power when the load change is 0.04pu

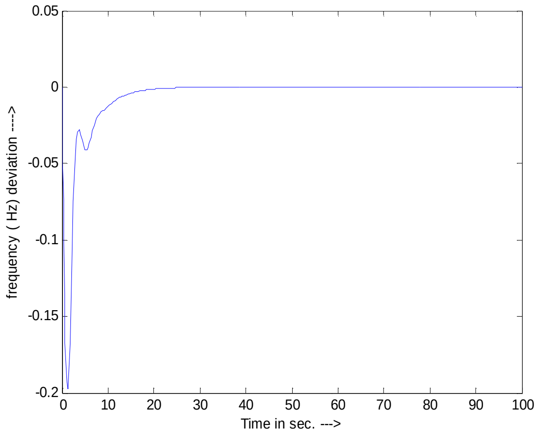

Figure 10. Frequency deviation when load change is 0.05pu

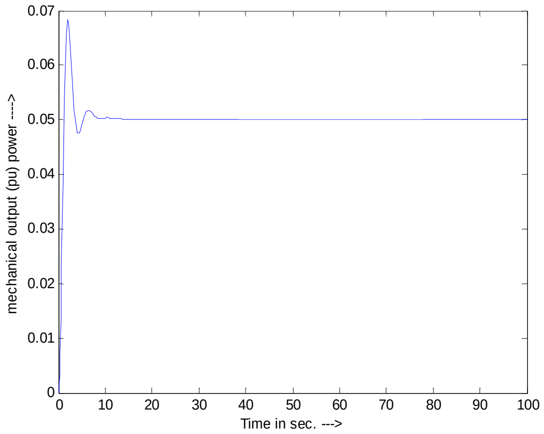

Figure 11 .Mechanical output power when the load change is 0.05pu

5. CONCLUSION

In this paper, we have proposed a **nonlinear feedback control** method for single area **reheat thermal power system**. The basic idea of the feedback linearization approach is to use a control consisting of two components: one component cancels out the plant's nonlinearities and the other controls the resulting linear system. Simulation results have demonstrated that the proposed control strategy ensure the better transient improvement after different load change conditions.

APPENDIX AND NOMENCLATURE

Let us put the appropriate values of the different parameters in the given system equation (20) to equation (30) respectively, we get

$$Z_1 = \Delta\delta \quad \text{(A-1)}$$

$$Z_2 = \Delta\omega \quad \text{(A-2)}$$

$$Z_3 = 6\Delta P_m - 6\Delta P_D - 0.05\Delta\omega \quad \text{(A-3)}$$

$$Z_4 = 19.98\Delta P_g - 20.28\Delta P_m + 0.3\Delta P_D + 0.0025\Delta\omega \quad \text{(A-4)}$$

$$Z_5 = 0.99\Delta P_{g1} - 69.515\Delta P_g + 67.547\Delta P_m - 0.0000125\Delta\omega + 9.99u - 0.015\Delta P_D - 0.4155 \quad \text{(A-5)}$$

Then, the original nonlinear system can be transformed into a linear and controllable system as:

$$\dot{Z}_1 = Z_2 \quad \text{(A-6)}$$

$$\dot{Z}_2 = Z_3 \quad \text{(A-7)}$$

$$Z'_3 = Z_4 \quad \text{(A-8)}$$

$$\dot{Z}_4 = Z_5 \quad \text{(A-9)}$$

$$\dot{Z}_5 = v \quad \text{(A-10)}$$

Then we can write it in compact form as:

$$Z' = Az + Bv \quad \text{(A-11)}$$

Where,

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \text{And} \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Gain vectors are:

$K_1 = 1, K_2 = 3.2361, K_3 = 5.2361, K_4 = 5.2361, K_5 = 3.2361$ are obtained from Algebraic Riccatti Equation and its solutions.(taken when default value of R=1)

Hence control input law is as follows:

$$u = \left\{ \left(-L_f^n h(x) + v \right) \Big/ \left(L_g L_f^{n-1} h(x) \right) \right\} \qquad \text{(A-12)}$$

Where,

$$v = -K_1 Z_1 - K_2 Z_2 - K_3 Z_3 - K_4 Z_4 - K_5 Z_5 \qquad \text{(A-13)}$$

Nomenclature used in this paper, as follows:
$T_g =$ Governor time constant, $T_r =$ Reheat turbine constant, $T_t =$ Turbine time constant, $K_p =$ Gain constant of power system, $K_r =$ Gain of reheat turbine, $\Delta P_g =$ Governor power, $\Delta P_t =$ Turbine power $\Delta P_m =$ Mechanical power, $\Delta P_D =$ Load change

Table1. Input System Parameters

$K_p = 120$	$T_p = 20$	$T_g = 0.08$	$T_t = 0.3$
$R = 2.40$	$K_r = 0.5$	$T_r = 10$	

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