Hybrid systems modelling and control using multiple mixed logical dynamical predictive model control: Application to a three-tank spherical system

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ABSTRACT

This study employs the mixed logical dynamical (MLD) framework for modelling, simulating, and controlling hybrid dynamical systems. Hybrid systems, which combine continuous-time dynamics and discrete logical events, pose significant challenges for conventional control strategies, such as proportional-integral-derivative (PID) controllers, particularly under complex operational constraints. To address these challenges, the MLD formalism provides a unified representation that integrates differential equations, logical rules, and inequality constraints. Based on the MLD model, a multivariable hybrid model predictive control (HMPC) approach is designed to optimize control system performance and operational efficiency over a prediction time horizon. At each sampling time step, a mixed quadratic programming (MIQP) optimization problem is solved online to determine the control law. The proposed control approach is applied to a three-spherical tank system, where simulation and experimental results demonstrate its effectiveness in ensuring stability, minimizing tracking errors, and satisfying physical constraints. These results underscore the relevance of MLD-based predictive control approaches for the optimization and advanced control of complex multivariable hybrid dynamical systems in industrial fields.

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1. INTRODUCTION

The applications of hybrid dynamical systems have been the subject of numerous studies in recent decades due to their ability to better model the transient dynamics of complex systems across various domains. There are two types of dynamics that occur simultaneously: continuous dynamics and discrete-event dynamics [1]–[3]. Systems exhibiting interaction between continuous dynamics and discrete logical decision-making are termed hybrid. The dynamics of these systems have been studied independently until now. The event-driven subsystem was modeled using automata [4] or Petri nets [5], while the continuous subsystem was modeled using differential equations.

In the industrial field area of process automation and control, the use of a hybrid dynamical system for modeling, analysis, and control, aimed at achieving high performance, requires a thorough analysis of all dynamic components and their interactions. Consequently, various approaches have been developed for large-scale installations, where programmable logic controllers (PLCs) have been used for control and protection within instrumentation and control systems, integrating different types of measurements and regulators. However, adopting intelligent and reliable strategies in industrial sectors such as oil, chemicals, and energy production requires efficient methods that inherently combine logical decision-making states and controllers for continuous dynamics.

The automatics and mathematics research communities have investigated hybrid dynamical systems to offer answers in a number of domains, including modeling and analysis [6]–[8] and stability [9], [10]. Numerous writers have examined control strategies [11], [12] and expanded them to nonlinear hybrid systems [13], [14]. The studies [15], [16] proposed a novel observer based on linear hybrid models, and numerous applications [17], [18] have investigated extending identification techniques from linear systems to hybrid dynamical systems. Finally, stochastic hybrid systems have been used when uncertainty parameters are present in the model [19].

This study presents a modeling and simulation approach for developing a model predictive control (MPC) law adapted to a specific kind of hybrid dynamical systems. The MLD modeling framework enables the integration of multi-model predictive control techniques and provides the mathematical foundation for the proposed modeling approach. The main objective is to assess the extent to which these formalisms and methods can address the challenges associated with hybrid system modeling, particularly in terms of the fidelity of dynamic behavior representation, design complexity, and computational burden.

This paper is organized into different sections as follows: The second section describes the modeling method based on the mixed dynamic logic (MDL) framework. Next, the multi-model MLD approach is developed in section 3, followed by section 4, which presents the hybrid predictive control (HMPC) strategy, which relies on the multi-MDL model and describes the mathematical transformation of the control low criteria into a mixed-integer quadratic optimization (MIQP). Section 5 describes the experimental setup, composed of three interconnected spherical tanks. Finally, section 6 concludes the work with remarks and future perspectives.

2. MULTIPLE MODEL MLD SYSTEMS

The multiple-MLD modeling framework is based on the principle of transforming logical relations, discrete dynamics, and logical subject to constraints into linear mixed inequalities. The integration of inequalities in a continuous-time dynamic will be modeled by linear difference equations. The mixed logical dynamic model developed for each subsystem is described by a state space and inequality constraints equations [11], [20].

$$\begin{cases} x_{k+1} = A_i x_k + B_1^i u_k + B_2^i \delta_k + B_3^i z_k \\ y_k = C_i x_k + D_1^i u_k + D_2^i \delta_k + D_3^i z_k \\ E_2^i \delta_k + E_3^i z_k \leq E_1^i u_k + E_4^i x_k + E_5^i \end{cases} \tag{1}$$

Where $A_i, \{B_j^i\}_{j=1\ldots3}, C_i, \{D_j^i\}_{j=1\ldots3}, \{E_j^i\}_{j=1\ldots5}$ $(i = 1 \ldots n_{mod})$ are matrices that describe the dynamics transient behavior of the system with known dimensions. The variables x represents the logical and continuous states, y the continuous and logical outputs, and $[u_c\; u_d]$ the continuous and logical inputs. $\delta \in \{0,1\}^{r_d}$ and z are auxiliary logical and continuous variables, respectively.

Index i denote the lookup of the model at the selected area, δ_i consists of binary auxiliary variables that are not part of $\Delta_{\alpha,j}^{\ell}$ and $z_i \in \mathcal{R}^{r_c}$ with $r_{i_c} \leq r_c$ and $u_i \in \mathcal{R}^{m_{ic}} \times \{0,1\}^{m_{i\ell}}$, where, $m_{i_c} \leq m_c$ and $m_{i\ell} \leq m_\ell$. Therefore, each simplified MLD model is associated with a set of constraints defined by Δ. The auxiliary variables will be introduced during the transformation of propositional logic into linear inequalities.

3. MODELING OF A SPHERICAL TANK BENCHMARK USING A HYBRID FRAMEWORK

The benchmark depicted in Figure 1 is complex, characterized by strong nonlinearity and significant coupling among the process's different states. It has been modeled using the MLD framework. The corresponding mathematical expressions were derived using standard techniques described in [20], [21], incorporating continuous and binary auxiliary variables introduced during the modeling stage.

The differential equations are discretized using the forward difference approximation, where $\dot{h_i}(t)$ is replaced by $\left(h_i^{k+1} - h_i^k\right)/T_s$, with T_s defining the sampling time. We define:

$$\begin{cases} x = [h_1 \;\; h_2 \;\; h_3]^T \\ u_c = [Q_1 \;\; Q_2]^T \\ u_d = [V_1 \; V_2 \; V_{13} \; V_{23}]^T \\ \delta^T = [\delta_{01} \;\; \delta_{02} \;\; \delta_{03}] \\ z = [z_{13} \;\; z_{23} \;\; z_{01} \; z_{02} \;\; z_{03} \;\; z_1 \; z_2]^T \end{cases} \quad (2)$$

We can write the resulting expression as (3).

$$\begin{cases} h_1^{k+1} = h_1^k + \frac{1}{A}(Q_1 - k_1 z_1^k - k_{13} z_{13}^k - k_{L1} z_{L1}^k) \\ h_2^{k+1} = h_2^k + \frac{1}{A}(Q_2 - k_1 z_2^k - k_{23} z_{23}^k - k_{L2} z_{L2}^k) \\ h_3^{k+1} = h_3^k + \frac{1}{A}(k_1 z_1^k + k_{13} z_{13}^k + k_1 z_2^k + k_{23} z_{23}^k - k_{N3} z_{N3}^k) \\ y^k = x^k \end{cases} \quad (3)$$

However, the elements of the matrices E_i, with $i = 1, \dots 5)$ include the constraints, as well as the inequalities arising from the logical statements of the three-tank system.

An organized set of transformation rules must be applied in order to formulate a hybrid system inside the MLD framework. The following elements are present in the final model:
- Three (03) states (three (3) continuous)
- Six (06) input variables (two (2) continuous, four (4) binary)
- Three (03) output variables (three (03) continuous, zero (0) binary)
- Seven (07) continuous variables of the auxiliary kind
- Three (03) binary variables of the auxiliary kind
- Forty-four 44 mixed-integer linear inequalities

The model in (3) is made up of three state variables, six inputs, and three outputs, along with auxiliary variables-three logical and seven continuous, and forty-four mixed integer linear inequality constraints.

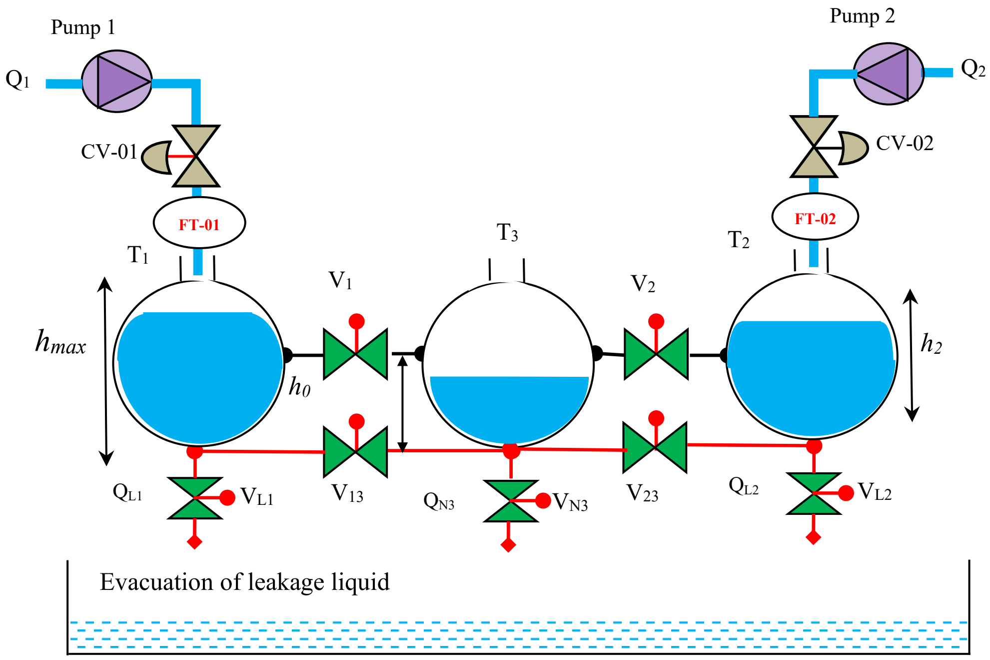

Figure 1. Three-tank spherical system (Gurski)

4. MULTIPLE-MODEL PREDICTIVE CONTROL DESIGN FOR HYBRID SYSTEMS

Numerous studies [11]–[13], [20] present MPC as a framework for hybrid dynamical systems that utilize an MLD representation combined with a mixed quadratic optimization solver. The MPC control approach uses a model to predict the future transient output y^k at a specified prediction time horizon, based on current observations or measurements [22]. At each sample interval, the prediction enables the controller to determine future control inputs by minimizing a relevant cost function while adhering to constraints. The ideal sequence is initially input to the plant at the subsequent time sampling step, after which the optimization criterion is repeated using revised process measurements to deliver the necessary feedback control action in real time.

The MLD model is used by multiple MPC predictive control strategies to synthesize a hybrid controller. For this, the index k denote the present time, $x(k)$ the current state-space, $(x_e\ ,u_e)$ an equilibrium state or reference value to reach, $k+N$ the final time prediction, we aim to compute the sequence of future control inputs $u_k^{k+N-1}=\{u(k),\dots u(k+N-1)\}$ go from the state $x(k)$ to x_e by minimizing the criterion defined as (4):

$$\min_{\begin{Bmatrix} u_k^{k+N_m-1} \\ \delta_k^{k+N_p-1} \\ z_k^{k+N_p-1} \end{Bmatrix}} j(x_k)=\sum_{i=0}^{N_p-1}\left(\left\|y_{k+i/k}-y_e\right\|_{Q_5}^2+\left\|\delta_{k+i/k}-\delta_e\right\|_{Q_2}^2+\left\|z_{k+i/k}-z_e\right\|_{Q_3}^2+\right.$$

$$\left.\left\|x_{k+i/k}-x_e\right\|_{Q_4}^2\right)+\sum_{i=0}^{N_u-1}\left\|u_{k+i/k}-u_e\right\|_{Q_1}^2 \tag{4}$$

Where $\|x\|_Q^2=x^TQx$, Q_i , $i=1,\dots 5$ define weighting matrices and $Q_i=Q_i^T\geq 0$ for i=1, 4.

N denotes the prediction time horizon for the output, whereas δ_e and z_e denote the values of the auxiliary variables related to the setpoint, determined by solving an MIQP problem subject to inequality constraints. The optimal solution is expressed by $\left\{u_k^{k+N-1}(j)\right\}_{j=0,\dots N-1}$ and can be computed at each sampling time. Nevertheless, solely the initial input $u(k)$ of this sequence is implemented in the system. The ideal input control $[u(k+1),\dots u(k+N-1)]$ is thereafter disregarded, and the entire optimization program is repeated at each instant ($k+1$). The weighting matrix Q_1, Q_2, Q_4 and Q_5 are crucial and particularly important, since they provide the trade-off between the exertion of control actions and the accuracy of following the setpoint tracking.

5. HYBRID MODEL PREDICTIVE CONTROL (HMPC)

The mathematical relationship of the optimization criterion on the output variables, expressed by (4), is reformulated as a MIQP problem. The quadratic criterion function penalizes deviations error between the reference trajectories and the predictive model. This criterion can be formulated as (5):

$$\begin{cases} F(\chi,x_k)=\min\limits_{x}\frac{1}{2}\chi^TH\chi+f^TH \\ A_{in}\chi\begin{Bmatrix} = \\ \leq \end{Bmatrix}b_{in} \end{cases} \tag{5}$$

where the optimization vector is defined as:

$$\chi=[u_k^T \quad u_{k+N-1}^T \quad \delta_k^T \quad \delta_{k+N-1}^T \quad z_k^T \quad z_{k+N-1}^T]$$

The expression for the matrix H and the vector f is given by the following relation [11]:

$$\begin{cases} H=P^TQP \\ f^T=Y_e^TQP \end{cases}$$

$$\begin{bmatrix} Q_4^N & 0 & 0 & 0 & 0 \\ 0 & Q_5^N & 0 & 0 & 0 \\ 0 & 0 & Q_1^N & 0 & 0 \\ 0 & 0 & 0 & Q_2^N & 0 \\ 0 & 0 & 0 & 0 & Q_3^N \end{bmatrix}$$

where Q_i^N is a matrix with diagonal element, m_d and m_c defines the binary controlled and auxiliary number of variables, respectively.

$$A_{in}^{E_i} = \begin{bmatrix} -E_i & & \cdots & 0 \\ -E_4 B_i & & -E_i & \mathrm{M} \\ \mathrm{M} & 0 & & 0 \\ -E_4 A^{N-2} B_i & \cdots & -E_4 B_i & -E_i \end{bmatrix}$$

$$A^{E_i} = \begin{bmatrix} A_{in}^{E_1} & A_{in}^{E_2} & A_{in}^{E_3} \end{bmatrix}$$

Here, $i = 1,2,3$.

$$b_{in}^T = [(E_4 x(k) + E_5)^T \quad (E_4 A x(k) + E_5)^T \cdots (E_4 A^{N-1} x(k) + E_5)^T]$$

$$P_{B_i} = \begin{bmatrix} B_i & & 0 & \cdots & 0 \\ AB_i & \ddots & B_i & & \vdots \\ \vdots & \ddots & \vdots & & \vdots \\ A^{N-1} B_i & & A^{N-2} B_i & & B_i \end{bmatrix}$$

$$P_{D_i} = \begin{bmatrix} D_i & \cdots & 0 & \cdots & 0 \\ CD_i & & D_i & & \vdots \\ \vdots & \ddots & \vdots & & \vdots \\ CA^{N-2} B_i & \cdots & CB_i & & D_i \end{bmatrix}, i \in \{1,2,3\}$$

$$P = \begin{bmatrix} P_{B_1} & P_{B_2} & P_{B_3} \\ P_{D_1} & P_{D_2} & P_{D_3} \\ & I_{m_v \times m_v} & \end{bmatrix}$$

Here, $m_v = N * (m + r_c + r_d)$

$$Y_e^T = \begin{bmatrix} (Ax(k) - x_e)^T (A^2 x(k) - x_e)^T \cdots (A^N x(k) - x_e)^T \\ (Cx(k) - y_e)^T (CAx(k) - y_e)^T \cdots (CA^{N-1} x(k) - y_e)^T \\ \underbrace{-u_e^T, -u_e^T \cdots - u_e^T}_{N}, \underbrace{-\delta_e^T, -\delta_e^T \cdots - \delta_e^T}_{N}, \underbrace{-z_e^T, -z_e^T \cdots - z_e^T}_{N} \end{bmatrix}$$

The value of $L = N * (m_d + r_d)$ determines the number of logical variables within the optimization problem, where the variables m_d and r_d denote the number of binary control and auxiliary variables, respectively. Conversely, the variable $m = m_c + m_d$ describes the continuous and binary number control variables. Where A_{in} is the constraint matrix, b_{in} is the constraint constant vector, f is the linear cost, and H defines the Hessian matrix of the quadratic criterion problem.

This control vector comprises both logical and continuous components. The MIQP algorithm is employed to solve the multi-model hybrid predictive control problem. At each sampling time, the MIQP problem has been calculated with the aim of determining the optimal control sequence:

$$u_k^{k+N-1} = \{u(k), \ldots u(k + N - 1)\}$$

In accordance with the receding time horizon concept, only the first control input is applied to the system, while the remaining inputs are discarded. The new control at (k+1) is then repeated at the next time step.

6. MIQP-BASED OPTIMIZATION CONTROL ALGORITHM

The algorithm of mixed integer quadratic programming typically relies on enumeration techniques, in which all possible solutions are systematically explored and evaluated to identify the one that minimizes the objective function. This method is commonly carried-out using a recurrent branch-and-bound algorithm [23]–[25]. The control scheme is depicted in Figure 2.

In the CPLEX solver, the *ctype* argument must be defined within the CPLEX MIQP function to specify the nature of each variable in the optimization vector. The corresponding expression is given as: The $ctype = [[C_{mc}^T \, C_{md}^T]_1^N, [I_{rd}^T]_1^N, [C_{rd}^T]_1^N]$ value is a string composed of characters {B,I,C,S,N}, representing logical, integer, continuous, semi-continuous, and semi-integer variables, respectively.

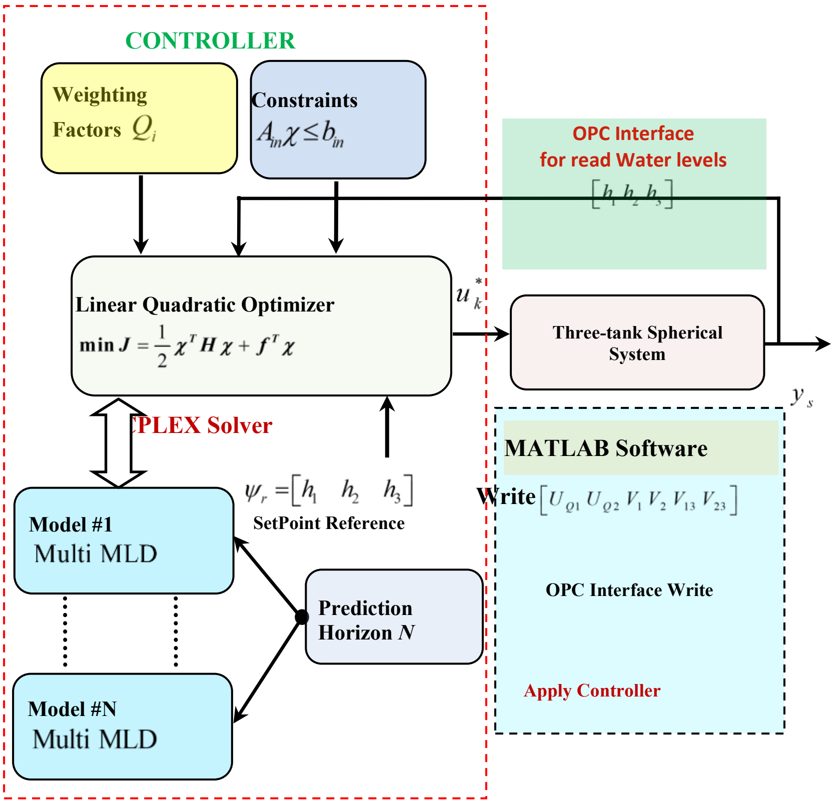

Figure 2. Execution of HMPC control strategy

7. RESULTS AND DISCUSSION

This section focuses on evaluating the control vector performance during both transient and steady-state phases, under variable step changes in the setpoints of the three water levels. The simulation results presented in Figure 3 demonstrate the system's trajectory tracking capabilities. The liquid levels [h_1 h_2 h_3] are controlled to follow a sequence of setpoint transitions: initially from [0 0 0] to [80 50 25]%, then to [80 50 25]% to [50 75 50]%, followed by [50 75 50]% to [80 62.50 62.50]% and finally to [62.50 75 50]%. This sequence of filling and emptying operations effectively engages the various operating modes of the tank system. The predictive control strategy demonstrates excellent performance in maintaining accurate trajectory tracking under all mode transitions.

The predictive control strategy provides accurate trajectory tracking across all operating modes. During the second mode — corresponding to the transition from [80 50 25]% to [50 75 50]% — it is observed that the flow and valves associated with the first tank [Q_1 V_1 V_2], after reaching steady-state, no longer participate in regulating tanks T_2 and T_3. Control efforts are instead shifted to flow Q_2, with frequent switching of its corresponding valve. Conversely, the transition from [80 62.50 62.50]% to [62.50 75 50]% places greater demand on the second tank T_2, which enters a filling phase, while the first tank T_1 transitions to a dumping phase. This dynamic interaction results in oscillations in tanks T_2 and T_3 during the steady-state phase, accompanied by a noticeable static error in the water level h_2. In this control case, the input control is applied using the control vector includes the outlet valve of tank T_3 maintained completely open, whereas the outlet valves of tanks T_1 and T_2 remain entirely closed in Figure 4.

The hybrid model predictive control (MPC) strategy is implemented based on multiple MLD models. The simulation scenario includes three distinct operating modes, denoted M_1, M_2 and M_3, each constrained within a region defined by the physical limitations of the benchmark system. Mode M_1 operates within the region [0-0.25] [0-0.25] [0.0-0.10] m^3; mode M_2 corresponds to the range [0.25-0.45] [0.25-0.45] [0.0-0.20] m^3; and mode M_3 is defined over the space [0.45-0.55] [0.45-0.55] [0.2-0.30] m^3. The simulation results demonstrate effective trajectory tracking across all three operating modes, along with a significant reduction in the computation time required to run the algorithm to retrieve the optimal solution to the MIQP optimization problem.

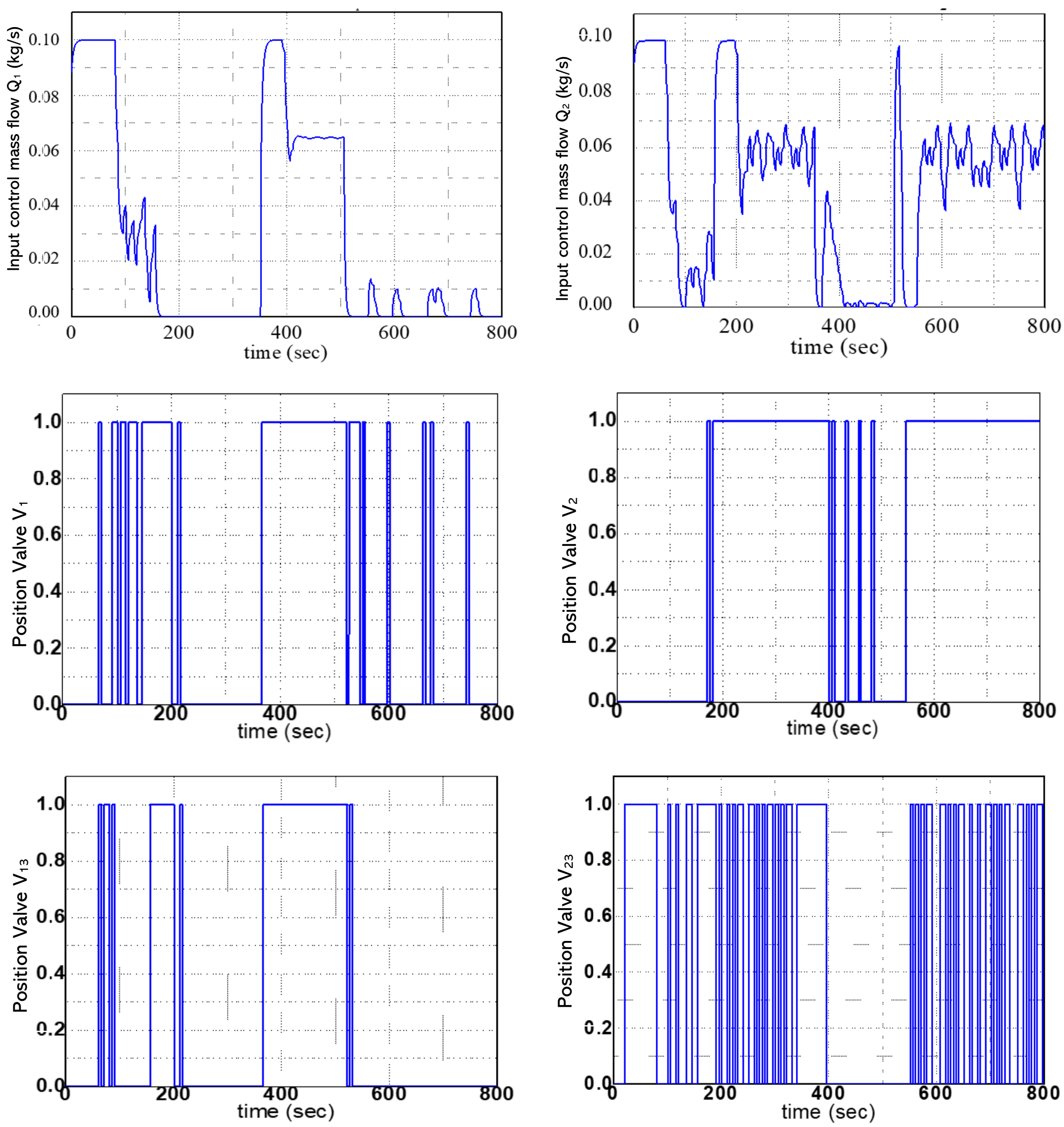

Figure 3. Model predictive control of liquid levels h_1, h_2 and h_3 (Prediction horizon N=3, Q_u=6000, Q_d=Q_z=0.01, valve VL_3 (=1) is held open with Step SetPoint)

The multiple model predictive control approach is transformed into a linear programming problem with 132 mixed linear constraints on a prediction time horizon chosen to be equal to the value N=3 to 48, continuous and binary variables. The input controller is calculated by the MIQP algorithm with CPLEX solver and transmitted under an OPC interface to follow the water liquid level specifications h_1=45 cm, h_2=35 cm, and h_3=10 cm. The MPC controller is reformulated as a linear programming problem with mixed-integer constraints over a prediction time horizon of N=3, involving 48 continuous and binary variables subject to 132 linear constraints. The optimal control inputs are computed using the MIQP formulation solved by the CPLEX solver, and implemented in real time via an OPC interface to meet the water liquid level setpoints: h_1=45 cm, h_2=35 cm and h_3=10 cm. On the other hand, the water level in tank T_1 shows oscillatory behavior, attributed to the switching actions of valves V_1 and V_{13}, which are tasked with maintaining the level h_1 and h_2 and h_3 around their respective reference trajectories. In the optimization criterion, the weighting matrix Q_y is carefully chosen to penalize deviations between measured water levels and their setpoints, thereby improving tracking performance.

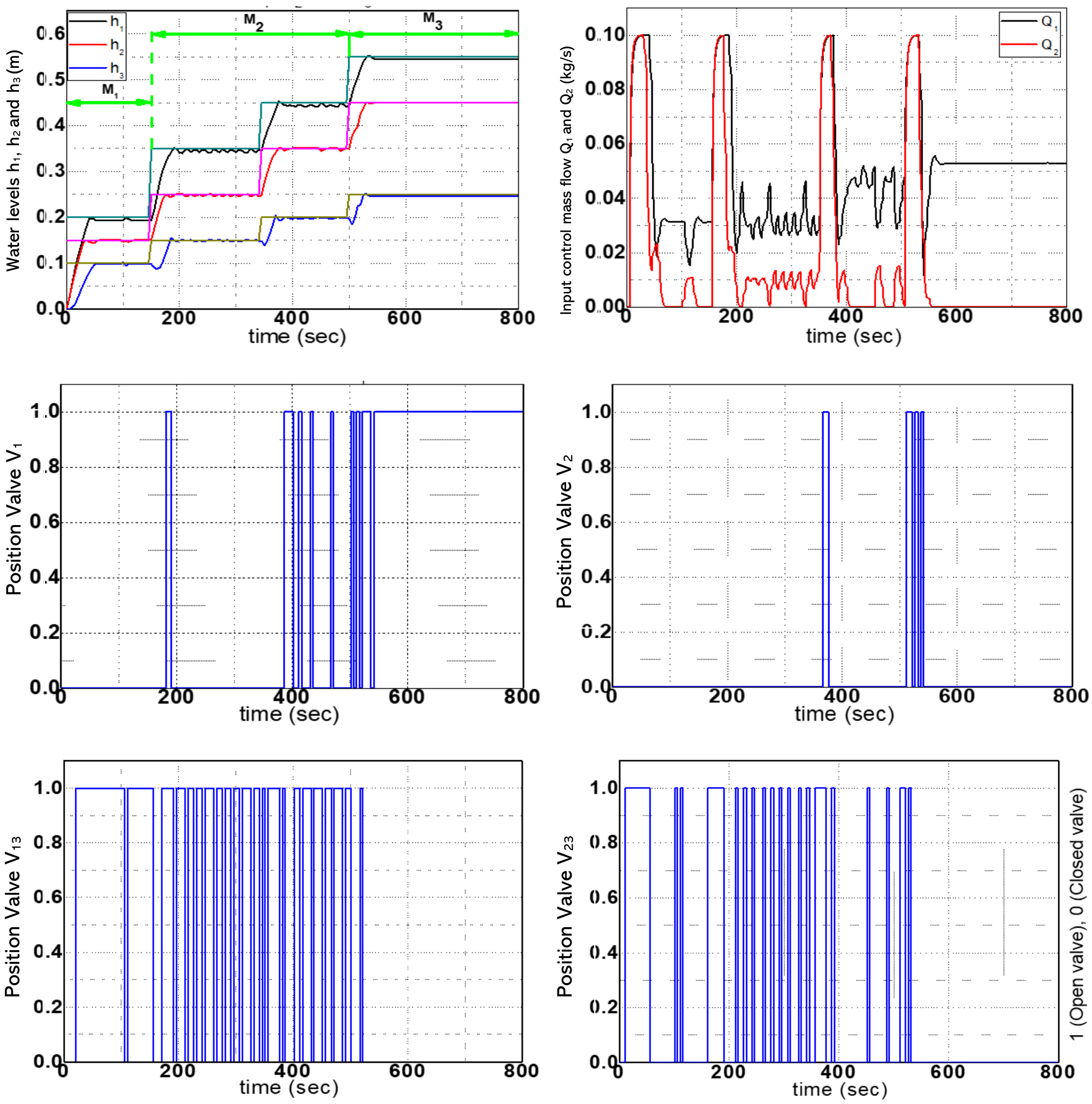

Figure 4. Multiple model predictive control of liquid levels h_1, h_2 and h_3 (Prediction horizon N=3, Q_u=6103, Q_d=Q_z=10^{-2}, valve VL_3 (=1) is held open)

8. CONCLUSION

This study successfully illustrated the modeling, simulation, and control of a hybrid dynamical benchmark using the multiple MLD framework and predictive control strategies. The application to a three-spherical tank system illustrated the robustness and effectiveness of the proposed methodology in achieving stable and optimized performance. By integrating logic rules with continuous dynamics, the model effectively handled operational constraints and control objectives. The results underscore the potential of hybrid predictive control methods for managing complex systems in various industrial applications.

While experimental validation has been successfully performed on a laboratory setup, the proposed control strategy has not yet been implemented on an autonomous or embedded system. Moreover, this approach uses the commercial CPLEX solver to find the optimal solution to mixed (integer) quadratic problem (MIQP), which may pose challenges for real-time execution in resource-constrained environments. Additionally, the modeling process based on the MLD formalism can be complex and sensitive to structural inaccuracies.

Future work will aim to adapt the control strategy for embedded real-time platforms, explore more computationally efficient solvers or approximation techniques, and investigate model simplification methods to improve scalability. Further extensions may include the application to large-scale or distributed hybrid systems, as well as the integration of learning-based components for enhanced adaptability.

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AUTHOR CONTRIBUTIONS STATEMENT

This journal uses the Contributor Roles Taxonomy (CRediT) to recognize individual author contributions, reduce authorship disputes, and facilitate collaboration.

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Djamel Boukhetala		✓					✓	✓		✓			✓	✓

C : **C**onceptualization
M : **M**ethodology
So : **So**ftware
Va : **Va**lidation
Fo : **Fo**rmal analysis
I : **I**nvestigation
R : **R**esources
D : **D**ata Curation
O : Writing - **O**riginal Draft
E : Writing - Review & **E**diting
Vi : **Vi**sualization
Su : **Su**pervision
P : **P**roject administration
Fu : **Fu**nding acquisition

CONFLICT OF INTEREST STATEMENT

The authors declare that they have no conflicts of interest.

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author, K.H., upon justified request.

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