

Analytic algebraic Riccati solution for a robust control system: application to 2-DOF arm robot

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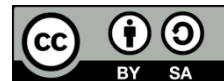
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ABSTRACT

An analytic solution to the Riccati algebraic equation has been investigated by employing eigenvalue–eigenvector techniques combined with the Gram–Schmidt orthogonality process. An analytic solution to the Riccati algebraic equation has been investigated by employing eigenvalue–eigenvector techniques combined with the Gram–Schmidt orthogonalization process. The applied method is used to improve robust control of second and third-order state-dependent systems by handling nonlinearities. An H_∞ controller is designed in this context via backstepping technique to enhance robustness and reduce computational effort. The effectiveness of this method has been demonstrated on a two-degree-of-freedom (2-DOF) robotic manipulator arm. Simulation results validate the performance of the controller, showing improved tracking accuracy, disturbance rejection, and overall system stability, thereby confirming the efficiency and applicability of the combined analytic Riccati algebraic equation and H_∞ backstepping approach for nonlinear robotic systems.

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1. INTRODUCTION

A Hamiltonian system is a mathematical formalism to describe the evolution equations of a physical system. They are characterized by the existence of a simplistic structure on a smooth even dimensional manifold [1]. This is not only a matter of convenience but also a powerful tool for finding invariants of the motion, and a fundamental feature of the Hamiltonian formulation [2], [3]. The Hamiltonian optimal control theory was developed by Lev Pontryagin as part of his maximum principle.

Note that Lyapunov equations are most useful in system analysis while algebraic Riccati equation (ARE) are most useful in control system synthesis; particularly in H_2 and H_∞ optimal control. The ARE with the Invariant graph subspaces under the Hamiltonian matrix has been well investigated by [4], [5]. The numerical solutions and conditioning of AREs have been also treated in [6] and developed in [7]. The Riccati equation, edited by Abou-Kandil *et al.* [8], is a succinct overview on the theory and applications of matrix Riccati equations in control and systems theory, including continuous and discrete-time formulations, and numerical solution methods. Ogata [9] presents the classical linear quadratic regulator (LQR) design based on the ARE. Note that the controller is only locally optimal, not for the full nonlinear system. It cannot handle disturbances or uncertainties without additional methods. In study [10] the linearized models may not

capture the full nonlinear dynamics of robotic manipulators, reducing accuracy in some operating regions. Rigatos *et al.* [11] proposes a control method for multi-joint robotic manipulators that works by simplifying the robot's complex nonlinear behavior into a linear form at each instant. It then solves an ARE to determine the optimal control gains needed to keep the system stable. However, if the robot's model is not perfectly accurate because of uncertainties in its parameters, the controller's performance can lead to instability. Additionally, the controller needs to achieve a compromise between tracking the desired motion quickly and accurately, while still remaining robust to disturbances and avoiding undesired oscillatory behavior.

In order to overcome certain difficulties related to the strategy and control weaknesses, Long [12] proposes a hybrid control strategy for robotic manipulators that combines Riccati equation-based gain design (LQR/ARE), sliding mode control (SMC) and adaptive observer for state estimation. From difficulties due to the complexity of the controller mathematically and computationally, additionally multiple parameters of observer and SMC gains must be carefully tuned. Roveda and Piga [13] shows the computational demands of solving the SDRE online pose a challenge, as real-time updating of control gains is required for effective force control. Overall, the paper demonstrates how SDRE-based variable impedance control can address these difficulties, enabling robust, sensorless force-tracking in dynamic robotic manipulation tasks. Çimen [14] Explains how state-dependent Riccati equation (SDRE) converts nonlinear systems into a pseudo-linear form, allowing real-time nonlinear optimal control. However, the difficulties arise in parameterization dependence; stability is not guaranteed globally; with complexity in tuning the weighting matrices. Xin and Balakrishnan [15] develop an SDRE-based control approach that incorporates robustness to handle model uncertainties and disturbances, improving stability and tracking performance in nonlinear robotic manipulators. Nekoo [16] proposes a model-reference adaptive SDRE controller for nonlinear uncertain systems and applies it specifically to regulation and tracking of free-floating space manipulators. On the other hand, the model uncertainties are not easy to handle; and the computational complexity associated with the online SDRE solution is a significant challenge. Shawky *et al.* [17], [18] presents an SDRE-based nonlinear H_∞ control scheme for flexible manipulators, enhancing robustness against disturbances while reducing vibrations and improving tracking performance. The problem arises in handling flexible-link vibrations and the computational effort for nonlinear SDRE, effectively handling vibration suppression and parameter sensitivity remains a difficult task. Korayem and Nekoo [19] develops an SDRE-based control method for time-varying nonlinear manipulators. The challenge lies in maintaining stability under nonlinear variations; computational complexity and sensitivity to parameter uncertainties. Moreover, Hoang and Khang [20] presents an adaptive Riccati-based control method for robotic manipulators, addressing nonlinearities and parameter uncertainties to ensure accurate trajectory tracking and robust system performance. The work by Saleem *et al.* [21] has several challenges. First, under-actuated systems are hard to control because there are fewer inputs than movements. The robot's nonlinear behavior also makes control more difficult. The adaptive weight adjustment needs careful tuning; in addition, the method requires high computation, which can limit real-time use. It can also be sensitive to noise and disturbances.

However, when the state number is important it is not easy to find the analytic ARE solution for a state dependent coefficient, since it is difficult to find eigenvalues-eigenvectors values, especially when the state number is higher. An analytical method for calculating eigenvalues-eigenvectors of the diffusion tensor directly from the diffusion tensor elements has been examined by [22].

Since the eigenvalues of any matrix are the same as those of its transpose and the eigenvalues of a matrix are the reciprocals of those of its inverse, it could be concluded that the eigenvalues of Hamiltonian matrix H can be written as stable part and unstable part with only sign changing. The stable eigenvalues of the matrix are related to the dynamics of the closed-loop optimal control system. Therefore, the unique stabilizing solution can be obtained by constructing an invariant subspace associated with the stable eigenvalues of the Hamiltonian matrix H [23]. Hence the Hamiltonian matrix has been introduced in this context to analyze the stabilized solution. Fortunately for two degrees (state number $n = 2$) and three degrees ($n = 3$) of freedom, there are always possibilities to find analytic eigenvalues and eigenvectors, hence the analytic solution of ARE can be computed. The derived controller then combines the attractive features of H_∞ optimal controller and the advantages of the backstepping technique. The backstepping technique used with H_∞ theory has been designed by breaking down complex nonlinear systems into smaller subsystems of two or three states. Performance issues of the controller are illustrated in a simulation study made for a 2-DOF system with state dependent coefficients. The paper is organized as follows: introduction illustrated in section 1. Hamiltonian matrix formalism in section 2. Eigenvalues and eigenvectors illustrated in section 3. Gram-Schmidt Flow chart in section 4. The application to two robot arms is illustrated in section 5. Finally, comments with a conclusion are presented.

2. HAMILTONIAN MATRIX

2.1. Definition 1

Let the square matrix $J \in R^{2n \times 2n}$ defined by $J = \begin{bmatrix} 0_n & I_n \\ -I_n & 0_n \end{bmatrix}$, with $0_n \in R^{n \times n}$ is a zero matrix, $I_n \in R^{n \times n}$ is the identity matrix, then a matrix $H \in R^{2n \times 2n}$ is called Hamiltonian if JH is symmetric, so: $H^T J + JH = 0$. Note that: $J^T = -J$.

2.2. Proposition 1

Let H a Hamiltonian and $p_H(x)$ is the characteristic polynomial of the matrix H , then: $p_H(x) = p_H(-x)$.

2.3. Definition 2

Let the dynamic system with a state dependent coefficient be:

$$\dot{x} = A(x)x + B(x)u + G(x)w$$

and the $2n \times 2n$ hamiltonian matrix be presented as:

$$H = \begin{bmatrix} A & C \\ Q & -A^T \end{bmatrix}$$

with $C = BR^{-1}B^T$, Q is a symmetric matrix ($Q^T = Q$) and R is a diagonal matrix. Let the columns of $[P_{11}^T, P_{21}^T]^T$, $P_{11}, P_{21} \in R^{n \times n}$ span a H -invariant, n -dimensional, then the following equation holds:

$$\begin{bmatrix} A & C \\ Q & -A^T \end{bmatrix} \begin{bmatrix} P_{11} \\ P_{21} \end{bmatrix} = \begin{bmatrix} P_{11} \\ P_{21} \end{bmatrix} Z, \quad Z \in R^{n \times n} \quad (1)$$

with P_{11} is assumed nonsingular, we obtain from (1):

$$AP_{11} + CP_{21} = P_{11}Z$$

$$P_{11}^{-1}AP_{11} + P_{11}^{-1}CP_{21} = Z$$

and

$$QP_{11} - A^T P_{21} = P_{21}Z = P_{21}(P_{11}^{-1}AP_{11} + P_{11}^{-1}CP_{21})$$

$$P_{21}P_{11}^{-1}AP_{11} + P_{21}P_{11}^{-1}CP_{21} - QP_{11} + A^T P_{21} = 0$$

$$-P_{21}P_{11}^{-1}A + P_{21}P_{11}^{-1}CP_{21}P_{11}^{-1} - Q - A^T P_{21}P_{11}^{-1} = 0$$

setting $-P_{21}P_{11}^{-1} = P$ we get:

$$PA + A^T P - PBR^{-1}B^T P + Q = 0 \quad (2)$$

The solution of the ARE is then obtained, and P is symmetric and stabilizing (2). Since the matrix H is real, it can be shown that the solution $P = -P_{21}P_{11}^{-1}$ is also real. Hence the following theorem hold:

Theorem 2.1: Suppose the pair (A, B) is controllable and the pair (Q, A) is observable. We assume that Q is positive semidefinite and $C = BR^{-1}B^T$, where R is positive definite.

1. Then the $2n \times 2n$ Hamiltonian matrix $\begin{bmatrix} A & C \\ Q & -A^T \end{bmatrix}$ has no pure imaginary eigenvalues. If λ is an eigenvalue of H , then $-\lambda$ is also an eigenvalue of H . Thus H has n eigenvalues in the open left half plane and n eigenvalues in the open right half plane.
2. If the $2n \times n$ matrix $\begin{bmatrix} P_{11} \\ P_{21} \end{bmatrix}$ has columns that comprise a basis for the invariant subspace of H associated with the n eigenvalues of H in the left half plane (the stable invariant subspace), then P_{11} is invertible and $P = -P_{21}P_{11}^{-1}$ is a solution to the algebraic Riccati equation. moreover P is symmetric and positive definite and the input.

$$u(t, x) = -R^{-1}B(x, t)P(x, t)x(t)$$

minimizes the cost function:

$$J = \int_{t_0}^{\infty} (x^T(t)Qx(t) + u^T(t)Ru(t))dt$$

For proof one can refer to [24].

3. EIGENVALUES-EIGENVECTORS FORMULATION

3.1. Eigenvalues

Case1: Let H is a 4×4 matrix i.e A is a 2×2 matrix. Then its characteristic equation can be expressed as:

$$p(\lambda) = \det(H - \lambda I)$$

where \det is the determinant and λ is the eigenvalues of H . For $n = 4$. The Cayley-Hamilton theorem [25] state that:

$$p(\lambda) = \lambda^4 + c_2\lambda^2 + c_0 = 0$$

The coefficients of λ can be directly written in terms of complete Bell polynomials by comparing this expression with the generating function of the Bell polynomial. Note that Bell polynomials provide a powerful tool in combinatorics and analysis, particularly for representing set partitions and for simplifying the computation of higher-order derivatives in nonlinear systems [26].

Differentiation of this expression with respect to λ allows the determination of the generic coefficients of the characteristic polynomial for general n , as determinants of $m \times m$ matrices, then:

$$c_{n-m} = \frac{(-1)^m}{m!} \det(C_m) \quad (3)$$

$$C_m = \begin{vmatrix} \text{tr}(H) & m-1 & 0 & \dots & 0 \\ \text{tr}(H^2) & \text{tr}(H) & m-2 & \dots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \text{tr}(H^{(m-1)}) & \text{tr}(H^{(m-2)}) & \dots & \dots & 1 \\ \text{tr}(H^m) & \text{tr}(H^{(m-1)}) & \dots & \dots & \text{tr}(H) \end{vmatrix}$$

with $\text{tr} = \text{trace}$ and $\text{tr}(H^k) = 0$ for $k = \text{odd}$ hence:

$$c_2 = -\frac{1}{2} \text{tr}(H^2)$$

$$c_0 = \det(H) = \left(\frac{1}{8} \text{tr}^2(H^2) - \frac{1}{4} \text{tr}(H^4) \right)$$

if T is a linear transformation from a vector space V over a field F into itself and v is a vector in V that is not the zero vector, then v is an eigenvector of T if $T(v)$ is a scalar multiple of v . This condition can be written as the equation:

$$T(v) = \lambda v$$

where λ is a scalar in the field F , known as the eigenvalue, characteristic value, or characteristic root associated with the eigenvector v . The solution of the characteristic equation can be represented as:

$$\lambda_{1,2,3,4} = \pm \left(\frac{1}{4} \text{tr}(H^2) \pm \frac{1}{2} \sqrt{\text{tr}(H^4) - \frac{1}{4} \text{tr}^2(H^2)} \right)^{\frac{1}{2}}$$

Case2: Let H is a 6×6 matrix *i.e.*, A is a 3×3 matrix. Then its characteristic equation can be expressed as:

$$\det(H - \lambda I) = \lambda^6 + c_4\lambda^4 + c_2\lambda^2 + c_0 = 0$$

c_i coefficients are defined by the determinant of the (3) defined above:

$$c_0 = -\frac{1}{48} \text{tr}^3(H^2) + \frac{1}{8} \text{tr}(H^2)\text{tr}(H^4) - \frac{1}{6} \text{tr}(H^6)$$

$$c_2 = \frac{1}{8} \text{tr}^2(H^2) - \frac{1}{4} \text{tr}(H^4)$$

$$c_4 = -\frac{1}{2} \text{tr}(H^2)$$

Let $\Lambda = \lambda^2$, then the characteristic equation can be replaced by:

$$\Lambda^3 + c_4\Lambda^2 + c_2\Lambda + c_0 = 0$$

The solution of the eigenvalues equation is as follow [22]:

$$\phi = \frac{\arccos\left(\frac{s}{v^2}\right)}{3}$$

$$v = \left(\frac{-c_4}{3}\right)^2 - \left(\frac{c_2}{3}\right) = -\frac{1}{72} \text{tr}^2(H^2) + \frac{1}{12} \text{tr}(H^4)$$

$$s = \left(\frac{-c_4}{3}\right)^3 + \frac{c_4 c_2}{6} - \frac{c_0}{2}$$

$$= \frac{1}{216} \text{tr}^3(H^2) - \frac{1}{24} \text{tr}(H^2)\text{tr}(H^4) + \frac{1}{12} \text{tr}(H^6)$$

The sorted eigenvalues: $(\lambda_1 = -\lambda_6) > (\lambda_2 = -\lambda_5) > (\lambda_3 = -\lambda_4)$ are then represented as follow [27]:

$$\lambda_{1,6} = \pm \sqrt{-\frac{c_4}{3} + 2v^{\frac{1}{2}}\cos(\phi)}$$

$$\lambda_{2,5} = \pm \sqrt{-\frac{c_4}{3} - 2v^{\frac{1}{2}}\cos\left(\frac{\pi}{3} + \phi\right)} \quad (4)$$

$$\lambda_{3,4} = \pm \sqrt{-\frac{c_4}{3} - 2v^{\frac{1}{2}}\cos\left(\frac{\pi}{3} - \phi\right)}$$

3.2. Eigenvectors

Given a Hamiltonian $2n \times 2n$ square matrix H of real numbers, an eigenvalue λ_i and its associated generalized eigenvector v_i are a pair obeying the relation:

$$(H - \lambda_i I)V_{\lambda_i} = 0$$

Given the square matrix H , by minor of an element a_{ij} , we mean the value of the determinant obtained by deleting the i^{th} row and j^{th} column of H matrix. It is denoted by H_{ij} . In order to find the i^{th} eigenvector V_{λ_i} computed for λ_i , we compute the determinants of the minors related to the i^{th} row of the square matrix, so we have to erase out a row and a column one by one at the time. The following steps are used to compute minors from a matrix:

Let the function:

$$H - \lambda_i I = \begin{bmatrix} A - \lambda_i I & BR^{-1}B^T \\ Q & -A^T - \lambda_i I \end{bmatrix}$$

Then the eigenvector will be represented as:

$$V_{\lambda_i} = [H_{i1} \ H_{i2} \ \dots \ H_{im}]^T$$

with $m = 2n$, $n = 1..2$, $i = 1..4$ for $H \in \mathbb{R}^{4 \times 4}$ and $n = 1..3$, $i = 1..6$ for $H \in \mathbb{R}^{6 \times 6}$.

4. ORTHOGONALITY AND MODIFIED GRAM–SCHMIDT ORGANIGRAM

The Gram–Schmidt process is a method for orthogonalizing a set of vectors in an inner product space, most commonly the Euclidean space R^n equipped with the standard inner product. The Gram–Schmidt process takes a finite, linearly independent set $S = \{V_{\lambda_1}, \dots, V_{\lambda_k}\}$ for $k \leq n$ and generates an orthogonal set $S' = \{U_{\lambda_1}, \dots, U_{\lambda_k}\}$ that spans the same k -dimensional subspace of R^n as S .

Definition 4.1

A set of vectors $\{V_i, 1 \leq i \leq n\}$ is orthogonal if $V_i \cdot V_j = 0$ whenever $i \neq j$. Hence

$$U_{\lambda_1} = V_{\lambda_1}$$

$$U_{\lambda_k} = V_{\lambda_k} - \sum_{j=1}^{k-1} \text{proj}_{U_{\lambda_j}}(V_{\lambda_k})$$

with

$$\text{proj}_{U_{\lambda_j}}(V_{\lambda_k}) = \frac{U_{\lambda_j} V_{\lambda_k}^T}{U_{\lambda_j}^T U_{\lambda_j}} U_{\lambda_j} = \alpha_{jk} U_{\lambda_j}$$

taking U_{λ} the matrix represented by:

$$U_{\lambda} = [U_{\lambda_1} \ U_{\lambda_2} \ \dots \ U_{\lambda_n}]$$

Note the eigenvalues have been sorted in an ascending way, so the eigenvectors will also be. Then U_{λ} can be represented in a matrix form as:

$$U_{\lambda} = \begin{bmatrix} P_{11n \times n} & P_{12n \times n} \\ P_{21n \times n} & P_{22n \times n} \end{bmatrix}$$

Finally, the positive definite solution for Riccati equation will be presented as:

$$P(x) = -P_{21}(x)P_{11}^{-1}(x)$$

5. APPLICATION

We consider a two-degree-of-freedom (2-DOF) planar robotic arm consisting of two rigid links: l_1 and l_2 with masses m_1 , and m_2 respectively. The two revolute joints are q_1 and q_2 with a moment of inertia I_1, I_2 respectively. The end-effector moves in a 2D plane (XY plane) Figure 1, with the goal of designing a robust controller to track desired trajectories. To simplify the dynamic model while retaining essential nonlinearities, the following assumptions are made:

- Mass distribution: The mass of each link is concentrated at its tip, so the center-of-mass distances are set to the link lengths: $r_1 = l_1$ and $r_2 = l_2$
- Neglect rotational inertia: The moments of inertia of the links about their centers of mass are assumed negligible.

Under these assumptions, the manipulator's equation of motion is expressed in standard robotic dynamic form:

$$M(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + G(\theta) = \tau \quad (5)$$

With $M \in \mathbb{R}^{2 \times 2}$, $C \in \mathbb{R}^{2 \times 2}$, $G \in \mathbb{R}^{1 \times 2}$

The velocity of link 1's center of mass is $v_1^2 = \dot{x}_1^2 + \dot{y}_1^2$ with $x_1 = l_1 \cos(\theta_1)$, $y_1 = l_1 \sin(\theta_1)$, and $\dot{x}_1 = -l_1 \dot{\theta}_1 \sin(\theta_1)$, $\dot{y}_1 = l_1 \dot{\theta}_1 \cos(\theta_1)$, hence $v_1^2 = l_1^2 \dot{\theta}_1^2$. The kinetic energy of link 1 will be:

$$T_1 = \frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_1 l_1^2 \dot{\theta}_1^2.$$

The velocity of link 2's center of mass is $v_2^2 = \dot{x}_2^2 + \dot{y}_2^2$ with $x_2 = l_1 \cos(\theta_1) + l_2 \cos(\theta_1 + \theta_2)$, $y_2 = l_1 \sin(\theta_1) + l_2 \sin(\theta_1 + \theta_2)$, and $\dot{x}_2 = -l_1 \dot{\theta}_1 \sin(\theta_1) - l_2(\dot{\theta}_1 + \dot{\theta}_2) \sin(\theta_1 + \theta_2)$, $\dot{y}_2 = l_1 \dot{\theta}_1 \cos(\theta_1) + l_2(\dot{\theta}_1 + \dot{\theta}_2) \cos(\theta_1 + \theta_2)$, hence $v_2^2 = l_1^2 \dot{\theta}_1^2 + l_2^2(\dot{\theta}_1 + \dot{\theta}_2)^2 + 2l_1 l_2 \dot{\theta}_1(\dot{\theta}_1 + \dot{\theta}_2) \cos(\theta_2)$. The kinetic energy of link 2 will be:

$$T_2 = \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_2 (l_1^2 \dot{\theta}_1^2 + l_2^2(\dot{\theta}_1 + \dot{\theta}_2)^2 + 2l_1 l_2 \dot{\theta}_1(\dot{\theta}_1 + \dot{\theta}_2) \cos(\theta_2)).$$

The total energy will be: $T = T_1 + T_2$ which can be presented as $T[\dot{\theta}_1 \quad \dot{\theta}_2] M \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$. With

$$M(1,1) = (m_1 + m_2)l_1^2 + m_2 l_2^2 + 2m_2 l_1 l_2 \cos(\theta_2)$$

$$M(2,1) = m_2 l_2^2 + m_2 l_1 l_2 \cos(\theta_2)$$

$$M(1,2) = m_2 l_2^2 + m_2 l_1 l_2 \cos(\theta_2)$$

$$M(2,2) = m_2 l_2^2$$

The Christoffel formula gives the coefficient of Coriolis $C_{ij} = \sum_{k=1}^2 \frac{1}{2} \left(\frac{\partial M_{ij}}{\partial \theta_k} + \frac{\partial M_{ik}}{\partial \theta_j} - \frac{\partial M_{kj}}{\partial \theta_i} \right) \dot{\theta}_k$ hence:

$$C(1,2) = -2m_2 l_1 l_2 \sin(\theta_2) \dot{\theta}_1 \dot{\theta}_2 - m_2 l_1 l_2 \sin(\theta_2) \dot{\theta}_2^2$$

$$C(2,1) = m_2 l_1 l_2 \sin(\theta_2) \dot{\theta}_1^2$$

The gravity torque is given by: $G(\theta) = \begin{bmatrix} \frac{\partial U}{\partial \theta_1} \\ \frac{\partial U}{\partial \theta_2} \end{bmatrix}^T$, U is the potential energy. Then:

$$G(1,1) = (m_1 + m_2)gl_1 \sin(\theta_1) + m_2 gl_2 \sin(\theta_1 + \theta_2)$$

$$G(1,2) = m_2 gl_2 \sin(\theta_1 + \theta_2)$$

Since M is invertible, the (5) can be rewritten as:

$$\ddot{\theta} = -M^{-1}C(\theta, \dot{\theta})\dot{\theta} - M^{-1}G(\theta) + M^{-1} \quad (6)$$

Taking $\theta_1 = x_1$; $\theta_2 = x_2$; $\dot{\theta}_1 = x_3$; $\dot{\theta}_2 = x_4$; let $x = [x_1, x_2, x_3, x_4]^T$ then the model can be represented as:

$$\dot{x} = A(x)x + B(x)u + G(x)w \quad (7)$$

With

$$A(x) = \begin{bmatrix} 0_{2 \times 2} & I_{2 \times 2} \\ 0_{2 \times 2} & -M^{-1}C \end{bmatrix}; B(x) = \begin{bmatrix} 0_{2 \times 2} \\ M^{-1} \end{bmatrix}; G(x) = \begin{bmatrix} 0_{2 \times 2} \\ G_1 \end{bmatrix}$$

$$G_1(x) = \begin{bmatrix} (m_1 + m_2)gl_1 & m_2gl_2 \\ 0 & m_2gl_2 \end{bmatrix}; \quad w = \begin{bmatrix} \sin(x_1) \\ \sin(x_1 + x_2) \end{bmatrix}$$

Let

$$X_1 = [x_1 \quad x_2]$$

Then the concatenation of the model into two parts can be represented as:

$$\dot{X}_1 = X_2 \tag{8}$$

$$\dot{X}_2 = AX_2 + B_2u + G_1w \tag{9}$$

The procedure of backstepping technique can be investigated in section 5.1 to 5.3.

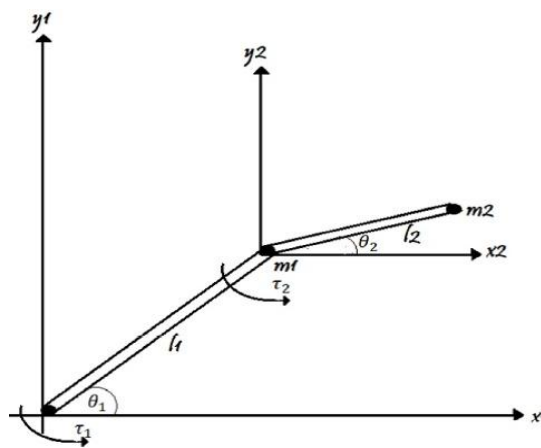


Figure 1. 2-DOF robot arm

5.1. Step 1

Let $\dot{X}_1 = X_2$ using backstepping technique [28] we get:

$$\dot{X}_1 = v_1$$

$$z_1 = X_{1d} - X_1$$

$$\dot{z}_1 = \dot{X}_{1d} - \dot{X}_1 = \dot{X}_{1d} - v_1$$

Let

$$\dot{z}_1 = A_1z_1 + B_1\xi_1 + G_{11}w_1$$

with

$$A_1 = 0; \quad B_1 = I; \quad G_{11} = 0; \quad \xi_1 = \dot{X}_{1d} - v_1$$

The control law leads to

$$\xi_1 = -R_1^{-1}B_1^T\lambda = -R_1^{-1}B_1^T P_{1a}z_1$$

for $A_1 = 0$; $B_1 = I$ and $G_{11} = 0$; in that case the riccati equation becomes

$$-\dot{P}_1 = -P_1R_1^{-1}P_a + Q_1 \tag{10}$$

with Q_1 is a symmetric matrix and R_1 is a diagonal matrix; then the control law is calculated as:

$$\xi_1 = -R_1^{-1}P_a z_1 = \dot{x}_{1d} - v_1$$

Then:

$$v_1 = (R_1^{-1}P_a z_1 + \dot{x}_{1d}) \quad (11)$$

The Hamiltonian matrix is $H_1 = \begin{bmatrix} A_1 & B_1 R_1^{-1} B_1^T \\ Q_1 & -A_1^T \end{bmatrix}$ with $R = \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix}$; $Q = \begin{bmatrix} q_1 & 0 \\ 0 & q_2 \end{bmatrix}$, so:

$$D = H_1 - \lambda I = \begin{bmatrix} -\lambda & 0 & \frac{1}{r_1} & 0 \\ 0 & -\lambda & 0 & \frac{1}{r_2} \\ q_1 & 0 & -\lambda & 0 \\ 0 & q_2 & 0 & -\lambda \end{bmatrix}$$

Computing the determinant of D will give:

$$\det(D) = \frac{(\lambda^2 r_2 - q_2)(\lambda^2 r_1 - q_1)}{(r_1 r_2)}$$

Hence the eigenvalues are:

$$\lambda_1 = -\sqrt{\frac{q_1}{r_1}}; \lambda_2 = -\sqrt{\frac{q_2}{r_2}}; \lambda_3 = \sqrt{\frac{q_1}{r_1}}; \lambda_4 = \sqrt{\frac{q_2}{r_2}}$$

The eigenvectors can be determined through the determinant of minors of D . It is denoted by H_{ij} . Let H_{11} be the minor of the hamiltonian matrix H by deleting the 1st row and 1st column. Hence for V_{1a} H_{1j} $j = 1..4$ are computed and we get:

$$V_{1a} = \begin{bmatrix} -\lambda_1^3 + \frac{q_2}{r_2} \lambda_1 \\ 0 \\ -q_1 \lambda_1^2 + \frac{q_1 q_2}{r_2} \\ 0 \end{bmatrix} = \begin{bmatrix} \sqrt{\frac{q_1}{r_1}} \left(\frac{q_1}{r_1} + \frac{q_2}{r_2} \right) \\ 0 \\ -\frac{q_1^2}{r_1} + \frac{q_1 q_2}{r_2} \\ 0 \end{bmatrix}$$

and V_{2a} H_{2j} $j = 1..4$ are computed and we get:

$$V_{2a} = \begin{bmatrix} 0 \\ \lambda_2^3 - \frac{q_2}{r_2} \lambda_2 \\ 0 \\ q_2 \lambda_2^2 - \frac{q_1 q_2}{r_2} \end{bmatrix} = \begin{bmatrix} 0 \\ \sqrt{\frac{q_2}{r_2}} \left(\frac{q_2}{r_2} - \frac{q_1}{r_1} \right) \\ 0 \\ \frac{q_2^2}{r_2} - \frac{q_1 q_2}{r_1} \end{bmatrix}$$

5.1.1 Orthogonalization

Let $U_{1a} = V_{1a}$ using Gram-Schmidt organigram leads to:

$$U_{2a} = V_{2a} - \frac{V_{1a}^T V_{2a}}{V_{1a}^T V_{1a}} V_{1a} = V_{2a}$$

Computing the two eigenvectors U_{1a} and U_{2a} is sufficient to find the matrix P_a . So:

$$[U_{1a} \ U_{2a}]_{4 \times 2} = \begin{bmatrix} P_{11a(2 \times 2)} \\ P_{21a(2 \times 2)} \end{bmatrix}$$

With

$$P_{11a} = \begin{bmatrix} \sqrt{\frac{q_1}{r_1}} \left(\frac{q_1}{r_1} - \frac{q_2}{r_2} \right) & 0 \\ 0 & \sqrt{\frac{q_2}{r_2}} \left(\frac{q_2}{r_2} - \frac{q_1}{r_1} \right) \end{bmatrix}$$

$$P_{21a} = \begin{bmatrix} -\frac{q_1^2}{r_1} + \frac{q_1 q_2}{r_2} & 0 \\ 0 & \frac{q_2^2}{r_2} - \frac{q_1 q_2}{r_1} \end{bmatrix}$$

Hence

$$P_a = -P_{21a} P_{11a}^{-1}$$

Finally

$$P_a = \begin{bmatrix} \sqrt{q_1 r_1} & 0 \\ 0 & \sqrt{q_2 r_2} \end{bmatrix}$$

Note that to avoid singularity for this case we choose

$$\frac{q_1}{r_1} \neq \frac{q_2}{r_2}$$

5.2. Step 2

Let the system be:

$$\dot{X}_2 = AX_2 + B_2 u + G_1 w \quad (12)$$

The application for (9) with $m_1 = 5 \text{ Kg}$; $m_2 = 2 \text{ Kg}$; $l_1 = l_2 = 0.34 \text{ m}$; $g = 9.81 \text{ ms}^{-2}$ the model is represented as:

$$d = -1011.5 + 289 \cos^2(x_2);$$

$$a_{11} = (-289(1 + \cos(x_2)) \sin(x_2) x_3^2) / d$$

$$a_{12} = (-578 \sin(x_2) x_4 (x_3 + .5x_4)) / d$$

$$a_{21} = (-144.5(-9 - 4 \cos(x_2)) \sin(x_2) x_3^2) / d$$

$$a_{22} = (578(1 + \cos(x_2)) \sin(x_2) x_4 (x_3 + .5x_4)) / d$$

$$g_{11} = 1250 \times 23.3478$$

$$g_{12} = 1250 \times 6.6708 - 8338.5(1 + \cos(x_2))$$

$$g_{21} = 1250 \times 23.3478(1 + \cos(x_2))$$

$$g_{22} = 1250 \times 6.6708 - 4169.25(-9 - 4 \cos(x_2))$$

Then

$$B_2 = \frac{1}{d} \begin{bmatrix} -1250 & 1250(1 + \cos(x_2)) \\ 1250(1 + \cos(x_2)) & 625(-9 - 4\cos(x_2)) \end{bmatrix}$$

$$A_2 = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}; G_1 = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}$$

Let the error be:

$$z_2 = v_1 - x_2;$$

Then

$$\dot{z}_2 = \dot{v}_1 - \dot{x}_2;$$

$$\dot{z}_2 = \dot{v}_1 - (A_2 X_2 + B_2 u + G_1 w)$$

$$\dot{z}_2 = \dot{v}_1 + A_2(z_2 - v_1) - B_2 u - G_1 w \quad (13)$$

Writing (13) as:

$$\dot{z}_2 = A_2 z_2 + B_{22} \xi_2 - G_1 w$$

with $B_{22} = I$, ξ_2 is obtained:

$$\xi_2 = \dot{v}_1 - A_2 v_1 - B_2 u \quad (14)$$

The optimal control law will give:

$$\xi_2 = -R_2^{-1} B_2^T P_b z_2$$

So, the control is:

$$\xi_2 = \dot{v}_1 - A_2 v_1 - B_2 u = -R_2^{-1} B_2^T P_b z_2$$

Hence

$$u = B_2^{-1} (\dot{v}_1 - A_2 v_1 + R_2^{-1} B_2^T P_b z_2) \quad (15)$$

Let the Hamiltonian matrix be presented as:

$$H_2 = \begin{bmatrix} A_2 & B_{22} R_2^{-1} B_2^T \\ Q_2 & -A_2^T \end{bmatrix}$$

Taking $r_3 = r_4 = r$, and $q_3 = q_4 = q$:

$$R_2 = \begin{bmatrix} r & 0 \\ 0 & r \end{bmatrix}; Q_2 = \begin{bmatrix} q & 0 \\ 0 & q \end{bmatrix}$$

then

$$\det(H_2) = \lambda^4 + c_2 \lambda^2 + c_0$$

with

$$c_0 = \det(H_2); c_2 = -\frac{1}{2} \text{tr}(H^2)$$

$$V_{1b} = \begin{bmatrix} v_{11b} \\ v_{12b} \\ v_{13b} \\ v_{14b} \end{bmatrix}; V_{2b} = \begin{bmatrix} v_{21b} \\ v_{22b} \\ v_{23b} \\ v_{24b} \end{bmatrix}$$

$$\begin{aligned}
v_{11b} &= -\lambda_1^3 - a_{11}\lambda_1^2 + (a_{22}^2 + a_{12}a_{21} + q/r)\lambda_1 \\
&+ (a_{11}a_{22}^2 - a_{12}a_{21}a_{22} + a_{11}q/r) \\
v_{12b} &= -a_{21}\lambda_1^2 - (a_{11}a_{21} + a_{21}a_{22})\lambda_1 \\
&+ (a_{12}a_{21}^2 - a_{11}a_{21}a_{22} + a_{12}q/r) \\
v_{13b} &= (-r\lambda_1^2 + a_{21}^2r + a_{22}^2r + q)\frac{q}{r} \\
v_{14b} &= q(a_{12} - a_{21})\lambda_1 - q(a_{21}a_{11} + a_{12}a_{22}) \\
\left[\begin{aligned}
v_{21b} &= -a_{12}\lambda_2^2 - (a_{11}a_{12} + a_{12}a_{22})\lambda_2 \\
&+ (a_{12}^2a_{21} - a_{11}a_{12}a_{22} + a_{21}q/r) \\
v_{22b} &= -\lambda_2^3 - a_{22}\lambda_2^2 + (a_{11}^2 + a_{12}a_{21} + q/r)\lambda_2 \\
&+ (a_{11}^2a_{22} - a_{11}a_{12}a_{21} + a_{22}q/r) \\
v_{23b} &= q(-a_{12} + a_{21})\lambda_2 - q(a_{21}a_{11} + a_{12}a_{22}) \\
v_{24b} &= (-r\lambda_2^2 + a_{11}^2r + a_{12}^2r + q)\frac{q}{r}
\end{aligned} \right]
\end{aligned}$$

5.2.1. Orthogonalization

$$U_{1b} = V_{1b}$$

$$U_{2b} = V_{2b} - \frac{V_{1b}^T V_{2b}}{V_{1b}^T V_{1b}} V_{1b} = V_{2b}$$

Computing the two eigenvectors U_{1b} and U_{2b} is sufficient to find the matrix P_b . So:

$$[U_{1b} \quad U_{2b}]_{4 \times 2} = \begin{bmatrix} P_{11b(2 \times 2)} \\ P_{21b(2 \times 2)} \end{bmatrix}$$

$$P_{11b} = \begin{bmatrix} u_{11b} & u_{21b} \\ u_{12b} & v_{u22b} \end{bmatrix}$$

$$P_{21b} = \begin{bmatrix} u_{13b} & u_{23b} \\ u_{14b} & u_{24b} \end{bmatrix}$$

$$P_b = -P_{21b} P_{11b}^{-1}$$

5.3. Simulation results

An initial state $x_0 = [0.5, 0.5, 0, 0]$ is introduced, which corresponds to both joints starting at nonzero positions (0.5 rad) with zero initial velocities, the dynamic responses of the system are illustrated in Figure 2 and Figure 3. Specifically, Figure 2 displays the position and velocity of the 1st arm, while Figure 3 illustrates the corresponding responses for the 2nd arm. These figures show how the positions and velocities of both arms change when a filtered step input with a constant time $\tau = 0.2 \text{ sec}$ is applied. Using a filtered step helps the controller's ability to handle smooth signal and evaluate its transient behavior, including rise time, overshoot, and settling. The results show that the proposed controller, which combines H_∞ control, backstepping, and a state-dependent coefficient (SDC) approach for computing the matrix P, performs well. The system smoothly moves from the initial state to the desired trajectory, with acceptable rise time, overshoot, and settling. This demonstrates that the controller effectively stabilizes the manipulator's nonlinear dynamics while maintaining accurate tracking.

Furthermore, to evaluate the system's capability to track continuous, time-varying trajectories, a sinusoidal input was applied. The performance for this sine input is presented in Figure 4 for the 1st arm and Figure 5 for the 2nd arm. As depicted in these figures, both arms exhibit excellent tracking capabilities; despite a slight initial deviation during the first few seconds, the measured position and velocity trajectories quickly converge and tightly follow the desired sinusoidal paths. Overall, the combination of H_∞ , backstepping, and the SDRE method provides strong robustness, fast response, and reliable performance for controlling nonlinear systems like a 2-DOF robotic arm. As a result, the proposed control strategy is well suited for handling system nonlinearities and uncertainties while ensuring accurate trajectory tracking and stable system behavior.

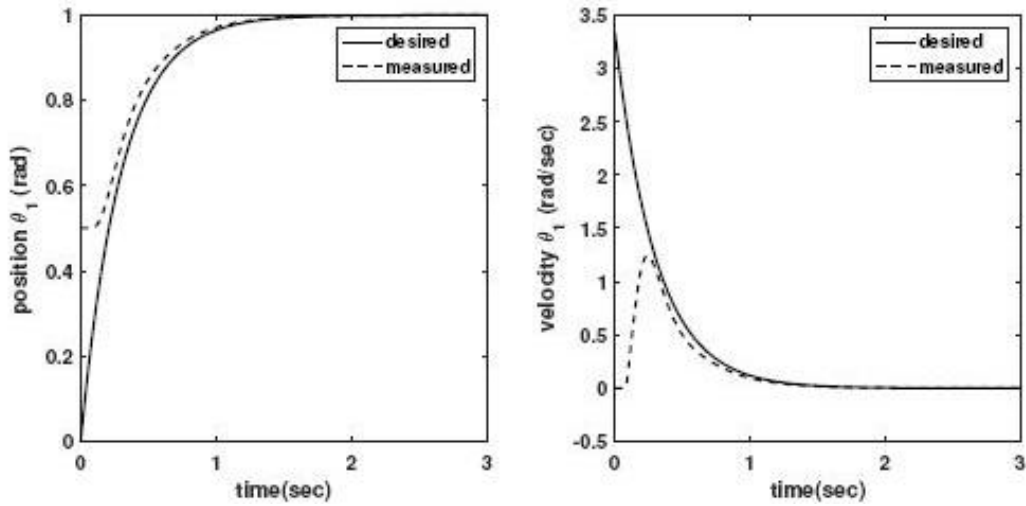


Figure 2. Position and velocity of 1st arm for a filtered step

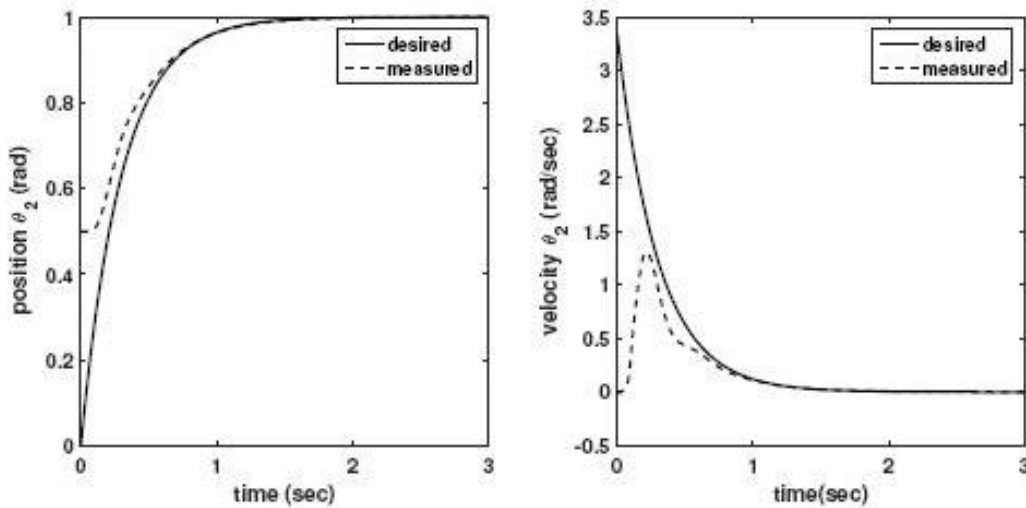


Figure 3. Position and velocity of 2nd arm for a filtered step

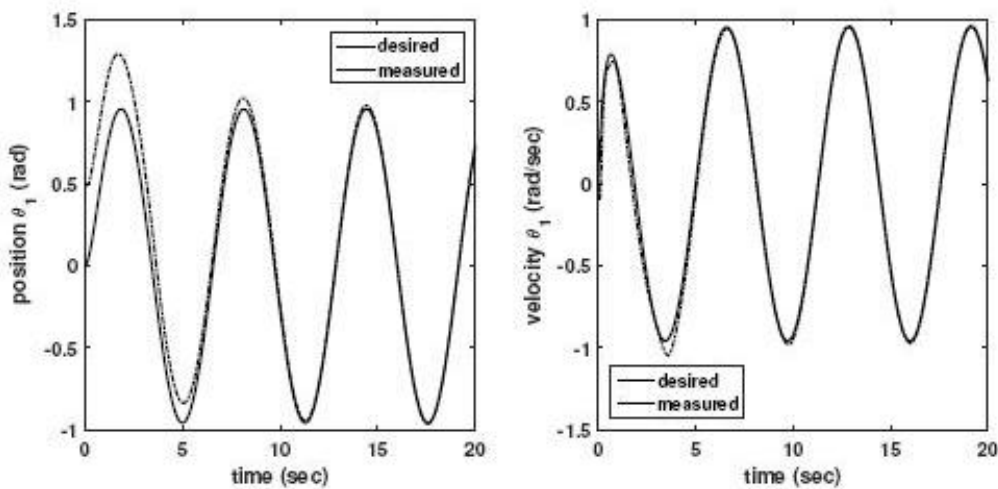


Figure 4. Position and velocity of 1st arm for a sine input

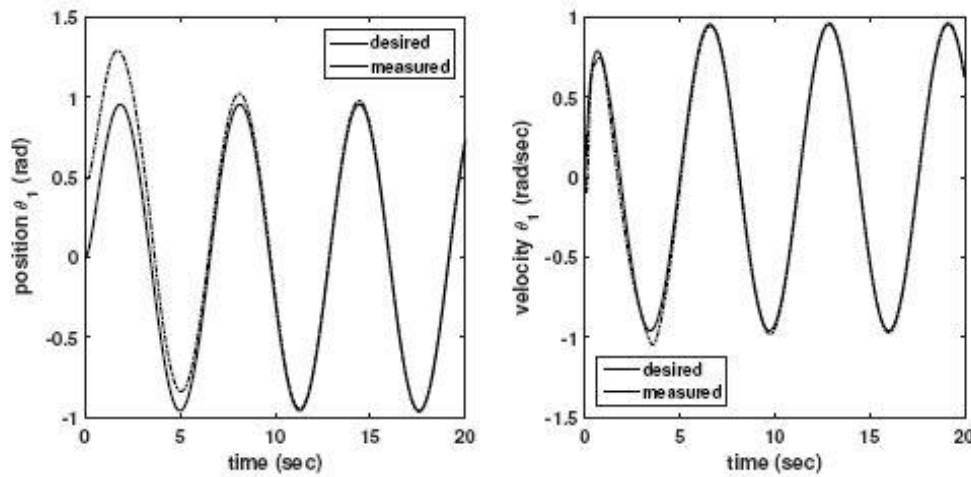


Figure 5. Position and velocity of 2nd arm for a sine input

6. CONCLUSION

An analytic Riccati algebraic equation solution using eigenvalues-eigenvectors tools and Gram–Schmidt orthogonality has been proposed with H_∞ controller based on backstepping technique for two degrees of freedom robot arm. Even though the mathematical model was highly nonlinear and the environmental disturbances were always present; the proposed model representation -which is state dependent coefficients- has made the methodology of the combined controller easier to track predefined position trajectories. It is shown that the all over system is able to track the predefined trajectory with a truncation in the model (taking a part as a perturbation vector w). Future research will focus on extending the proposed control method from the current 2-DOF manipulator to 3-DOF and more complex robotic systems.

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C : Conceptualization
 M : Methodology
 So : Software
 Va : Validation
 Fo : Formal analysis

I : Investigation
 R : Resources
 D : Data Curation
 O : Writing - Original Draft
 E : Writing - Review & Editing

Vi : Visualization
 Su : Supervision
 P : Project administration
 Fu : Funding acquisition

CONFLICT OF INTEREST STATEMENT

Authors state no conflict of interest.

INFORMED CONSENT

We have obtained informed consent from all individuals included in this study.




DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author, [AM], upon reasonable request.




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