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# Stochastic planning of multi-bus hydrothermal systems using the scenario tree technique

**Martha Patricia Camargo-Martínez[1], Ricardo Rincón Ballesteros[1], Fabian Salazar-Caceres[1], Andrés F. Panesso H.[1], Harrynson Ramírez-Murillo[1], Osvaldo Añó[2]**
[1]Programa de Ingeniería Eléctrica, Universidad de La Salle, Bogotá, Colombia
[2]IEE-CONICET, Universidad Nacional de San Juan, San Juan, Argentina

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**ABSTRACT**

Hydrothermal operation planning (HTOP) is a complex, large-scale optimal control problem. Traditionally, mathematical programming is used to solve it; however, metaheuristic techniques have emerged as an alternative approach. However, even in the context of current technological developments, the models developed to date generally require simplifications in the formulation. In particular, in medium-term planning, they have used a deterministic model or simplified transmission lines into a single bus. However, this approach leads to conservative and unrealistic solutions that may result in either oversizing or underutilization of resources. Therefore, this work proposes a methodology for incorporating uncertainties into the HTOP problem with a multi-bus topology. It was tested in a three-bus system, where linear functions are applied to simplify the production of hydroelectric plants and the cost of thermal units. The methodology incorporated well-established techniques in an implicit stochastic optimization (ISO) model, using a tree of 50 scenarios to model the hydrological series, which is solved with linear programming (LP). The results were validated with the costs of the 10000 generated series, showing an error of 5.07%. Additionally, the solutions were compared with an adapted metaheuristic technique for this problem to explore models applicable to more complex formulations.



***Corresponding Author:***

Martha Patricia Camargo-Martínez
Programa de Ingeniería Eléctrica, Universidad de La Salle
Carrera 2 #10-70 Bogotá, D.C., La Candelaria, 111711, Colombia
Email: mpcamargo@unisalle.edu.co

## 1. INTRODUCTION

An essential requirement for the operation of any electric power system is the energy supply to meet the demand economically, *i.e.*, maximizing social benefit or, under certain assumptions, minimizing operating costs [1]. To achieve this goal, systems require careful planning that meets standards for quality, security, environmental protection, and reliability, ensuring sustainable and efficient power system operation.

The type of primary resource used to obtain electric energy decisively determines the complexity of planning. Hydropower, which relies on water reservoirs, is significantly affected by seasonal changes and climate phenomena such as the El Niño-Southern Oscillation (ENSO). On the other hand, thermal power plants, typically powered by fossil fuels, offer a stable energy supply but result in environmental harm. A hydrothermal system, therefore, balances both resources by considering that the water resource has no direct monetary cost; thus, the operating cost is determined by the amount of thermal generation used. However, the physical capacity of reservoirs limits the storage of water resources. Combined with the unpredictable nature of water inflow, this creates a dilemma about whether to use the resource immediately or conserve it for future use, resulting in a temporary interdependence between these choices. Thus, hydrothermal operation planning (HTOP) is a dynamic process that can be modeled as an optimization problem, where the objective is to assess the amount of water that could replace thermal energy within a time horizon [2].

In addition to these dynamic and stochastic characteristics, HTOP is regarded as a complex optimization problem because of i) its non-linear, non-separable, and non-convex nature [3]; ii) it includes several constraints, such as energy balance, limits of water reserve volume, total outflows, hydroelectric generation functions, and water balance equations; iii) it involves many variables with different time discretization; and iv) it is hard to consider a long-time horizon divided into several short intervals. For all these reasons, it is necessary to hierarchically disaggregate the problem into long-term, medium-term, and short-term horizons. The coupling between horizons is carried out from the largest to the smallest, based on the availability of water resources or other parameters resulting from previous planning, such as the value of water [4].

Conventionally, HTOP problems are solved by mathematical programming [5], such as linear programming (LP), dynamic programming (DP), nonlinear programming (NLP), or stochastic dynamic programming (SDP) and stochastic dual dynamic programming (SDDP) [6], [7]. These procedures obtain the global optimal solution if the necessary optimality conditions are satisfied; however, in complex systems, they require being combined with decomposition [8], [9] linear approximations [10], [11] or simulation techniques (*e.g.*, Monte Carlo) [12], [13] and also need to consider significant simplifications, even though technological advances in processors and software have reduced the number of these reductions. Another emerging approach in recent decades to solving optimization problems is metaheuristic techniques. Although an optimal global solution is not guaranteed, especially for complex and high-dimensional problems, they provide near-optimal solutions in a reasonable time [14]. Given their features such as simplicity, adaptability, and robustness, metaheuristic tools have been applied to solve HTOP problems, showing great potential to address their complex formulation and to model them without extensive simplifications [15], [16]. In [17], a comprehensive and up-to-date overview of the metaheuristic tools applied to HTOP in the short term is provided. Regarding the development of tools applied in medium-term planning problems, the works [18]–[21] remark that different tools, such as genetic algorithm (GA) or particle swarm (PSO), are implemented to solve deterministic and single-bus problems.

Mathematical and metaheuristic models developed so far to solve the HTOP problem simplify the nature of the objective function, the restrictions, or the variables. For example, studies such as [11] assume the costs of thermal power plants as linear functions. Alternatively, works like [22] and different versions of SDDP reduce the objective function to a piecewise linear form, but it is still convex. Regarding stochastic modeling, [23] assumes uncertainty with a two-stage model, while [24] includes a stochastic environment only in the final stages. In addition, some models condense the system into a single bus [25], and others simplify the reservoirs using aggregation methodology [26]. On the metaheuristics side, works such as [27]–[29] consider non-convex cost functions stochastically and non-linear water production functions, but employ a single-bus model. Instead, [30]–[32] take on the transmission network but assume a deterministic optimization problem.

Depending on the time horizon studied, each simplification could affect the results and decision-making. For instance, in the medium-term horizon, electrical transmission constraints are significant in large, loosely meshed systems such as those in South America, because the lines are more vulnerable to exceeding their operating limits. Furthermore, it is important to stochastically model specific input parameters, such as renewable primary resources and electricity demand, since uncertainty propagates over longer planning horizons [33]. Taking into account the above-mentioned aspects, and considering that implementing realistic economic dispatch models that reflect the physical restrictions and unpredictability of the generation is essential to avoid either assignment of load that cannot be produced or oversizing or underutilization of resources, the core research question addressed in this work is how to model the inherent uncertainty of variables associated with renewable generation resources within a medium-term HTOP, while simultaneously accounting for the transmission network and ensuring the resulting methodology can solve the problem despite complex formulations. In this regard, this work proposes a novel optimization methodology that can be applied to medium-term problems, where the network and the stochasticity of water inputs are included. An implicit stochastic optimization (ISO) strategy was applied to address uncertainty in hydro unit inflows, using a scenario tree with the progressive clustering method (PCM) [34], which was reduced through an algorithm based on particle swarm optimization [35]. The proposed methodology was tested in a case study in which the locations of the hydro and thermal power generators, as well as the load demand, were considered at different system buses, and the inflows of the hydro plant were modeled using a reduced scenario tree with 50 scenarios. Linear functions were used to represent hydro plant production and thermal unit costs, and a linear programming (LP) tool was implemented to solve the optimization; however, the methodology incorporated a meta-heuristic technique to validate the case and to provide a tool that can treat non-linear and non-convex problems.

In summary, this research established some theoretical ideas in power system optimization, stochastic modeling, metaheuristic techniques, and sustainable energy planning to compute an optimal solution presented through a novel methodology to address the HTOP problem, offering a practical tool for medium-term planning in hydrologically variable regions, in which, moreover, an improvement over classical approaches that consider single-bus simplifications is included, leading to a particular analysis of meshed grids with high hydropower potential. The development of this efficient scenario reduction technique, which enables the solution of a complex stochastic optimization problem critical to electrical grid reliability, directly aligns with IJECE's scope by presenting a significant computer engineering solution to the field of electrical power systems engineering.

The structure of this paper is shown as follows: Initially, section 2 explains the theoretical description of the stochastic model, outlining how uncertainties in hydrothermal operation planning are addressed through scenario-based approaches using an ISO model. Subsequently, the ISO formulation for HTOP is introduced as a minimization problem to optimize total operating costs while accounting for system constraints and including the power flow for lines and stochastic variables. Then, the most relevant characteristics of the metaheuristic technique used to validate the results are described. Section 3 describes the application of the proposed methodology to the case study. Section 4 presents the results, comparing the proposed scenario tree model against a baseline approach and highlighting its effectiveness in terms of cost and reliability. Finally, section 5 presents the conclusions, showing the benefits for power system management and suggesting directions for future work.

## 2. THE COMPREHENSIVE THEORETICAL BASIS

### 2.1. Stochastic programming

Optimization model solutions that include explicit uncertainties in parameters or variables are developed through stochastic programming (SP), where the uncertainty of a random variable is described using a continuous probability density function, which implies the presence of expected values within the formulation. This approach generally has a complex evaluation, but one way to overcome this difficulty is to approximate the continuous function to a discrete distribution function. Therefore, if a random variable is observable over time, standard analysis used to evaluate its behavior is performed by defining scenarios or steps.

Two methods are employed to perform this optimization. In the first approach, explicit stochastic optimization (ESO) is an approach where the uncertainty is considered within the formulation of the problem. This optimization is based on the two-stage model [34]. In the first stage, a decision is taken, assuming a priori information (*i.e.*, a value for the random variable is established). Due to the uncertainty, the assumed value needs corrective actions (resources) in the second stage. This model can be expanded to include more resources–a multistage problem–which would correspond to the evolution of the uncertainty over time. The decisions for each stage depend only on this observable data, and the decisions at any stage are independent of the next one. This characteristic is named non-anticipative of the decisions [36]. The second approach involves using implicit stochastic optimization (ISO) [37], which includes indirect uncertainty. In this case, different scenarios model the variable, and each one is run independently. Then, a multivariate analysis is required to obtain the optimal solution. These models are known as Monte Carlo or Stochastic Simulation (SS). While they do not consider the non-anticipative principle, its implementation implies a reduction in the variables of the problem and, therefore, less computation time.

#### 2.1.1. Scenario tree

In a multistage problem, the uncertainty in each stage $t$ of the random variable $x$ can be sampled, generating realizations that can be organized using a scenario tree. For its construction, it is assumed that discrete values may represent the probability distribution of the variable at each stage.

In the tree, a scenario $S_j$ is defined as any path from the value of the variable in the first stage $\alpha_1$ (root), taking discrete values (nodes $\alpha_i$) in each stage $1, 2, 3, \dots, T$. It means that each scenario, $S_j$, has $T$ nodes. The structure of the scenario is exemplary in Figure 1. In this case, the random variable $x$ is represented by a tree of $T = 3$ stages, with 4 scenarios and 7 nodes. For example, it can be observed that $S_1$ is described by nodes $\alpha_1, \alpha_2$ y $\alpha_4$, where each one belongs to a stage 1, 2, and 3, respectively. Each node $\alpha_i$ branches into a set of successive nodes $\alpha_i^+,$ where the path between them has a transition probability $\pi_{\alpha_i^+/\alpha_i} > 0$ (represented in the figure by the blue boxes). It represents the probability that successor node $Ni$ becomes $\alpha_i^+$. The probability $\pi_{\alpha_i^+}$ of node $\alpha_i^+$ is calculated by (1),

$$\pi_1 = 1 \text{ for } \alpha_i^+ = 1 \quad (1)$$

$$\pi_{\alpha_i^+} = \pi_{\alpha_i^+/\alpha_i} \cdot \pi_{\alpha_i} \text{ for } \alpha_i^+ \neq 1$$

where nodes in the last stage have a probability of each scenario $\pi^i$; *e.g.*, in Figure 1, $\pi_5$ corresponds to the probability of $\alpha_5$, and also defines the probability $\pi^2$ of $S_2$.

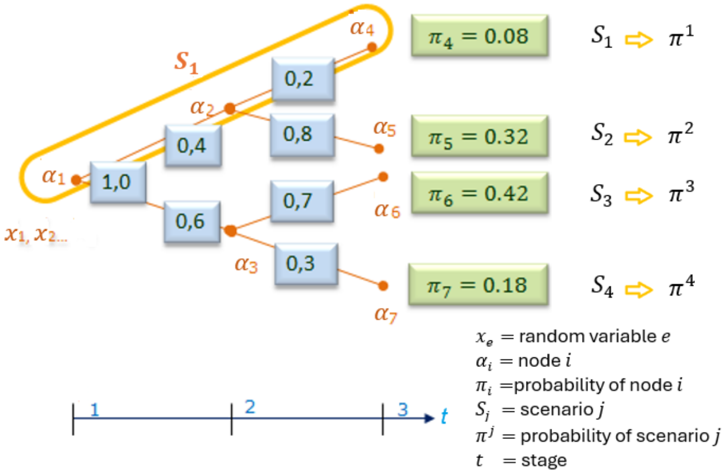


Figure 1. Example of a scenario tree

It is worth observing that in a tree, the scenarios share nodes in the early stages and branch out to reflect the growth of uncertainty over time. This behavior occurs because a random variable can assume both reduced and predictable values in the present. In contrast, the set of possible values in the later stages, *i.e.*, the number of nodes in the future, is greater. On the other hand, this branching structure guarantees that the non-anticipative characteristic is explicitly and naturally represented, because a node must be the same across all scenarios that share that history [36].

### 2.1.2. Building a scenario tree

Generating a scenario tree requires data to represent the occurrence of a random process, which is described by a random variable family and its temporal evolution. There are different methods in the literature for building a tree [34], where its structure is generally proposed as an input parameter; that is, the number of nodes per stage is established a priori, and therefore, the total number of scenarios $J$ of the tree. PCM [38] was used in this work, as described below.

a. Progressive clustering method (PCM)

The random process realizations are denoted as $\{\omega^R\}$ and are considered broken into stages $\{\omega_t\}$, as it is shown in Figure 2. The scenario tree is defined by $\{S^j\}$ scenarios, where $\{S^j\} \in \{\omega^R\}$, each one has a probability $\pi^j$ for $j = 1, 2, \dots, J$, and $\{\alpha_i\}$ nodes for $i = 1, 2, \dots, I$, as previously stated. This method starts by defining a root node $\alpha_1$ to represent the first component $\omega_1$, *e.g.*, using the mean value of the whole series set $\{\omega_1^r\}$ for $r = 1, 2, \dots, R$. The process continues by conditional clustering of the second component $\omega_2$ into as many clusters as the number of nodes in the tree at stage 2. The centroids resulting from the clustering process are used to establish the values of the nodes. In stage 3, the series $\{\omega_3^r\}$ are now grouped according to their connection with the nodes of the second stage. Then, each group is clustered with respect to the branches defined for each node in the third stage. Again, the node values are based on the centroids found during clustering. The process continues in the same way until reaching the last stage.

b. Scenario tree reduction

Generally, scenario reduction techniques minimize the distance between the original and the reduced tree. The established distance function defines the objective function, which selects the best-reduced tree composed of the most representative scenarios among the entire set. This high-level optimization problem has a combinatorial nature, and it is essentially a set cover problem [7]. Given the benefits of metaheuristics and the ease of expressing the problem in terms of a fitness function, the model cited in [35] proposes a scenario tree reduction using the PSO technique.

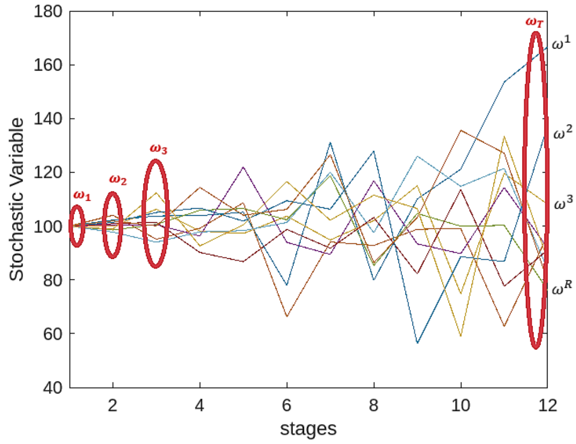


Figure 2. Example of a random process realization

In this problem, the search space consists of the set of all $J$ scenarios of the original tree, where $\Im$ scenarios of the reduced tree are established a priori as input data. The objective of the swarm is to determine $\Im \in J$ scenarios that are the furthest from each other, *i.e.*, those that maximize the distance to their neighbors. Each $\kappa$-th particle of the swarm is measured in terms of its adaptability or fitness function using the minimum multivariate normalized Euclidean distance $\beth$ of the swarm, multiplied by its probability $\pi^{\kappa}$, as described in (2).

$$fitness\ particle\ \kappa\ = \min_{\beth \in \Im} \pi^{\kappa} \left\{ \frac{1}{X} \sum_{x=1}^{X} \frac{\sqrt{\sum_{t=1}^{T} \left(\alpha_{x,t}^{\kappa} - \alpha_{x,t}^{\beth}\right)^2}}{\psi_x} \right\} \tag{2}$$

where $\Im$ is number of scenarios in the reduced tree (*i.e*, number of particles of the swarm), $\kappa$ is index for the particle or scenario for the reduced tree, $\beth$ is index for ($\Im - 1$) different scenarios to scenario $\kappa$, $\pi^{\kappa}$ is probability of scenario $\kappa$ in the reduced tree, $X$ is number of random variables, $T$ is Total number of stages, $\alpha_{x,t}^{\kappa}$ is value of random variable $x$ corresponding to particle $\kappa$ at stage $t$, $\alpha_{x,t}^{\beth}$ is value of random variable $x$ corresponding to particle $\beth$ at stage t, and $\psi_x$ and normalized factor for random variable $x$.

The applied algorithm is summarized in the next steps:

- Define parameters for PSO: search space, number of particles, and initialization of swarm.
- Calculate the probability of each scenario-particle κ of the swarm: This is obtained by comparing each particle with the original tree; thus, the probability of the $\kappa$-th scenario corresponds to the probabilities of the scenarios in the original tree that were closest to that scenario. Since the swarm is updated in each iteration, the probability values vary, so particles have a dynamic probability.
- Assign to each particle $\kappa$ a fitness function value, (2). The objective function is the maximization of the distance between the particles in the swarm, *i.e.*,

$$\text{Objective}(\text{particle})_{\kappa} = \max\ (\text{fitness}\ \kappa)$$

- Save the best position of each particle κ and that of its neighborhood, using (3):

$$\upsilon_{\kappa}^{i+1} = \varpi_{\kappa}^{i} \upsilon_{\kappa}^{i} + \phi_1 r_1 \left(\chi_{\kappa}^{pbest} - \chi_{\kappa}^{i}\right) - \phi_1 r_2 \left(\chi_{\kappa}^{gbest} - \chi_{\kappa}^{i}\right) \tag{3}$$

where $\upsilon_{\kappa}^{i+1}$ expresses that the new velocity of particle $\kappa$ and iteration $i+1$, *i.e.*; $\upsilon_{\kappa}^{i+1}$ is influenced by its previous velocity $\upsilon_{\kappa}^{i}$, the constant inertia weight $\varpi_{\kappa}^{i}$, the distance from its previous best performance $\chi_{\kappa}^{pbest}$ the distance from its nearest neighbor $\chi_{\kappa}^{gbest}$, its actual position $\chi_{\kappa}^{i}$, and the acceleration coefficients $\phi_1$ and $\phi_2$, and finally, $r_1$ and $r_2$ are independent random variables sampled from a uniform distribution $Unif(0,1)$. With the new velocity, the position is updated in each iteration, as is shown in (4):

$$\chi_{\kappa}^{i+1} = \chi_{\kappa}^{i} + \upsilon_{\kappa}^{i+1} \tag{4}$$

The process is carried out until the stopping criterion defined by the user is achieved [39].

### 2.2. Implicit stochastic optimization formulation for the hydro-thermal operation planning

In general, HTOP is written as a minimization function whose aim is to determine the optimal combination of available generation resources to provide the demand to minimize the sum of production costs associated with thermal units and penalty costs due to non-supplied energy for a given horizon, which is usually subdivided into several intervals [3]. The annual horizon is divided into successive weekly intervals to reduce computational effort. To consider the stochastic nature of water inflows within the optimization problem, this work employed the ISO strategy. In this way, the formulation for the HOTP problem is written in terms of an objective function for each scenario $k$ ($OF_k$), defined by $T$ nodes, as shown in (5), and its associated restrictions, described by (5):

$$OF_k = \min \sum_{t=1}^{T} \left[ \sum_{n=1}^{N} C_n\left(g_{n,t}^k\right) + \sum_{\beta=1}^{B} \tilde{C}_\beta\left(\tilde{g}_{\beta,t}^k\right) \right] \quad (5)$$

subject to:

$$\sum_{n=1}^{N} g_{n,t}^k + \sum_{e=1}^{E} p_{e,t}^k + \sum_{\beta=1}^{B} \tilde{g}_{\beta,t}^k = \sum_{\beta=1}^{B} D_{\beta,t}^k, \quad (6a)$$

$$p_{e,t}^k = \sum_{m \in e} p_{m,t}^k, \quad (6b)$$

$$p_{m,t}^k = \eta_m \cdot f\left(v_{e,t}^k, q_{e,t}^k\right), \quad (6c)$$

$$v_{e,t+1}^k = v_{e,t}^k + \left[ y_{e,t}^k - \sum_{m \in e} q_{e,t}^{k,m} - s_{e,t}^k + \sum_{u \in e}\left(q_{e,t}^{k,u} + s_{e,t}^{k,u}\right) \right] \cdot \tau, \quad (6d)$$

$$f_{l,t}^k = \mathcal{A}_{\mathcal{L} \times \mathcal{B}} \cdot \left(G_{\beta,t}^k + P_{\beta,t}^k + \tilde{g}_{\beta,t}^k - D_{\beta,t}^k\right), \quad (6e)$$

$$V_{e_{min}} \leq v_{e,t}^k \leq V_{e_{max}}, \quad (6f)$$

$$Q_{m_{min}} \leq q_{e,t}^k \leq Q_{m_{max}}, \quad (6g)$$

$$P_{m_{min}} \leq p_{e,t}^{k,m} \leq P_{m_{max}}, \quad (6h)$$

$$G_{n_{min}} \leq g_{n,t}^k \leq G_{n_{max}}, \quad (6i)$$

$$F_{l_{min}} \leq f_{l,t}^k \leq F_{l_{max}}, \quad (6j)$$

where:
$k$ : Scenarios index ($K$: total number of scenarios of the tree to be optimized)
$t$ : Stages index ($T$: total number of stages)
$n$ : Thermal units index ($N$: total number of thermal plants)
$e$ : Reservoirs index ($E$: total number of reservoirs)
$m$ : Hydro plants index ($M$: total number of hydro plants)
$u$ : Transmission lines index ($L$: total number of lines)
$l$ : Index to the set of plants directly upstream of reservoir $e$
$\beta$ : Index of system buses ($B$: total number of buses)
$\tau$ : Time duration of each stage $t$ [h]
$g_{n,t}^k$ : Thermal generation of unit n at stage $t$ of scenario $k$ in [MW]
$\tilde{g}_{\beta,t}^k$ : Power not supplied in bus $\beta$ at stage $t$ of scenario $k$ in [MW]
$C_n\left(g_{n,t}^k\right)$ : Cost function of unit $n$ at stage $t$ of scenario $k$ in [\$]
$\tilde{C}_\beta\left(\tilde{g}_{\beta,t}^k\right)$ : Cost of not served energy in [\$] associated to bus $k$ at stage $t$ of scenario $k$
$G_{\beta,t}^k$ : Power output in [MW] of thermal units connected to bus $k$ at stage $t$ of scenario $k$
$P_{\beta,t}^k$ : Power output in [MW] of hydro plants connected to bus $\beta$ at stage $t$ of scenario $k$
$D_{\beta,t}^k$ : Load demand in [MW] of bus $\beta$ at stage $t$ of scenario $k$
$p_{m,t}^k$ : Power output in [MW] of hydro plant $m$ associated to reservoir $e$ at stage $t$ of scenario $k$
$p_{e,t}^k$ : Power output in [MW] of hydro plants associated to reservoir $e$ at stage $t$ of scenario $k$
$v_{e,t}^k$ : Volume in [hm$^3$] of water stored in reservoir $e$ at stage $t$ of scenario $k$

$y_{e,t}^{k}$ : Water inflow in [$hm^3$/h] arriving at reservoir $e$ at stage $t$ of scenario $k$
$q_{e,t}^{k}$ : Turbined outflow in [$hm^3$/h] of a hydro plant associated to reservoir $e$ at stage $t$ of scenario $k$
$s_{e,t}^{k}$ : Spillage in [$hm^3$/h] of reservoir $e$ at stage $t$ in scenario $k$
$\eta_m$ : Productivity factor of hydro plant m in [$MWh/m^3$]
$f_{l,t}^{k}$ : Power flow for line l at stage $t$ of scenario $k$ in [MW]
$\mathcal{A}_{\mathcal{L}\times\mathcal{B}}$ : Branch-to-bus incidence matrix

Keep in mind that in the ISO model, each scenario is optimized independently. Then, they are weighted by their probability of obtaining the expected value (EV) described by (7).

$$EV = \sum_{k=1}^{K} OF_k \cdot \pi^k \quad (7)$$

### 2.3. Metaheuristics techniques

Most metaheuristic algorithms are based on evolution algorithms such as GA, PSO, differential evolution and evolutionary strategies (ES) [17]. Other evolution algorithms were recently developed, *e.g.*, median-variance mapping optimization (MVMO) [40]. Previous works showed a favorable adaptation of the algorithm in comparison to similar techniques when applied to the HTOP problem [41]. MVMO has some fundamental conceptual similarities to other heuristic approaches. However, it exploits the statistical attribute of search dynamics by using a unique mapping function for mutation operations based on the mean and variance of the $\chi$-best solutions achieved so far and saved in a continuously updated archive. In addition, the basic implementation of MVMO is characterized by a single-particle approach whose trade-off between search diversification and intensification translates into fast progress rates with reduced risk of premature convergence.

The recent variant of the mean-variance mapping optimization (MVMO-SH) algorithm enhances its efficiency by incorporating a multi-parent crossover strategy, increasing population diversity, and improving solution quality. As described in [42], the algorithm begins by initializing its parameters and generating a normalized initial population of $\chi$ particles. It then evaluates the population fitness and applies local search techniques when necessary to improve solutions while maintaining a counter to track the number of iterations. By updating an individual archive, the algorithm maintains a set of good and bad particles, thereby guiding future search steps. During the offspring generation phase, the algorithm applies single-parent crossover to bad particles based on the local best solutions to exploit nearby promising regions. On the other hand, the mutation is applied by mapping selected dimensions using the local mean and variance, ensuring variability in the generated offspring. The algorithm evaluates if the stopping conditions are met, such as completing the maximum number of iterations or achieving a solution of the required quality. If the convergence criteria are not accomplished, the process repeats from the fitness evaluation step until the algorithm stops and outputs the best solution found.

## 3. METHOD

The primary objective of this section is to empirically validate the concepts, assertions, and theoretical ideas described above, exhibiting the complex characteristics of the HTOP problem that were included in the proposed methodology. The power system and experimental process are detailed below.

### 3.1. System

We propose a simulated three-bus power system which considers three transmission lines, as shown Figure 3. The transmission system is assumed to be lossless. Bus $B_1$ is supplied by two thermal generators: $GT_1$and $GT_2$; bus $B_2$ by a hydraulic generator $GH_1$; and bus $B_3$ by a hydraulic generator $GH_2$. In each bus, fictitious generators $GF_1$, $GF_2$, and $GF_3$ are incorporated to consider cases where the system cannot supply energy. Thermal power plants are modeled by a linear cost function, given in (8).

$$C_n\left(g_{n,t}^{k}\right) = b_n \cdot g_{n,t}^{k} \quad (8)$$

Their costs and main technical parameters are presented in Table 1. Similarly, a linear cost function 1500 $\tilde{g}_{\beta,t}^{k}$ [$] was assumed for power not supplied by generators.

On the other hand, hydroelectric plants are modeled with the constant production function described in (9). In this case, each reservoir $e$ only has one hydro-plant associated with it, so $m$ was used as a subscript in the expressions.

$$p\left(g_{\mathrm{m},t}^{k}\right) = b_n \cdot g_{n,t}^{k} \quad (9)$$

All parameters for hydro-plants are described in Table 2. The parameters of the transmission network are shown in Table 3.

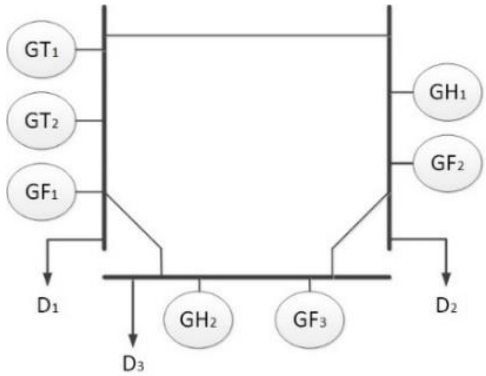


Figure 3. Single-phase diagram of the power system

Table 1. Parameters of thermal generators

| Thermal Plant ($n$) | Bus ($\beta$) | $P_{nmín}$ [MW] | $P_{nmax}$[MW] | $b_n$[$/MWh] |
|---|---|---|---|---|
| $GT_1$ | 1 | 0 | 100 | 90.22 |
| $GT_2$ | 1 | 0 | 50 | 135.32 |
| $GF_1, GF_2, GF_3$ | 1,2,3 | 0 | 1000 | 15000 |

Table 2. Parameters of hydraulic generators

| Hydro Plant($m$) | Bus($\beta$) | $V_{mmín}$ [hm$^3$] | $V_{mmax}$[hm3] | $V_0$[hm$^3$] | $V_{eT}$[hm$^3$] | $\eta_m$[MW/m$^3$/s] | $Q_{mmáx}$[m$^3$/s] | $P_{mmáx}$[MW] |
|---|---|---|---|---|---|---|---|---|
| $GN_1$ | 2 | 300 | 1200 | 500 | 500 | 0.72 | 300 | 216 |
| $GN_2$ | 3 | 300 | 800 | 400 | 400 | 0.72 | 150 | 108 |

Table 3. Parameters of transmission network

| Line ($l$) | Bus connection | Susceptance [p.u] | $F_{lmax}$ [MW] |
|---|---|---|---|
| 1 | 1-2 | 0.047 | 60 |
| 2 | 1-3 | 0.023 | 60 |
| 3 | 3-2 | 0.064 | 60 |

With respect to load demand, it is modeled by $D_1$, $D_2$, and $D_3$ using a load duration curve (LDC) with four (4) load levels for each one, as illustrated in Figure 4. The LDC graph depicts the power supplied over time, with distinct monthly load segments (blocks) corresponding to different power levels, all with the same durations of 40, 300, 270, and 120 hours per month. Note that the time horizon for the optimization is defined as one year, divided into twelve monthly intervals.

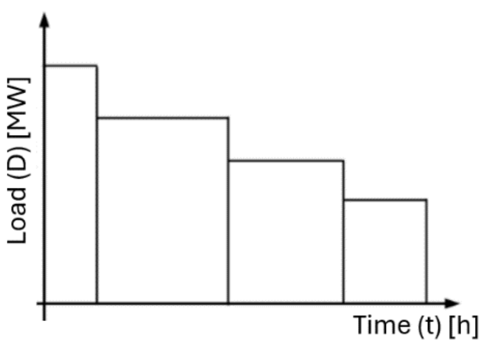


Figure 4. LDC with four load levels

### 3.2. Methodology

This section provides a clear and structured exposition of the methods applied. This approach integrates generation time series, scenario tree construction and reduction, and optimization techniques to address the complexities of HTOP under uncertainty.

#### 3.2.1. Time series generation

A total of 10000 time series of water inflows for the 2 reservoirs were generated during the 12 stages, using an auto-regressive model [43], as shown in (10).

$$Y_{e,t} = \mu_{e,t} + \sigma_{e,t} X_{e,t} \tag{10}$$

where $e$ is index respect to storage, $Y_{e,t}$ is water inflow of reservoir $e$ at stage $t$, $\mu_{e,t}$ is periodic mean of water inflow of reservoir $e$ at stage $t$, $\sigma_{e,t}$ is periodic standard deviation of water inflow of reservoir $e$ at stage $t$; $X_{e,t}$ is random variable.

In this case, $X_{e,t}$ represents the inflows in reservoir $e$ at stage $t$. Each one can be modeled with a first order auto-regressive model described by (11):

$$X_{e,t} = \rho_1 X_{e,(t-1)} + \sqrt{1-{\rho_1}^2}\varepsilon_t \tag{11}$$

By replacing (11) in (10), it is obtained:

$$Y_{e,t} = \mu_{e,t} + \sigma_{e,t}\left(\rho_1 \frac{Y_{e,(t-1)} - \mu_{e,(t-1)}}{\sigma_{e,(t-1)}} + \sqrt{1-{\rho_1}^2}\varepsilon_t\right)$$

where $\rho_1$ is the autocorrelation coefficient in $t = 1$ of the random variable $X$ and $\varepsilon_t$ is the independent random residue. $\mu_{e,t}$ and $\sigma_{e,t}$ are estimated from historical water inflow. It was assumed that $\rho_1$ was equal to 0.5. $\varepsilon_t$ is a random number taken from a normal distribution $\mathcal{N}(0,1)$. The original data and the obtained series are shown in Figure 5.

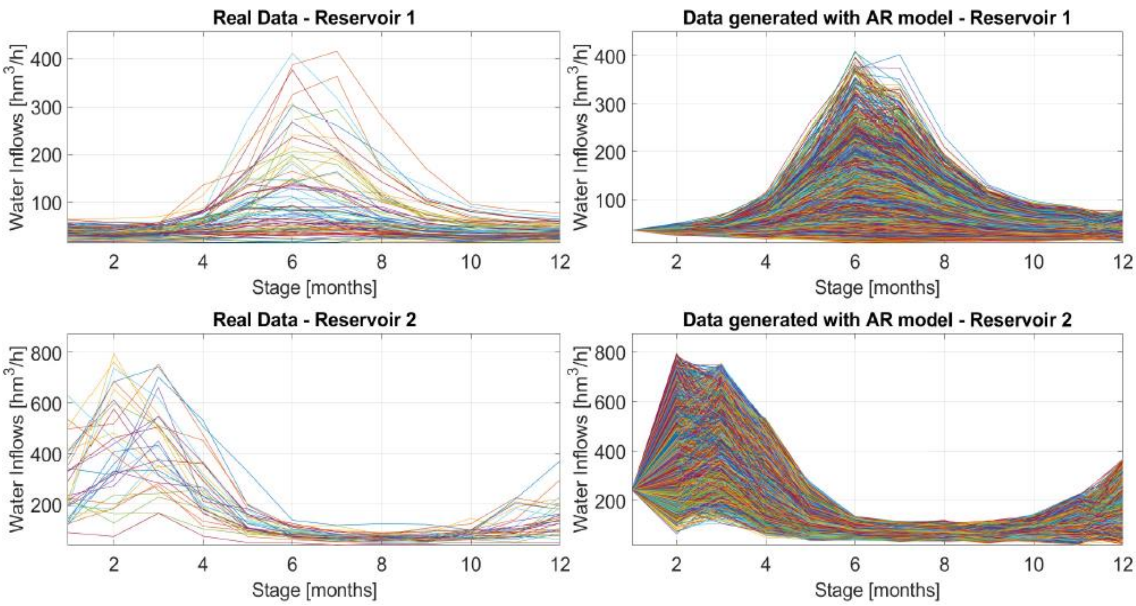


Figure 5. Original data and the obtained series for the reservoirs of $GH_1$ and $GH_2$

#### 3.2.2. Scenarios tree

The PCM described in section 2.1.2. was applied to generate a scenario tree. The proposed structure is as follows: the first stage has one branch; from the 2$^{nd}$ to the 12$^{th}$ stage, each node branches into two to obtain a tree (with two dimensions, one for each reservoir) with $2^{11}$ scenarios.

#### 3.2.3. Scenario reduction

The scenario reduction PSO mentioned above was applied, where the search space comprises the complete set of scenarios from the original tree. Fifty particles were selected as input parameters for the swarm. In this way, $(2^{11})^2$ scenarios from the original tree were reduced only to 50 scenarios for each reservoir, as is depicted in Figure 6.

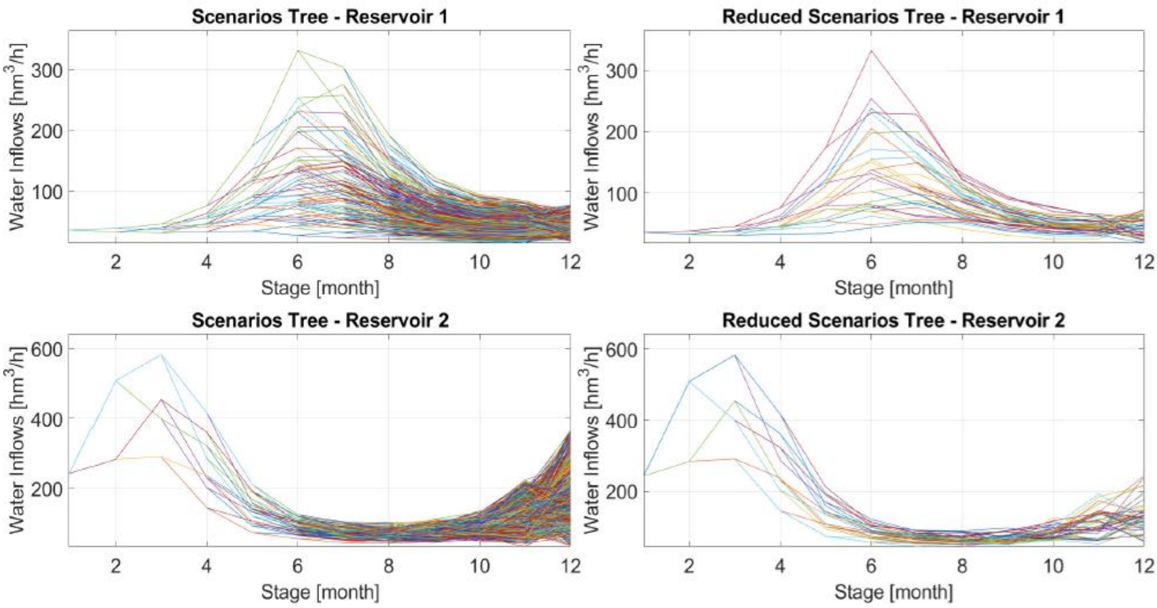


Figure 6. Original and reduced tree for the reservoirs of $GH_1$ and $GH_2$

#### 3.2.4. Optimization model for each scenario

Each scenario was solved by means of two techniques:

a. LP: Using the $linprog$ function of MATLAB, the formulation was written in matrix terms. Due to the mathematical fundamentals of this kind of optimization, the global solution is obtained.
b. MVMO: The adaptability of metaheuristic techniques facilitates the division of HOTP into hydro and thermal sub-problems. This result was used as validation and introduced a tool that could be used in non-linear or non-convex problems. This procedure is described below.

– The storage variables $v_{e,t}^k$ are defined as optimization variables or individuals. In this way, every $\chi$ individual has a length of $t \cdot E = 12 \cdot 2 = 24$. A LP algorithm is used to handle water balance constraints instead of using a penalty scheme. Given that inflows $y_{e,t}^k$ are predetermined in each scenario and that the optimization algorithms propose the $v_{e,t}^k$ values within the search boundaries, the purpose of using a LP is to determine the values of $s_{e,t}^k$ and $q_{e,t}^k$ such that (6d) is satisfied. Nevertheless, the fulfillment of the maximum bounds of $s_{e,t}^k$ and $q_{e,t}^k$ is not guaranteed when using LP. To overcome this problem, a heuristic rule is applied after performing LP to ensure that $q_{e,t}^k \leq Q_{m_{max}}$. Considering that $s_{e,t}^k \geq 0$ is the only condition for $s_{e,t}^k$, the rule is defined as follows:

$$s_{e,t}^k = \begin{cases} 0, & 0 < q_{e,t}^k \leq Q_{m_{max}} \\ q_{e,t}^k - Q_{m_{max}}, & q_{e,t}^k > Q_{m_{max}} \end{cases}$$

with $s_{e,t}^k$ and $q_{e,t}^k$ in [hm$^3$/h] from hydroelectric generation 6d and the production factor $\eta_m$ from Table 2, the total hydraulic energy in each stage $EH_t$ can be calculated.

– From the $EH_t$ values, thermal generations $g_{n,t}^k$ and hydroelectric generations $p_{m,t}^k$ in each load demand box are obtained through a DC load flow using LP. Values of $p_{m,t}^k$ are replaced in the cost function from (8), where the outcomes are considered as the fitness function of the individual $\chi$. The MVMO algorithm runs until the convergence criterion is accomplished.

To enhance the robustness and clarity of our methodology, thereby enabling its seamless and accurate application, a step-by-step exposition of the experimental procedure employed in this case study is provided, as outlined below. Figure 7 shows the flow of the methodology.

a. Generate 10000 time series of reservoir inflows using the auto-regressive model (10)-(11) with parameters $\rho_1 = 0.5$, normal distribution $\mathcal{N}(0,1)$, and historical means/std deviations.
b. Build a scenario tree via PCM: Start with the root node (mean of the first stage), and cluster subsequent stages into two branches per node, yielding $2^{11}$ scenarios for twelve stages and two reservoirs.
c. Reduce to 50 scenarios using PSO: Initialize the particles in the search space of original scenarios; update velocities/positions; maximize fitness, (8), until convergence.
d. Each reduced scenario is solved using LP to obtain its global solution.
e. To introduce metaheuristic techniques, each reduced scenario is solved using the MVMO tool, where each particle $\chi$ is defined in terms of storages $v_{e,t}^k$ corresponding to 24 decision variables ($E = 2, T = 12$) for scenario $k$; use LP to compute spills/turbines satisfying water balance (6d); apply heuristic for bounds ($q_{e,t}^k \leq Q_{m_{max}}$); then, compute total hydraulic energy $EH_t$. Subsequently, the thermal and hydro generations for each load block are determined using DC-LP. These power values are used to evaluate the total cost, defining the particle's fitness $\chi$. Run MVMO until 10000 iterations.
f. Each solution $k$ from the LP and MVMO is weighted by its probability $\pi^k$ to compute the EV (7).

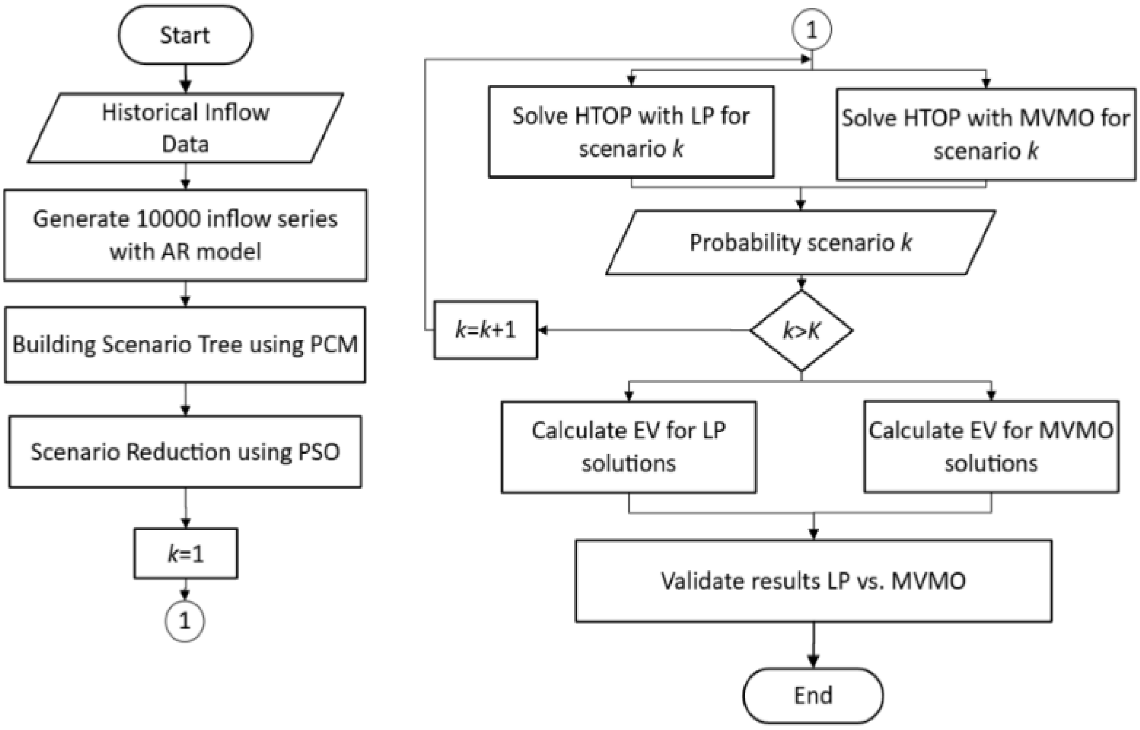


Figure 7. Flow chart of the proposed methodology

## 4. RESULTS AND DISCUSSION

The results for each of the 50 scenarios optimized with LP are shown in the third and eighth columns, while the costs with MVMO are presented in the fourth and ninth columns of Table 4. Besides, the error of MVMO techniques compared with the LP solution is described in the fifth and tenth columns, showing that the costs obtained with the metaheuristic tool are close to the optimum achieved using LP. Note that the probability $\pi^k$ shown in the second and seventh columns is obtained through the application of the scenario tree and reduction techniques.

The solution of the 10000 original time series without outliers, obtained by LP, was $\$111579.81 \times 10^3$. We compare this value with the EV of the cost finding with LP $\$105208.70 \times 10^3$, resulting in an error of 5.70%. We position our work within network-constrained hydrothermal scheduling under uncertainty. On one side, scenario tree reduction is a well-established technique in stochastic programming to improve tractability while preserving solution quality. [38] shows that reducing a scenario tree by 50% can still maintain around 90% accuracy, based on Fortet–Mourier probability metrics. Conversely, while prior studies demonstrate the transmission-constrained stochastic dispatch and SDDP-based policies, they employ the ESO approach for non-convex problems [44]. Recent medium-term models focus on multistage stochasticity and water valuation, frequently with simplified network representations [45].

Furthermore, compared to prior studies, our hybrid methodology demonstrates improved performance in handling stochastic uncertainties in multi-bus hydrothermal systems that could be applied to non-simplification problems. For instance, deterministic metaheuristic approaches in short-term HTOP, as reviewed in [17] reported errors up to 5–10% in cost estimations for simplified single-bus models. Moreover, the findings show effective uncertainty management, with low average errors confirming the approach's reliability. Nevertheless, higher errors (∼6%) in scenarios 17 and 35 indicate sensitivity to extreme inflows, which advanced reduction techniques could address. Readers can learn that scenario tree reduction efficiently captures 94.3% of original inflow variability using only 50 scenarios, avoiding the computational burden of full Monte Carlo simulations. In context, this advances medium-term HTOP beyond deterministic models ramp [18]–[21] by addressing uncertainty propagation, enabling more realistic planning in multi-bus systems. For future applications, this methodology is handy for integrating renewables (*e.g.*, wind, as in [24]), modeling more realistic systems, and including scalable tools for larger systems via parallel computing, all in favor of supporting the policy on reservoir management amid climate change and ensuring that operating decisions can be executed.

Table 4. Results of proposed methodology compared with LP solution

| Sce | Prob. [%] | Cost with LP [$ ×10³] | Cost with LP [$ ×10³] | Error [%] | Sce | Prob. [%] | Cost with LP [$ ×10³] | Cost with LP [$ ×10³] | Error [%] |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 4.23 | 106029.92 | 106289.62 | 0.24 | 26 | 1.46 | 109498.19 | 110809.82 | 1.20 |
| 2 | 0.42 | 91109.26 | 93380.17 | 2.49 | 27 | 6.00 | 131500.91 | 137946.44 | 4.90 |
| 3 | 0.97 | 105672.47 | 105676.62 | 0.00 | 28 | 2.35 | 96911.26 | 100122.04 | 3.31 |
| 4 | 3.00 | 119569.63 | 119616.83 | 0.04 | 29 | 1.33 | 86923.46 | 89985.55 | 3.52 |
| 5 | 1.42 | 80351.32 | 82110.38 | 2.19 | 30 | 4.39 | 108217.82 | 108675.75 | 0.42 |
| 6 | 0.97 | 87900.37 | 88705.32 | 0.92 | 31 | 0.63 | 108142.27 | 108198.56 | 0.05 |
| 7 | 1.46 | 81428.60 | 82861.59 | 1.76 | 32 | 3.88 | 105202.91 | 106381.66 | 1.12 |
| 8 | 0.92 | 111192.94 | 111271.10 | 0.07 | 33 | 0.84 | 106405.66 | 107606.37 | 1.13 |
| 9 | 2.01 | 116989.85 | 117160.27 | 0.15 | 34 | 3.50 | 108588.07 | 108822.22 | 0.22 |
| 10 | 5.32 | 117920.07 | 117969.09 | 0.04 | 35 | 0.8 | 70608.12 | 74693.93 | 5.79 |
| 11 | 2.33 | 103050.40 | 104395.61 | 1.31 | 36 | 3.09 | 124392.32 | 124781.60 | 0.31 |
| 12 | 1.85 | 104975.83 | 106850.45 | 1.79 | 37 | 0.55 | 92318.53 | 92753.49 | 0.47 |
| 13 | 1.51 | 85954.68 | 88608.42 | 3.09 | 38 | 0.98 | 98862.47 | 100940.76 | 2.10 |
| 14 | 1.18 | 99932.39 | 100571.58 | 0.64 | 39 | 0.63 | 101199.49 | 101752.35 | 0.55 |
| 15 | 1.28 | 95230.83 | 97761.24 | 2.66 | 40 | 1.30 | 92697.10 | 93462.87 | 0.83 |
| 16 | 2.76 | 110386.61 | 110397.83 | 0.01 | 41 | 0.44 | 92091.89 | 93781.46 | 1.83 |
| 17 | 0.79 | 87617.69 | 92914.58 | 6.05 | 42 | 1.16 | 117948.52 | 118034.12 | 0.07 |
| 18 | 4.36 | 78120.47 | 79827.29 | 2.18 | 43 | 1.37 | 83105.65 | 85342.51 | 2.69 |
| 19 | 0.66 | 102443.71 | 102713.10 | 0.26 | 44 | 1.24 | 93724.34 | 96952.57 | 3.44 |
| 20 | 1.77 | 110548.87 | 110748.32 | 0.18 | 45 | 1.44 | 94444.89 | 97499.24 | 3.23 |
| 21 | 1.03 | 96473.18 | 96541.80 | 0.07 | 46 | 2.40 | 107127.66 | 107627.55 | 0.47 |
| 22 | 2.07 | 120341.61 | 120401.09 | 0.05 | 47 | 1.66 | 116026.54 | 116264.92 | 0.21 |
| 23 | 0.88 | 85443.77 | 86254.36 | 0.95 | 48 | 1.30 | 77111.98 | 78993.13 | 2.44 |
| 24 | 1.67 | 106985.55 | 106991.38 | 0.01 | 49 | 2.23 | 80283.84 | 82085.97 | 2.24 |
| 25 | 5.30 | 107189.84 | 107540.43 | 0.33 | 50 | 4.82 | 119142.82 | 120440.69 | 1.09 |
| | | | | | | EV | 105208.70 | 106540.92 | 0.13 |

## 5. CONCLUSION

A novel methodology to optimize hydrothermal operation planning, incorporating stochastic water inflows via ISO by multidimensional scenario trees reduced through PSO. Additionally, it considers transmission constraints and multi-bus configurations and highlights the adaptability of the models to large and sparsely meshed systems, such as those in South America. Key points include the generation of 10,000 time series, tree reduction to 50 scenarios, and its application on a 3-bus system case study, where the constraints and objective function were assumed linear. A LP was implemented to run both the scenario tree and the original time series with the Monte Carlo technique to validate the results, yielding an EV of $\$105,208.70 \times 10^3$ with average errors of 5.7%, showing the robustness of this methodological approach. Besides, this work proposes and validates a model using a metaheuristic tool (MVMO-SH), achieving near-optimal costs with low errors in medium-term horizons and supplying a technique that could treat non-linear and non-convex problems.

Summarizing, this methodology allows the use of the scenario tree with a few scenarios in exchange for a Monte Carlo technique, which requires a lot of these, obtaining similar results when we run them as ISO models, and allows for modeling complex systems when it is combined with a MVMO tool. However, ISO modeling does not consider the non-anticipative principle, used in an ESO models, which is key because it ensures that hydrothermal operating decisions are made based on the information available at any given time, avoiding fictitious solutions that use information from the future. As future work, we expect to incorporate the ESO strategy into the methodology.

Other implications for future studies include extensions to incorporate renewables, such as wind and solar, and considering models for real-time applications. In practice, this serves as an operational tool for utilities in regions like South America, optimizing costs and reliability in sparse networks. For policy, it informs regulations on reservoir management under climate variability (*e.g.*, ENSO), promoting sustainable energy planning.

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| Name of Author | C | M | So | Va | Fo | I | R | D | O | E | Vi | Su | P | Fu |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Martha Patricia Camargo-Martínez | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | | | ✓ | ✓ | ✓ | | | |
| Ricardo Rincón Ballesteros | ✓ | | | ✓ | | ✓ | | | | ✓ | ✓ | | | |
| Fabian Salazar-Caceres | ✓ | | | | ✓ | | | ✓ | | ✓ | | ✓ | | |
| Andrés F. Panesso H. | | | | | | ✓ | | | | ✓ | | | | |
| Harrynson Ramírez-Murillo | | ✓ | | | ✓ | | | ✓ | | ✓ | | ✓ | | |
| Osvaldo Añó | | ✓ | | | ✓ | | ✓ | | ✓ | | | | | |

C : **C**onceptualization
M : **M**ethodology
So : **So**ftware
Va : **Va**lidation
Fo : **Fo**rmal analysis
I : **I**nvestigation
R : **R**esources
D : **D**ata Curation
O : Writing - **O**riginal Draft
E : Writing - Review & **E**diting
Vi : **Vi**sualization
Su : **Su**pervision
P : **P**roject administration
Fu : **Fu**nding acquisition

## CONFLICT OF INTEREST STATEMENT

Authors state no conflict of interest.

## INFORMED CONSENT

We have obtained informed consent from all participants involved in this study.

## DATA AVAILABILITY

The authors confirm that the data supporting the findings of this study are available within the article. The data, which contain information that could compromise the privacy of research participants, are not publicly available due to certain restrictions.

## REFERENCES

[1] F. Marzbani and A. Abdelfatah, "Economic dispatch optimization strategies and problem formulation: A comprehensive review," *Energies*, vol. 17, no. 3, 2024, doi: 10.3390/en17030550.
[2] A. Carneiro, "Contribuição ao Planejamento da Operação Energética de Sistemas Hidrotérmicos de Potencia," Unicampahnas, Capinas, 1995.
[3] S. Soares, C. Lyra, and H. Tavares, "Optimal generation scheduling of hydrothermal power systems," *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-99, no. 3, pp. 1107–1118, May 1980, doi: 10.1109/TPAS.1980.319741.

[4] X. Zeng, A. T. Hammid, N. M. Kumar, U. Subramaniam, and D. J. Almakhles, "A grasshopper optimization algorithm for optimal short-term hydrothermal scheduling," *Energy Reports*, vol. 7, pp. 314–323, 2021, doi: 10.1016/j.egyr.2020.12.038.

[5] J. Torres-Riveros and S. Rivera-Rodriguez, "Optimal energy dispatch in multiple periods of time considering the variability and uncertainty of generation from renewable sources," *Prospectiva*, vol. 16, no. 2, pp. 75–81, 2018.

[6] X. Zhang, T. Ding, Y. Xiao, H. Zhang, J. Liu, and Y. Wang, "Data-driven multistage distributionally robust programming to hydrothermal economic dispatch with renewable energy sources," *IEEE Transactions on Sustainable Energy*, vol. 15, no. 4, pp. 2322–2335, 2024, doi: 10.1109/TSTE.2024.3416210.

[7] F. Beltrán, E. C. Finardi, and W. de Oliveira, "Two-stage and multi-stage decompositions for the medium-term hydrothermal scheduling problem: A computational comparison of solution techniques," *International Journal of Electrical Power and Energy Systems*, vol. 127, p. 106659, 2021, doi: 10.1016/j.ijepes.2020.106659.

[8] C. A. Junior, B. Dias, and A. Diniz, "Integrated two-stage Benders decomposition and dual dynamic programming for hydrothermal-wind planning with intra-stage cost functions," *IEEE Transactions on Sustainable Energy*, vol. 15, no. 4, pp. 2263–2275, 2024, doi: 10.1109/TSTE.2024.3411577.

[9] B. Colonetti, E. Finardi, P. Larroyd, and F. Beltrán, "A novel cooperative multi-search benders decomposition for solving the hydrothermal unit-commitment problem," *International Journal of Electrical Power and Energy Systems*, vol. 134, p. 107390, 2021, doi: 10.1016/j.ijepes.2021.107390.

[10] M. Reolon Scuzziato, E. Cristian Finardi, and A. Frangioni, "Solving stochastic hydrothermal unit commitment with a new primal recovery technique based on Lagrangian solutions," *International Journal of Electrical Power and Energy Systems*, vol. 127, p. 106661, 2021, doi: 10.1016/j.ijepes.2020.106661.

[11] Z. Zhang, M. Liu, M. Xie, and P. Dong, "A mathematical programming–based heuristic for coordinated hydrothermal generator maintenance scheduling and long-term unit commitment," *International Journal of Electrical Power and Energy Systems*, vol. 147, p. 108833, 2023, doi: 10.1016/j.ijepes.2022.108833.

[12] P. Mundotiya, M. Bhadu, and H. P. Tiwari, "Hydro-thermal scheduling under RE uncertainties using an improved cheetah optimization," *Electrical Engineering*, vol. 106, no. 4, pp. 4339–4370, 2024, doi: 10.1007/s00202-023-02218-2.

[13] C. J. López-Salgado, A. Helseth, O. Añó, and D. M. Ojeda-Esteybar, "Stochastic daily hydrothermal scheduling based on decomposition and parallelization," *International Journal of Electrical Power and Energy Systems*, vol. 118, Jun. 2020, doi: 10.1016/j.ijepes.2019.105700.

[14] K. Rajwar, K. Deep, and S. Das, "An exhaustive review of the metaheuristic algorithms for search and optimization: taxonomy, applications, and open challenges," *Artificial Intelligence Review*, vol. 56, no. 11, pp. 13187–13257, Apr. 2023, doi: 10.1007/s10462-023-10470-y.

[15] P. Mundotiya *et al.*, "A novel hybrid approach for hydrothermal scheduling using mathematical and metaheuristic search methods," *IEEE Access*, vol. 12, pp. 22420–22432, 2024, doi: 10.1109/ACCESS.2024.3364099.

[16] B. Postolov and A. Iliev, "New metaheuristic methodology for solving security constrained hydrothermal unit commitment based on adaptive genetic algorithm," *International Journal of Electrical Power and Energy Systems*, vol. 134, p. 107163, 2022, doi: 10.1016/j.ijepes.2021.107163.

[17] M. S. Fakhar *et al.*, "Conventional and metaheuristic optimization algorithms for solving short term hydrothermal scheduling problem: A review," *IEEE Access*, vol. 9, pp. 25993–26025, 2021, doi: 10.1109/ACCESS.2021.3055292.

[18] P. C. B. Rampazzo, A. Yamakami, and F. O. De Franca, "Operation planning of hydroelectric systems: Application of genetic algorithms and differential evolution," in *Proceedings - 2013 12th International Conference on Machine Learning and Applications, ICMLA 2013*, 2013, vol. 2, pp. 232–237, doi: 10.1109/ICMLA.2013.128.

[19] A. A. F. M. Carneiro, P. T. Leite, D. Silva Filho, and A. C. P. L. F. Carvalho, "Genetic algorithms applied to hydrothermal system scheduling," in *POWERCON 1998 - 1998 International Conference on Power System Technology, Proceedings*, 1998, vol. 1, pp. 547–551, doi: 10.1109/ICPST.1998.729024.

[20] P. T. Leite, A. A. F. M. Carneiro, and A. C. P. L. F. Carvalho, "Hybrid genetic algorithm applied to the determination of the optimal operation of hydrothermal systems," in *Proceedings of the Ninth Brazilian Symposium on Neural Networks, SBRN'06*, 2006, pp. 84–89, doi: 10.1109/SBRN.2006.22.

[21] V. Miranda, D. Srinivasan, and L. M. Proença, "Evolutionary computation in power systems," *International Journal of Electrical Power and Energy Systems*, vol. 20, no. 2, pp. 89–98, Feb. 1998, doi: 10.1016/S0142-0615(97)00040-9.

[22] O. Dowson, D. P. Morton, and A. Downward, "Bi-objective multistage stochastic linear programming," *Mathematical Programming*, vol. 196, no. 1–2, pp. 907–933, 2022, doi: 10.1007/s10107-022-01872-x.

[23] J. L. Morillo, L. Zéphyr, J. F. Pérez, C. Lindsay Anderson, and Á. Cadena, "Risk-averse stochastic dual dynamic programming approach for the operation of a hydro-dominated power system in the presence of wind uncertainty," *International Journal of Electrical Power and Energy Systems*, vol. 115, p. 105469, 2020, doi: 10.1016/j.ijepes.2019.105469.

[24] R. E. C. Gonalves, E. C. Finardi, and E. L. Da Silva, "Applying different decomposition schemes using the progressive hedging algorithm to the operation planning problem of a hydrothermal system," *Electric Power Systems Research*, vol. 83, no. 1, pp. 19–27, 2012, doi: 10.1016/j.epsr.2011.09.006.

[25] P. T. Ha, D. T. Tran, T. M. Phan, and T. T. Nguyen, "Maximization of total profit for hybrid hydro-thermal-wind-solar power systems considering pumped storage, cascaded systems, and renewable energy uncertainty in a real zone, Vietnam," *Sustainability (Switzerland)*, vol. 16, no. 15, 2024, doi: 10.3390/su16156581.

[26] P. T. M. Lira, A. L. Diniz, and C. L. T. Borges, "Multi-plant hydro power production function modeling: Application to the stochastic mid-term hydrothermal planning problem," *Electric Power Systems Research*, vol. 235, p. 110800, 2024, doi: 10.1016/j.epsr.2024.110800.

[27] L. Huu Pham, B. Hoang Dinh, T. Trung Nguyen, and V. D. Phan, "Optimal operation of wind-hydrothermal systems considering certainty and uncertainty of wind," *Alexandria Engineering Journal*, vol. 60, no. 6, pp. 5431–5461, 2021, doi: 10.1016/j.aej.2021.04.025.

[28] K. Dasgupta, P. K. Roy, and V. Mukherjee, "Solution of short term integrated hydrothermal-solar-wind scheduling using sine cosine algorithm," *Energy Strategy Reviews*, vol. 40, p. 100824, 2022, doi: 10.1016/j.esr.2022.100824.

[29] S. Hazra and P. K. Roy, "Optimal dispatch using moth-flame optimization for hydro-thermal-wind scheduling problem," *International Transactions on Electrical Energy Systems*, vol. 30, no. 8, pp. 1–24, 2020, doi: 10.1002/2050-7038.12460.

[30] V. P. Sakthivel, K. Thirumal, and P. D. Sathya, "Quasi-oppositional turbulent water flow-based optimization for cascaded short term hydrothermal scheduling with valve-point effects and multiple fuels," *Energy*, vol. 251, p. 123905, 2022, doi: 10.1016/j.energy.2022.123905.

[31] N. Fang, J. Zhou, R. Zhang, Y. Liu, and Y. Zhang, "A hybrid of real coded genetic algorithm and artificial fish swarm algorithm for short-term optimal hydrothermal scheduling," *International Journal of Electrical Power and Energy Systems*, vol. 62,

pp. 617–629, 2014, doi: 10.1016/j.ijepes.2014.05.017.
[32] A. Kumar and J. S. Dhillon, “Environmentally sound short-term hydrothermal generation scheduling using intensified water cycle approach,” *Applied Soft Computing*, vol. 127, p. 109327, 2022, doi: 10.1016/j.asoc.2022.109327.
[33] S. Soares, “Programação matemática de grande porte: Uma aplicação a sistemas hidrotérmicos,” Universidad Estadual de Campinas, Brasil, 1978.
[34] J. M. Latorre, S. Cerisola, and A. Ramos, “Clustering algorithms for scenario tree generation: Application to natural hydro inflows,” *European Journal of Operational Research*, vol. 181, no. 3, pp. 1339–1353, 2007, doi: 10.1016/j.ejor.2005.11.045.
[35] V. Pappala, “Aplication of PSO for optimization of power systems under uncertainty,” 2009.
[36] S. Cerisola and A. Ramos, “Node aggregation in stochastic nested Benders decomposition applied to hydrothermal coordination,” in *PMAPS2000: 6th International Conference on Probabilistic Methods Applied to Power Systems*, 2000, p. 84.
[37] R. Fourer and L. Lopes, “A management system for decompositions in stochastic programming,” *Annals of Operations Research*, vol. 142, no. 1, pp. 99–118, 2006, doi: 10.1007/s10479-006-6163-1.
[38] J. Dupačová, G. Consigli, and S. W. Wallace, “Scenarios for multistage stochastic programs,” *Annals of Operations Research*, vol. 100, no. 1–4, pp. 25–53, 2000, doi: 10.1023/a:1019206915174.
[39] A. Corominas, “On deciding when to stop metaheuristics: Properties, rules and termination conditions,” *Operations Research Perspectives*, vol. 10, p. 100283, 2023, doi: 10.1016/j.orp.2023.100283.
[40] I. Erlich, G. K. Venayagamoorthy, and N. Worawat, “A mean-variance optimization algorithm,” in *2010 IEEE World Congress on Computational Intelligence, WCCI 2010 - 2010 IEEE Congress on Evolutionary Computation, CEC 2010*, Jul. 2010, pp. 344–349, doi: 10.1109/CEC.2010.5586027.
[41] M. P. Camargo, J. L. Rueda, I. Erlich, and O. Añó, “Comparison of emerging metaheuristic algorithms for Optimal hydrothermal system operation,” *Swarm and Evolutionary Computation*, vol. 18, pp. 83–96, 2014, doi: 10.1016/j.swevo.2014.04.001.
[42] J. L. Rueda and I. Erlich, “Hybrid mean-variance mapping optimization for solving the IEEE-CEC 2013 competition problems,” in *2013 IEEE Congress on Evolutionary Computation, CEC 2013*, Jun. 2013, pp. 1664–1671, doi: 10.1109/CEC.2013.6557761.
[43] D. Ojeda, “Coordinación integrada de sistemas eléctricos y de gas natural de mediano plazo,” Universidad Nacional de San Juan, 2010.
[44] T. Lohmann, A. S. Hering, and S. Rebennack, “Spatio-temporal hydro forecasting of multireservoir inflows for hydro-thermal scheduling,” *European Journal of Operational Research*, vol. 255, no. 1, pp. 243–258, 2016, doi: 10.1016/j.ejor.2016.05.011.
[45] A. Navarro, M. Favereau, Á. Lorca, D. Olivares, and M. Negrete-Pincetic, “Medium-term stochastic hydrothermal scheduling with short-term operational effects for large-scale power and water networks,” *Applied Energy*, vol. 358, p. 122554, 2024, doi: 10.1016/j.apenergy.2023.122554.

## BIOGRAPHIES OF AUTHORS

**Martha Patricia Camargo Martínez** electrical engineer and electrical engineer Master in Distribution area. She is currently finishing her Ph.D. studies at the National University of San Juan, Argentina. She works in areas such as high voltage, power quality, distribution, and energy markets. She is interested in linking to the electrical sector and contributing her experience in research, group work, and consulting development, in the professional field. She can be contacted at email: mpcamargo@unisalle.edu.co.

**Ricardo Rincón Ballesteros** received his bachelor’s and master’s degrees in electrical engineering from Universidad Nacional de Colombia, Bogotá, in 2004 and 2014, respectively. He is currently working toward Ph.D. degree from Universidad Nacional de Colombia, and he also is currently an assistant professor in the electrical engineering program at Universidad de La Salle, Bogotá D.C., Colombia. His main research interests include power system stability, particularly voltage stability, energy management systems, and renewable energy. He can be contacted at email: rrinconb@unisalle.edu.co.

**Fabian Salazar-Caceres** received his bachelor’s degree in design engineering and electronic automation from the Universidad de La Salle, Bogotá, and the master’s degree in electrical engineering from the Universidad de los Andes, Bogotá, in 2009 and 2014 respectively. He is currently working toward the Ph.D. degree part-time from Universidad Nacional de Colombia, and he is also currently a Professor in the Facultad de Ingeniería, Programa de Ingeniería Eléctrica, Universidad de La Salle, Bogotá D.C., Colombia. His main research interests include dynamical systems and control, machine learning, and optimization. He can be contacted at email: jfsalazar@unisalle.edu.co

**Andrés F. Panesso H.** received his B.E., and M.Sc. degrees in electrical engineering from the Universidad Tecnológica de Pereira, Colombia, in 2010 and 2013, respectively. Currently, he is pursuing his doctorate degree in Agrosciences from Universidad de La Salle, Colombia. He is an assistant professor at the Electrical Engineering Program, Universidad de La Salle (Colombia), and a researcher of AVARC-GI group. His research interests are in the areas of fault analysis and location, power distribution modeling, reliability, education in engineering, and rural development. He can be contacted at email: afpanesso@unisalle.edu.co.

**Harrynson Ramírez-Murillo** received bachelor's and master's degrees in electrical engineering from the Universidad Tecnológica de Pereira (UTP), Pereira, Colombia, in 2007 and 2010, respectively, and the master's and Ph.D. degrees in electronic engineering from the Universitat Rovirai Virgili (URV) de Tarragona, Tarragona, Spain, in 2011 and 2015, respectively. He is currently a professor in the Facultad de Ingeniería, Programa de Ingeniería Eléctrica, Universidad de La Salle, Bogotá, Colombia. His main research interests include fuel cells, DC-DC power converters, renewable energy resources, energy management systems, and machine learning. He can be contacted at email: hramirez@unisalle.edu.co.

**Osvaldo Añó** received the electromechanical engineering degree in 1983 from the National University of San Juan, Argentina, where he is currently a tenured professor and the leader of a power systems optimization group within the Institute of Electric Energy (IEE). His main research interests are in the area of competitive markets of energy and reserve. He can be contacted at email: oagno@iee-unsjconicet.org.