

Fractional-order chaos modelization and sliding mode control in a biological enzyme system

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ABSTRACT

This paper proposes two main contributions to fractional-order modeling and control of biological systems that may exhibit chaotic behavior. First, a fractional-order chaotic model is designed to represent a biological enzyme using bifurcation diagrams and fractional orders tuning inspired by the available integer order model. This new approach improves the biological model by introducing physical properties specific to fractional order systems such as the memory effect, fractal properties, tissue heterogeneity and non-local behavior. Furthermore, this makes the use of a more effective, robust and powerful fractional-order control easier and more natural. The second main contribution is to propose a fractional-order sliding mode surface in order to derive a sliding mode control (SMC) controller that is able to stabilize this fractional-order biological system asymptotically. We successfully performed the stability analysis using the Lyapunov theory. Numerical simulations using MATLAB are given to demonstrate the efficiency of the proposed fractional-order controller with a drastic improvement in convergence time comparatively to the integer-order counterpart.

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1. INTRODUCTION

The study of chaos in biological systems has attracted and fascinated many researchers in the past. Several publications describe complex phenomena that are almost unpredictable or random, but which are governed by highly deterministic mathematical models [1]–[3]. One of their most distinctive properties is their dependency on initial conditions, the change of which leads to very different behavior. This stems from an inescapable fact: biological systems are governed by highly nonlinear patterns between their variables. These behaviors have been observed in many biological systems like population dynamics [4], brain activity [5], heart rhythms [6], and development of certain organisms [7]. This growing research field has important applications in medical sciences for example deal with evolutionary disease processes [8] or to detect and predict sickly chaotic behavior like epileptic seizures and cardiac arrhythmias [9]. This is why much work in the literature has focused on the detection and modeling of biological systems with chaotic behavior [10].

The analysis of biological systems such as cell is usually achieved by measuring the biological behavior and properties of their elements such as proteins. One can use the velocity of displacement and geometry to observe this behavior. One possibility is to use chaos theory to calculate these modifications or cancer development [11].

Another important aspect of this research is the control and synchronization of chaotic biological systems using various strategies like Backstepping control [12]–[14]. This has very interesting applications such as the understanding and diagnosis of several complex phenomena and their treatment, for example, epilepsy [15]. Nowadays, the use of fractional order systems has invaded several sectors of scientific research, notably for their intrinsic properties which are very profitable for the modeling of several physical or biological phenomena and their advantageous properties for the treatment, control and robustness towards changes in certain parameters [16]–[19].

One of the control methods that obtained widespread support among automation engineers is sliding mode control (SMC), due to its robustness and ease of implementation [20], [21]. Furthermore, its stability analysis is straightforward, ensuring that the closed-loop system converges quickly to the desired reference despite disturbances and uncertainties. The specialized literature proposes several variations that improve control performance and signal quality by reducing chatter.

Among these techniques, we can mention: adaptive neuro-fuzzy inference system-based SMC [22], optimized backstepping fuzzy SMC [23], [24], adaptive fuzzy SMC [25], [26], fuzzy SMC [27], super twisting SMC [28], adaptive fuzzy super-twisting SMC [29], robust fractional order controller [30], terminal fractional-order SMC (FOSMC) [31], adaptive particle swarm optimization (PSO) based gain optimization of sliding mode control [32], and robust PSO tuned FOSMC [33].

In this work, we will propose a fractional-order mathematical model to model the behavior of a given enzyme by demonstrating that this model exhibits chaotic behavior for certain values of its parameters. We will then propose a sliding-mode control law to synchronize and control this system, demonstrating its convergence and stability using Lyapunov's theorem extended to fractional-order systems.

The content of this article will be developed as: Section 2 includes some basic concepts of fractional calculus. Section 3 is dedicated to the fractional-order modeling of the biological enzyme system with chaotic behavior. Section 4 defines the proposed fractional-order SMC controller to stabilize the biological chaotic system based on the semi-fractional-order model. Simulation results are presented and discussed in section 5 to validate the proposed controller. Finally, concluding comments are given in section 6.

2. ELEMENTS OF FRACTIONAL CALCULUS

Experimental analysis of several natural phenomena has shown the existence of natural fractional-order patterns. This fact has been confirmed for dielectric polarization impedance, transmission lines, cardiac rhythm, interfaces, spectral density of physical wave [20]. These systems can be represented by differential equations of non-integer order. Fractional integrals and derivatives have been of great interest in the past to illustrious mathematicians who have left us various definitions for these operators, the most popular of which are those of Riemann-Liouville, Caputo and Grünwald-Letnikov [21].

2.1. Basic concepts

The Riemann-Liouville's definition of fractional order integral (RL) of order $\eta > 0$ for a function $x(t)$ is,

$${}_{RL}I_{t_0}^{\eta} x(t) = \frac{1}{\Gamma(\eta)} \int_{t_0}^t (t - \tau)^{\eta-1} x(\tau) d\tau \quad (1)$$

while the RL fractional-order derivative of order $\eta > 0$ is expressed as (2),

$${}_{RL}D_{t_0}^{\eta} x(t) = D^n I^{n-\eta} x(t) \quad (2)$$

here $\Gamma(\cdot)$ represents the Euler's gamma function, with $(n - 1 < \eta < n, n \in \mathbb{N})$.

The Caputo's fractional-order (C) derivative of order $\eta > 0$ is given by (3),

$${}_CD_{t_0}^{\eta} x(t) = \frac{1}{\Gamma(n-\eta)} \int_{t_0}^t (t - \tau)^{n-\eta-1} x^{(n)}(\tau) d\tau \quad (3)$$

where μ is a real number such that: $n - 1 < \eta < n$.

Grünwald-Letnikov (GL) definition is as (4):

$${}_{GL}D_{t_0}^{\eta} x(t) = \lim_{h \rightarrow 0} \sum_{j=0}^k \omega_j^{(\eta)} x(kh - jh) \quad (4)$$

where h is the sampling time, and the coefficients $\omega_j^{(\eta)}$ are computed as (5):

$$\omega_j^{(\eta)} = \frac{(-1)^j \Gamma(\eta+1)}{\Gamma(j+1)\Gamma(\eta-j+1)}, j=0, 1, \dots, k \quad (5)$$

One can remark that this last definition serves also as numerical approximation for the fractional order differentiation by fixing the discrete time $t = kh$.

3. PROPOSED FRACTIONAL-ORDER BIOLOGICAL MODEL

In this paper, a new fractional-order biological model, based on the integer model proposed in [22] which consists of enzyme substrate reaction in both autonomous and non-autonomous case. This system has been extensively analyzed in [11]. The activated enzyme molecules were originally expressed using the following second order model,

$$\begin{cases} \dot{x} = y \\ \dot{y} = \mu(1 - x^2 + \alpha x^4 - \beta x^6)y - x + E \cos(\gamma t) \end{cases} \quad (6)$$

where the variable x is the system's state corresponding to the enzyme concentration, μ is an index of nonlinearity. E is a measure of the external excitation amplitude, and γ is the electromagnetic excitation frequency. α and β are two positive parameters. This model exhibits chaotic behavior for the following parameters values: $\alpha = 2.55$; $\beta = 1.7$; $\gamma = 3.465$; $\mu = 2.001$; $E = 8.27$. The parameters μ and E were determined based on bifurcation diagrams [11].

3.1. Global fractional-order model

In this study, we introduce a fractional-order mathematical model to represent the chaotic behavior of the enzyme. Modeling the biological system using fractional-order differential equations allows for the inclusion of very interesting physical properties (often neglected in integer-order models) such as the memory effect, fractal properties, tissue heterogeneity and non-local behavior. One of the main motivations for this method of describing real systems is the possibility of naturally introducing fractional-order control into a model that is already fractional. This allows us to benefit from the robustness and improved performance properties of these models. Inspired by the integer order model (4) the proposed representation of this physical phenomenon is given as,

$$\begin{cases} x^{q_1} = y \\ y^{q_2} = \mu(1 - x^2 + \alpha x^4 - \beta x^6)y - x + E \cos(\gamma t) \end{cases} \quad (7)$$

where q_1 and q_2 are positive real numbers.

We explored several regions for the valuation of the different parameters of the model by fixing the parameters α , β , γ and μ to their initial values in the integer-order model (6):

$$\alpha = 2.55; \beta = 1.7; \gamma = 3.465; \mu = 2.001.$$

Thus, we were able to obtain chaos for the fractional orders $(q_1, q_2) = (0.98, 0.99)$.

Figure 1 illustrates the chaotic behavior of the fractional-order enzyme model for the external excitation parameter E value 8.27. Figure 1(a) represents the phase plane behavior and Figure 1(b) gives the time series plot of the biological system. Figure 2 shows the system response for E equal to 11.40. Figure 2(a) represents the phase plane behavior and subfigure 2(b) gives the time series plot of the biological system. Figure 3 presents the chaotic response of the fractional-order enzyme model for the external excitation parameter E value 17.50. Figure 3(a) represents the phase plane behavior and Figure 3(b) illustrates the time series plot of the biological system.

3.2. Semi-fractional-order model

The chaotic behavior is also obtained for the fractional order values $(q_1, q_2) = (1, 1.2)$ leading to the fractional-order model:

$$\begin{cases} \dot{x} = y \\ D^q y = \mu(1 - x^2 + \alpha x^4 - \beta x^6)y - x + E \cos(\gamma t) \end{cases} \quad (8)$$

where the external excitation parameter $E=8.27$. The system behavior is illustrated in Figure 4 for the fractional derivative orders $(q_1, q_2)=(1, 1.2)$: Figure 4(a) presents the phase plane behavior and Figure 4(b) the time series plot of the biological system. These different possibilities and chaotic behaviors make it possible to represent varied situations and behaviors of the biological enzyme and make it possible to study new situations that can arise in the reality of the biological body.

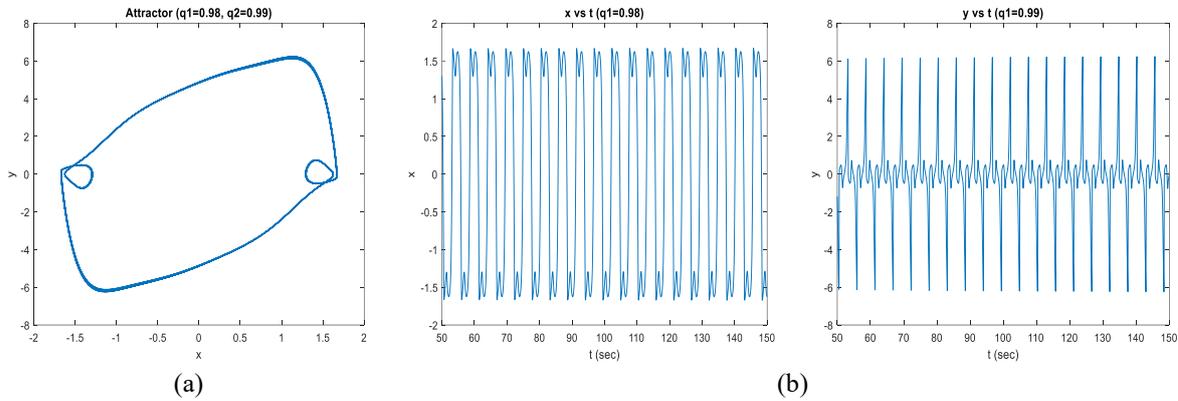


Figure 1. Fractional-order chaotic behavior with $(q_1, q_2)=(0.98, 0.99)$ and the parameter $E = 8.27$: (a) phase plane behavior and (b) time series plot of the biological system

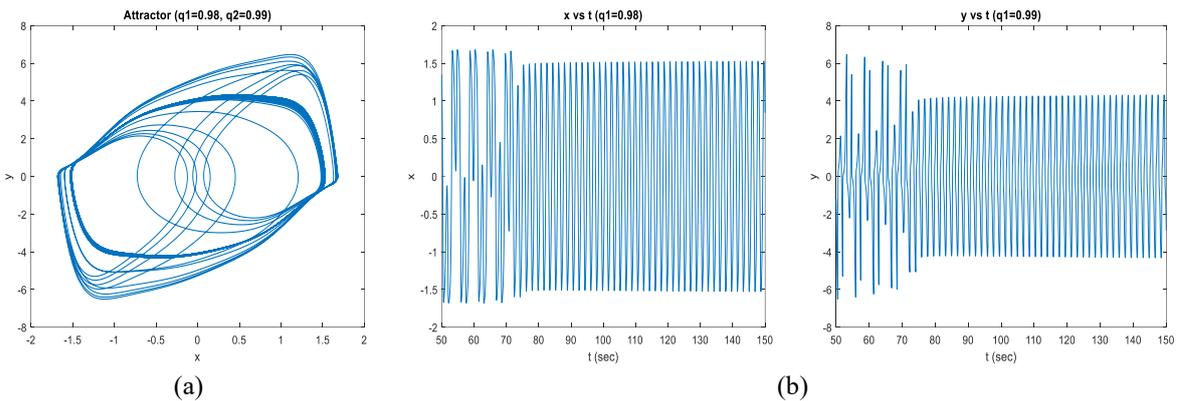


Figure 2. Fractional-order chaotic behavior with $(q_1, q_2)=(0.98, 0.99)$ and the parameter $E = 11.40$: (a) phase plane behavior and (b) time series plot of the biological system

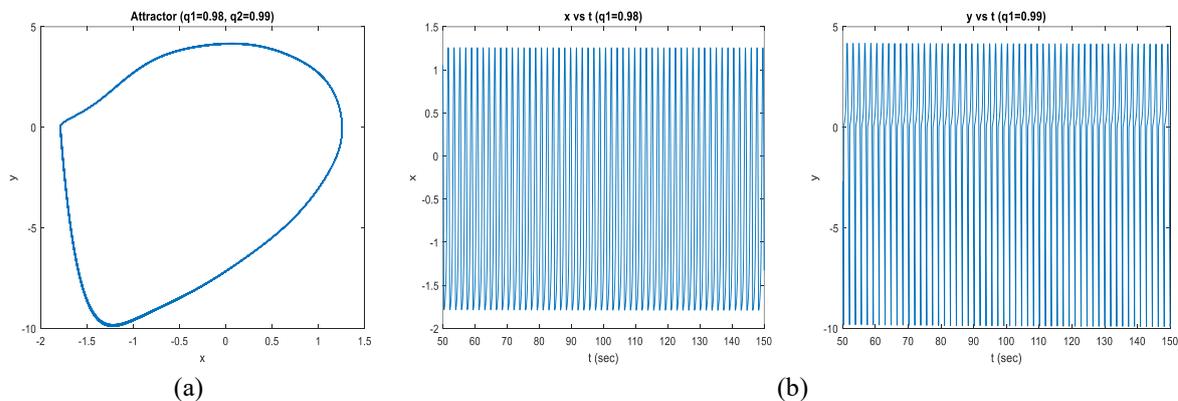


Figure 3. Fractional-order chaotic behavior with $(q_1, q_2)=(0.98, 0.99)$ and the parameter $E = 17.50$: (a) phase plane behavior and (b) time series plot of the biological system

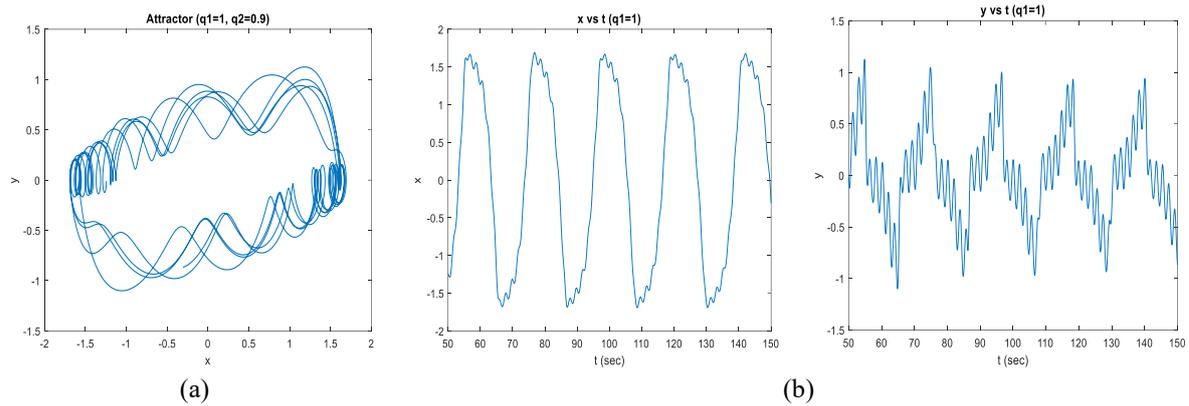


Figure 4. Fractional-order chaotic behavior with $(q_1, q_2)=(1, 1.2)$ and the parameter $E = 8.27$:
 (a) phase plane behavior and (b) time series plot of the biological system

4. FRACTIONAL-ORDER SMC CONTROL DESIGN

In the medical field, the stabilization of such chaotic phenomena is an absolute necessity, hence the interest in this research. The reason is that a good chaos control may be used for diagnostic as well as in modern therapies for many complex diseases. One can cite the problem information alteration of the neuron within the brain area [23], [24]. This motivates the present research work, which aims to stabilize this kind of chaotic behavior in a type of biological enzyme crucial for the living body. The control action is performed using appropriate molecules that can block or promote enzyme function.

The augmented fractional-order enzyme model (8) is expressed as (9):

$$\begin{cases} x^{q_1} = y \\ y^{q_2} = \mu(1 - x^2 + \alpha x^4 - \beta x^6)y - x + E \cos(\gamma t) + u \end{cases} \tag{9}$$

where $q_1=1$ and $q_2=q$ is the fractional derivative order, and the system can be rewritten as (10):

$$\begin{cases} x^{(1)} = y \\ y^q = \mu(1 - x^2 + \alpha x^4 - \beta x^6)y - x + E \cos(\gamma t) + u \end{cases} \tag{10}$$

Our objective is to design the control $u(t)$ such that the states x and y can converge to the origin. Now, we need to design a SMC law $u(t)$ to achieve the asymptotic stability of the fractional-order system dynamics (10) [25]. Let the fractional-order sliding surface $s(t)$ be defined as (11):

$$s(t) = cD^{-q}x(t) + y(t) \tag{11}$$

where $c > 0$.

The equivalent sliding mode control is obtained by taking the fractional order derivative of (10) as:

$$s^q(t) = cD^{-q+q}x(t) + y^q(t) = 0 \tag{12}$$

thus,

$$cx(t) + \mu(1 - x^2 + \alpha x^4 - \beta x^6)y - x + E \cos(\gamma t) + u = 0 \tag{13}$$

$$u_{eq}(t) = -\mu[1 - x^2 + \alpha x^4 - \beta x^6]y + (1 - c)x - E \cos(\gamma t) \tag{14}$$

choosing the following switch control law

$$u_{sw} = -KD^{q-1}tanh(s) \tag{15}$$

we obtain,

$$u(t) = u_{eq}(t) + u_{sw}(t) \tag{16}$$

and,

$$u(t) = -\mu[1 - x^2 + \alpha x^4 - \beta x^6]y + (1 - c)x - E \cos(\gamma t) - KD^{q-1} \tanh(s) \quad (17)$$

The sliding mode motion occurrence is permitted by an adequate control law $u(t)$ is designed in (17).

Then, we can state the following main result:

Theorem:

Consider the fractional order biological chaotic system (9). Then, the sliding mode control law (17) guaranties the converge of the enzyme system trajectories to the sliding surface $s(t) = 0$ and the state variables vanish asymptotically.

Proof of the theorem:

Let us consider the following Lyapunov candidate function:

$$V = \frac{1}{2} s^2 \quad (18)$$

Its time derivative is given as (19):

$$\begin{aligned} \dot{V} &= s\dot{s} = sD^{1-q}\{s^q\} \\ &= sD^{1-q}\{cD^q(D^{-q}x) + y^q\} \\ &= sD^{1-q}\{cx + \mu(1 - x^2 + \alpha x^4 - \beta x^6)y - x + E \cos(\gamma t) + u\} \end{aligned} \quad (19)$$

where

$$u(t) = u_{eq}(t) + u_{sw}(t) \quad (20)$$

The control $u_{eq}(t)$ is chosen as (21):

$$u_{eq}(t) = -\mu(1 - x^2 + \alpha x^4 - \beta x^6)y + (1 - c)x - E \cos(\gamma t) \quad (21)$$

Thus \dot{V} can be expressed as:

$$\dot{V} = sD^{1-q}\{u_d\} \quad (22)$$

choosing:

$$u_{sw} = -[K \operatorname{sgn}(s)]^{q-1} \quad (23)$$

where K is a strictly positive number. We have from (22),

$$\begin{aligned} \dot{V} &\leq -sD^{1-q}\{[K \operatorname{sgn}(s)]^{q-1}\} \\ &\leq -K|s| \\ &\leq 0 \end{aligned} \quad (24)$$

This concludes the proof.

As the sign function is discontinuous at 0, we replace u_{sw} by (25),

$$u_{sw} = -KD^{q-1} \tanh(s) \quad (25)$$

In fact, u_{sw} is designed using $\tanh(s)$ as a continuous approximation of the sign function to eliminate the chattering phenomenon in input signals. Thus, the final formula for the control law is given by (26):

$$u(t) = u_{eq}(t) + u_d(t) = -\mu(1 - x^2 + \alpha x^4 - \beta x^6)y + (1 - c)x - E \cos(\gamma t) - KD^{q-1} \tanh(s) \quad (26)$$

5. RESULTS AND DISCUSSION

Consider the semi-fractional-order model of the enzyme biological system given by (10) with the parameters values:

$$\alpha = 2.55; \beta = 1.7; \gamma = 3.465; \mu = 2.001, E = 8.27; q = 1.2.$$

leading to the model:

$$\begin{cases} x^{(1)} = y \\ y^{(1.2)} = \mu(1 - x^2 + \alpha x^4 - \beta x^6)y - x + E \cos(\gamma t) + u \end{cases} \quad (27)$$

Under these conditions, this system exhibits chaotic behavior as illustrated in Figure 5. Enzymes have to be carefully controlled because they regulate and guide the metabolism of the body cells. By applying the fractional-order SMC control law (19) at the time $t=70$ s, we obtain the numerical simulation results illustrated in Figures 6 to 8.

Figure 6 presents the states behavior and convergence to the origin in a finite time. The sliding surface s is illustrated in Figure 7 and clearly is set to zero after the application of the control signal at $t=70$ s. We observe that the system stabilizes rapidly after the launch of the control signal, with zero steady-state error and the absence of any oscillation. Figure 8 presents the control signal and shows a slight chattering phenomenon after the control beginning instant. Fortunately, the specialized literature offers several techniques for reducing, or even eliminating, this chattering phenomenon, such as higher-order SMC, adaptive gain scheduling, super-twisting SMC. In order to highlight the level of performance achieved by the proposed control scheme, we will make use of the integral of squared error (ISE) J as,

$$J = \sqrt{\int_{t_c}^{t_f} (x^2 + y^2) dt} \quad (28)$$

where t_c is the time of control application and t_f is the simulation time duration. Table 1 illustrates the performance of the proposed SMC control system (response time τ_r and ISE).

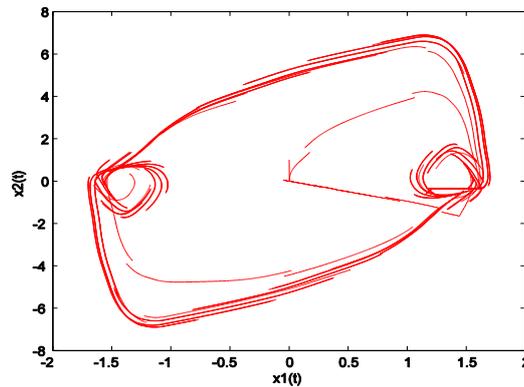


Figure 5. Fractional-order chaotic attractor

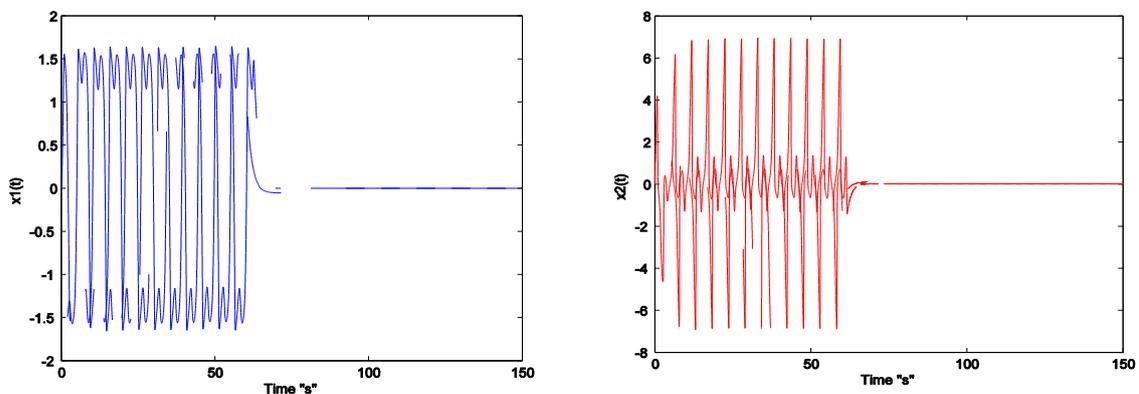


Figure 6. Controlled state variables of the biological system

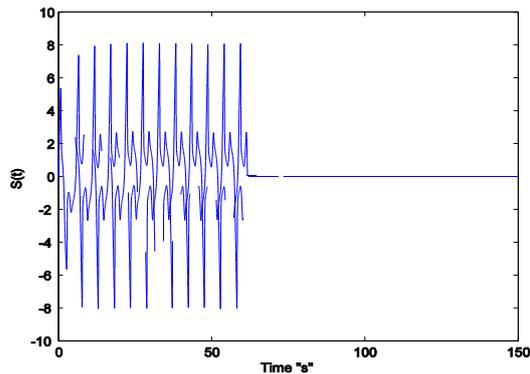
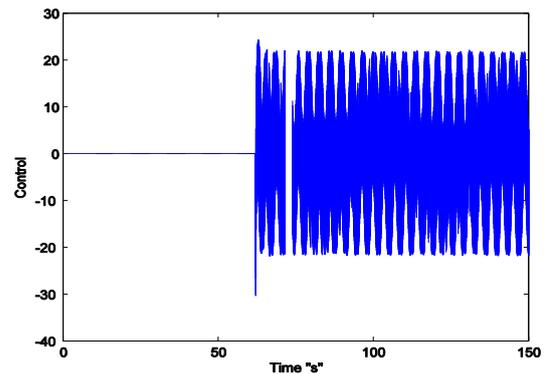
Figure 7. Sliding surface $S(t)$ 

Figure 8. Fractional-order SMC control signal

Table 1. Control system performance

| t_c : Time of control application (s) | t_f : Simulation time duration (s) | τ_r : Response time (s) | J : (ISE) |
|---|--------------------------------------|------------------------------|-------------|
| 70 | 150 | 5 | 9.2 |

The simulation results demonstrate the efficiency of the proposed SMC control method to achieve the stabilization of the fractional-order chaotic biological enzyme system. The proposed control made it possible to drastically reduce the convergence time of the system compared to the integer order model and SMC control. However, it is important to mention the limitations of this study on a biological enzyme due to sensitivity to parameters, unmodeled dynamics and other biological interactions.

6. CONCLUSION

In this study, the modelization of a biological enzyme system, which can exhibit chaotic behavior using a fractional-order model, is proposed and a fractional-order sliding mode control is designed for its stabilization. The proposed fractional-order model is inspired from the original integer order model presented in the literature. This innovation has made it possible to introduce several features that are more realistic and advantageous properties to the proposed model, such as the memory effect, faster and more complex dynamics, and a natural robustness against noise and disturbances.

By applying a fractional-order SMC controller, we were able to guarantee the asymptotic stabilization of the designed biological enzyme model using the Lyapunov theory. Simulation results in a MATLAB environment illustrated the efficiency of the proposed SMC controller to stabilize the biological system in a finite time. Future work will involve applying this control strategy to a more complex, multi-enzyme network and implementing it on a real-time system for experimental validation.

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AUTHOR CONTRIBUTIONS STATEMENT

Sakina Benrabah contributed to methodology, software, validation, investigation, data curation, writing-original draft and visualization. Bachir Bourouba contributed to software, investigation, data curation and visualization. Samir Ladaci contributed to conceptualization, methodology, software, validation, formal analysis, investigation, resources, data curation, writing (original draft and review & editing), visualization, supervision, project administration, and funding acquisition. All authors have read and approved the final manuscript.

| Name of Author | C | M | So | Va | Fo | I | R | D | O | E | Vi | Su | P | Fu |
|-----------------|---|---|----|----|----|---|---|---|---|---|----|----|---|----|
| Sakina Benrabah | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | | ✓ | ✓ | | | | ✓ | |
| Bachir Bourouba | | ✓ | ✓ | | ✓ | ✓ | | ✓ | | | ✓ | | | |
| Samir Ladaci | ✓ | | ✓ | ✓ | ✓ | | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |

| | | |
|-----------------------|-------------------------------|----------------------------|
| C : Conceptualization | I : Investigation | Vi : Visualization |
| M : Methodology | R : Resources | Su : Supervision |
| So : Software | D : Data Curation | P : Project administration |
| Va : Validation | O : Writing -Original Draft | Fu : Funding acquisition |
| Fo : Formal analysis | E : Writing - Review &Editing | |

CONFLICT OF INTEREST STATEMENT

Authors state no conflict of interest.

INFORMED CONSENT

We have obtained informed consent from all individuals included in this study.

ETHICAL APPROVAL

This research does not require ethical approval as it does not involve human participants, animal subjects, or sensitive data.

DATA AVAILABILITY

The data supporting this study's findings are available from the corresponding author, RB, upon reasonable request.

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