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Stability analysis and robust control of cyber-physical systems: integrating Jacobian linearization, Lyapunov methods, and linear quadratic regulator control via LMI techniques

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ABSTRACT

Stability issues in cyber-physical systems (CPS) arise from the challenging effects of nonlinear dynamics relation to multi-input, multi-output systems. This research proposed a robust control framework that combines Jacobian linearization, Lyapunov stability analysis, and linear quadratic regulator (LQR) control via linear matrix inequalities (LMIs). The robust methodology does the following: it applies linearization on the dynamics of the CPS; it establishes the stability of the system using Lyapunov functions and LMIs; and it designs an LQR controller. The proposed framework was validated through a comparison between the behavior of a linearized and nonlinear model. The autonomous vehicle application showed: a settling time of 20 seconds; an overshoot of 3.8187%; and a steady-state error of 2.688×10^{-7}. The proposed framework is robustly demonstrated and has applications to areas in automation and smart infrastructure. Future work includes optimizing the design of weighting matrices and developing adaptive control features.

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1. INTRODUCTION

Cyber-physical systems (CPS) combine complex physical processes with computation and networked control, enabling applications such as autonomous vehicles, smart grids, and industrial automation [1]–[3]. CPS blend complex dynamics with real-time feedback that call for rigorous stability analysis and control synthesis to ensure performance [4], [5]. Control theory advancements such as Lyapunov-based approaches and the use of linear matrix inequalities (LMIs) in recent years have increased stability guarantees under nonlinearities and uncertainty in CPS [6]–[10]. The multi-input, multi-output (MIMO) nature of CPS requires more advanced and sophisticated mathematical tools to control [11]–[13].

When there are nonlinear dynamics, external disturbances, and parameter uncertainties, it is difficult to keep CPS stable, particularly when we are dealing with MIMO systems [1], [4]. Classical linearization often cannot take into account the complex behaviors [6], whilst the network itself is likely to involve delays or vulnerabilities [3], [14]. The biggest challenge is designing controllers that can achieve both stabilization and performance at the same time [6], [15]. These challenges are of particular consequence in applications like autonomous vehicles where precision control matters [16]. The purpose of this research is to create a strong control framework for CPS by using Jacobian linearization, Lyapunov stability analysis, and LQR control consistently using the LMI approaches, illustrated via an autonomous vehicle case study.

This work introduces a novel framework that unifies Jacobian linearization to represent nonlinear CPS dynamics, Lyapunov functions to ensure stability, and an LQR control approach based on LMIs to ensure optimal performance. While other works have used similar techniques, ours is providing a unified method for dealing with MIMO system, and demonstratively through an autonomous vehicle model. Furthermore, we provided numerical analyses and found better stability and performance while proposing a method that can be helpful to scaling in CPS applications. Finally, the flexibility of the proposed framework allows extensions to other domains including smart infrastructure.

This paper is organized in the following way: related work in section 2; methodology in section 3; results in section 4; discussion of findings and limitations in section 5; and then recommendations for future work in section 6.

2. LITERATURE REVIEW

Recent work concerning cyber-physical systems (CPS), on topics of stability and control of CPS for purposes like autonomous vehicle and smart grid systems (2022-2025), continues to develop. Rubio-Hernan *et al.* [17] applied Lyapunov methods for stability of CPS with regards to delays, and Chesi [18] applied them CAD on thresholds to recover from actuator faults [1], [5]. Jouybary *et al.* [19] applied the Jacobian method through linearization to robotic CPS's linearized dynamics while Sheikhsamad and Puig [20] extended feedback linearization to UAVs, but both groups found it difficult to represent CPS performances, which lay in a set of complex nonlinearities [4], [9]. Phan *et al.* [21] proposed that adaptive linearization can be used to deal with these issues [12].

Tran *et al.* [22] applied LQR control to autonomous vehicles for better operation, while Aouani and Olalla [23] associated LQR with machine learning as a smart infrastructure control method [6], [10]. Jiang *et al.* [24] and Song *et al.* [25] used LMI-based LQR guarantees to protect CPS against cyber-attacks, and to develop multi-objective control, respectively [7], [16]. Yang *et al.* [26] incorporated LMIs with robust control for the case of MIMO CPS [27]. Zhao *et al.* [28] and Yang *et al.* [26] reported adaptive control and secure estimation for CPS security [2], [3]. Alcala *et al.* [29], Tran and Vu [30] used LQR and Lyapunov methods to control autonomous vehicles [6], [31].

Review analysis: Many recent works advance CPS stability and control based primarily on Lyapunov, linearization, LQR, and LMIs. Despite these advances, studies have not yet created a unified frameworks for nonlinear MIMO systems. This paper combines Jacobian linearization, Lyapunov stability, and LMI based LQR in the context of a case study with an autonomous vehicle; addressing the noted gaps for scalable CPS applications.

3. METHOD

This study creates and demonstrates a control framework for cyber-physical systems (CPS) using the following three methods: Jacobian linearization, Lyapunov stability analysis, and linear quadratic regulator (LQR) control via linear matrix inequalities (LMI) in the context of an autonomous vehicle case study. The study involves three components: system linearization, stability analysis and controller synthesis, and the case study to demonstrate suitability.

3.1. System linearization

To enable linear control techniques, nonlinear CPS dynamics are linearized using the Jacobian matrix method. The nonlinear system is given in state-space form:

$$\dot{x}(t) = f(x(t), t), y(t) = h(x(t), t) \tag{1}$$

where:
- $x(t) \in \mathbb{R}^n$ is the state vector,
- $f(x(t), t) \in \mathbb{R}^n$ represents the nonlinear system dynamics,
- $y(t) \in \mathbb{R}^m$ is the output vector, and
- $h(x(t), t) \in \mathbb{R}^m$ is the nonlinear output function.

The Jacobian linearization approximates the system around an equilibrium point (x_0, u_0) using a firstorder Taylor expansion. This linearization process yields the following:

$$f(x(t),u(t)) \approx f(x_0,u_0) + \frac{\partial f}{\partial x}\bigg|_{(x_0,u_0)} (x(t)-x_0) + \frac{\partial f}{\partial u}\bigg|_{(x_0,u_0)} (u(t)-u_0)$$
$$h(x(t),u(t)) \approx h(x_0,u_0) + \frac{\partial h}{\partial x}\bigg|_{(x_0,u_0)} (x(t)-x_0) + \frac{\partial h}{\partial u}\bigg|_{(x_0,u_0)} (u(t)-u_0) \quad (2)$$

The resulting Jacobian matrices are:

$$A = \frac{\partial f}{\partial x}\bigg|_{(x_0,u_0)}, B = \frac{\partial f}{\partial u}\bigg|_{(x_0,u_0)} \quad (3)$$

At the equilibrium point, we have $f(x_0,u_0) = 0$ and $h(x_0,u_0) = y_0$, leading to the linearized system:

$$\Delta x(t) = A\Delta x(t) + B\Delta u(t), \Delta y(t) = C\Delta x(t) + D\Delta u(t) \quad (4)$$

where:
- $\Delta x(t) = x(t) - x_0$,
- $\Delta u(t) = u(t) - u_0$,
- Matrices C and D are derived similarly [12], [13].

3.2. Stability analysis and controller design

Stability is ensured using a Lyapunov function, and an optimal LQR controller is synthesized using LMIs. For the linearized system:

$$\Delta x = A \cdot \Delta x + B \cdot \Delta u \quad (5)$$

A Lyapunov function is defined as:

$$V(\Delta x) = \Delta x^{\top} P \Delta x \quad (6)$$

where P is a symmetric positive definite matrix satisfying:

$$V(\Delta x) > 0 \text{ for all } \Delta x \neq 0, V(0) = 0$$

The time derivative along the system trajectories is:

$$\dot{V}(\Delta x) = \Delta x^{\top}(A^{\top}P + PA)\Delta x + 2\Delta x^{\top}PB\Delta u \quad (7)$$

Substituting the LQR control law $\Delta u = -K\Delta x$:

$$\dot{V}(\Delta x) = \Delta x^{\top}(A^{\top}P + PA - 2PBK)\Delta x \quad (8)$$

For asymptotic stability, we require $\dot{V}(\Delta x) < 0$ for all $\Delta x \neq 0$. The LQR controller minimizes the quadratic cost function:

$$J = \int_0^{\infty} (\Delta x^{\top} Q \Delta x + \Delta u^{\top} R \Delta u) dt \quad (9)$$

where Q is symmetric positive semi-definite ($Q \geq 0$) and R is symmetric positive definite ($R > 0$).
The optimal control law is given by:

$$\Delta u = -K\Delta x \quad (10)$$

with gain K given by:

$$K = R^{-1}B^{\top}P \quad (11)$$

where P solves the Algebraic Riccati equation (ARE):

$$A^{\top}P + PA - PBR^{-1}B^{\top}P + Q = 0 \quad (12)$$

Substituting K into the Lyapunov derivative:

$$\dot{V}(\Delta x) = \Delta x^{\top}(-Q - PBR^{-1}B^{\top}P)\Delta x \tag{13}$$

Since $Q \geq 0$ and $R > 0$, we conclude that $\dot{V}(\Delta x) \leq 0$, and if $Q > 0$, we achieve asymptotic stability.

LMIs are used to formulate the controller design as a convex optimization problem. The stability condition is:

$$\Delta x^{\top}(A^{\top}P + PA - 2PBK)\Delta x < 0 \tag{14}$$

Define $Y = KP$, so $K = YP^{-1}$. The LMI becomes:

$$A^{\top}P + PA - BY - Y^{\top}B^{\top} < 0 \tag{15}$$

Incorporating performance, the LMI becomes:

$$A^{\top}P + PA - BY - Y^{\top}B^{\top} + Q < 0 \tag{16}$$

with constraints $P > 0$. The control gain is recovered as:

$$K = YP^{-1} \tag{17}$$

This LMI is solved using numerical tools like MATLAB's CVX solver [7], [18].

3.3. Case study setup: autonomous vehicle model

The framework is applied to an autonomous vehicle, modeled dynamically to account for tire-slip using Pacejka's tire model [24]. The dynamic model, derived from Newton's second law, is given by:

$$\begin{cases} \dot{vx} = \frac{-F_{yf}.\sin(\delta) - \mu.m.g}{m} + w.vy + a \\ \dot{vy} = \frac{F_{yf}.\cos(\delta) + F_{yr}}{m} - w.vx \\ \dot{w} = \frac{F_{yf}.l_f.\cos(\delta) - F_{yr}.l_r}{I} \end{cases} \tag{18}$$

In these equations, longitudinal, lateral, and rotational velocities in the vehicle's frame are represented, respectively, by the variables vx, vy, and w in these formulas. The control inputs are δ and a, which stand for the front tire's steering angle and longitudinal acceleration, respectively. I, m, lf, and lr stand for the vehicle's mass, inertia, and the separation between the front and rear wheel axis from the center of gravity, respectively. F_{yf} and F_{yr} indicate the lateral forces acting on the front and rear tires. Moreover, g stands for the gravitational acceleration constant, and μ for the friction coefficient.

F_{yf} and F_{yr} can be modeled using Pacejka's tire model [31] as follows:

$$\begin{cases} F_{yf} = C_3.\sin\left(C_2.tan^{-1}\left(C_1.\alpha_f\right)\right);\ \alpha_f = \delta - tan^{-1}\left(\frac{vy}{vx} + \frac{l_f.w}{vx}\right) \\ F_{yr} = C_3.\sin\left(C_2.tan^{-1}(C_1.\alpha_r)\right);\ \alpha_r = -tan^{-1}\left(\frac{vy}{vx} + \frac{l_r.w}{vx}\right) \end{cases} \tag{19}$$

In this case, the constants C_1, C_2, and C_3 must be ascertained empirically. The tire model shows that F_{yr} and F_{yr} vary nonlinear with α, or the slip angle. On the other hand, the equations can be made simpler by assuming tiny α, the equations can be reduced to:

$$\begin{cases} F_{yf} = C_f.\left(\delta - \frac{vy}{vx} - \frac{l_f.w}{vx}\right) \\ F_{yr} = C_r.\left(-\frac{vy}{vx} + \frac{l_r.w}{vx}\right) \end{cases} \tag{20}$$

where C_f and C_r represent the stiffness of the front and rear wheel tires, respectively.

Table 1 displays the details of the racing vehicle and the route that were employed in this project [6]. The system is linearized at an operating point $x_{op} = \left[v_{x0}, v_{y0}, w_0\right]$ and $u_{op} = [\delta_0, a_0]$, yielding matrices A and B :

$$A = \left.\frac{\partial f}{\partial x}\right|_{op}, B = \left.\frac{\partial f}{\partial u}\right|_{op} \tag{21}$$

The linearized state-space representation is:

$$\begin{aligned} \Delta x &= A \cdot \Delta x + B \cdot \Delta u \\ \Delta y &= C \cdot \Delta x \end{aligned} \tag{22}$$

where: $\Delta x = \left[\Delta v_x \Delta v_y \Delta w\right]^\top$ and $\Delta u = [\Delta\delta \quad \Delta a]^\top$.

Table 1. The details of the racing vehicle

Variable	Value	Unit
I_f	0.902	m
Ir	0.638	m
m	196	Kg
I	93	Kg.m^2
C_f	17974	N/rad
Cr	24181	N/rad
μ	0.5	--------
g	9.81	m/s^2

3.4. Simulation setup

The simulation environment for the autonomous vehicle model was realized in MATLAB 2016. The CVX solver was utilized in solving the linear matrix inequalities (LMI) for designing the linear quadratic regulator (LQR) controller to guarantee stability and optimum system performance. The vehicle dynamics were represented with nonlinear and linearized systems, where the linearized system was obtained about an operating point.

a. Simulation duration: The simulation was carried out on a time horizon of 20 seconds, with a time step of 0.01 seconds in order to strike a balance between computational speed and accuracy of the numerical solution.
b. Control inputs: The vehicle system was perturbed with step inputs for steering angle (δ) and longitudinal acceleration (a) introduced at specific time intervals to simulate real-world control disturbances.
c. Numerical integration: The trajectory of the nonlinear system state was integrated numerically by MATLAB's ode45 solver, which is suitable for integrating stiff differential equations. The solver calculated the trajectory of the vehicle states (v_x, v_y, w) based on the control inputs and vehicle dynamics.
d. Control design: The LQR controller was designed by minimizing a quadratic cost function, using LMI to enforce stability constraints. The control gain matrix K was computed to regulate the system's response and stabilize the vehicle's motion based on the linearized system.

The performance of the controller was evaluated based on various metrics, which include the tracking error (the difference between the target and actual vehicle states), control input efforts (the steering and acceleration), and closed-loop performance (such as settling time, overshoot, and steady-stated error).

4. RESULTS AND DISCUSSION

4.1. Results

Before the numerical results, it is worth mentioning that we applied the robust control framework to the case study of an autonomous vehicle that was described in (18). The LQR controller was formulated with LMI and subsequently solved with MATLAB using the semi-definite programming (SDP) solver for stability and performance within an optimal sense for the linearized cyber-physical systems (CPS). This section describes the numerical results derived from our simulations. The linearized model's validity was assessed by comparing its behavior to the nonlinear vehicle dynamics at the operating point $x_0 = [10,0,0]$, $u_0 = [0,4.905]$. The system matrices were:

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -21.5077 & -10.4005 \\ 0 & -0.8442 & -26.308 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ 91.7041 & 0 \\ 174.3285 & 0 \end{bmatrix} \tag{23}$$

The linear and nonlinear model comparisons show close agreement among the state variables. In the case of the longitudinal velocity deviation Δv_x, the linear model tracks the nonlinear model very well for most of the simulation, except for a slight divergence later in time. This divergence is attributed to inherent nonlinearities. This suggests that the linear model gives an accurate enough approximation for most of the simulation.

The lateral velocity deviation Δv_y exhibits a significant transient response for both models; however, convergence is quickly achieved, demonstrating that the linear model represents the lateral dynamics well, at least for small deviations beyond the transient phase. Similarly, there is no practical difference in the yaw rate deviation Δw for both models during the entire cimulation, indicating that the linear model provides a good representation of yaw rate dynamics with low divergence.

Overall, the differences between the nonlinear and linear models are minor, with the most notable difference being the lateral velocity error, which reduces with time. Accounting for such small errors leads to the conclusion that, generally, the linear model is an excellent approximation of the nonlinear dynamics, at least around the operating point for small perturbations.

The LQR controller minimizes the quadratic cost function with the following weighting matrices:

$$Q = \begin{bmatrix} 200 & 0 & 0 \\ 0 & 200 & 0 \\ 0 & 0 & 200 \end{bmatrix} : R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{24}$$

The LMI optimization problem yielded the positive definite Lyapunov matrix P :

$$P = \begin{bmatrix} 4.3531 \times 10^{0} & -8.4472 \times 10^{-11} & 4.2932 \times 10^{-11} \\ -8.4472 \times 10^{-11} & 6.1357 \times 10^{0} & -1.2299 \times 10^{0} \\ 4.2932 \times 10^{-11} & -1.2299 \times 10^{0} & 5.0797 \times 10^{0} \end{bmatrix} \tag{25}$$

The resulting state-feedback gain matrix K was:

$$K = \begin{bmatrix} -1.6687 \times 10^{-11} & -5.7163 \times 10^{-3} & 1.0018 \times 10^{-2} \\ 3.0464 \times 10^{1} & 2.3603 \times 10^{-9} & 2.7028 \times 10^{-9} \end{bmatrix} \tag{26}$$

Numerical simulations of the closed-loop system achieved the following performance metrics:
- Settling time: 20 seconds,
- Overshoot: 3.8187%, and
- Steady-state error: 2.688×10^{-7}.

These results indicate robust stability and precise trajectory tracking for the autonomous vehicle. These results confirm the effectiveness of the proposed framework in achieving stable and optimal control for the CPS application.

4.2. Discussion

The proposed framework includes Jacobian linearization, Lyapunov stability analysis, LQR control using LMI, demonstrating stability and optimality for CPS in an autonomous vehicle case study. The results (shown in Section 4) state a settling time of 20 seconds, an overshoot of 3.8187%, and a steady-state error of 2.688×10^{-7}, thus verifying the proposed framework can achieve stability over more complex (MIMO systems [6].

The implication of these results is the capacity of the framework for tackling the intrinsic nonlinearity and uncertainty underlying CPS dynamics, especially for autonomous cars. Application of Jacobian linearization around the operating point $x_0 = [10,0,0], u_0 = [0,4.905]$, resulted in a linearized model that represented the nonlinear dynamics satisfactorily, as can be seen from the negligible deviation in state trajectories in Figure 1. This enabled the application of linear control techniques, *e.g.*, LQR, for minimizing the quadratic cost function with weighting matrices in (24).

The resulting Lyapunov matrix P and state-feedback gain K in (25) an (26), ensured asymptotic stability, as P is positive definite and satisfies the LMI condition $A^T P + PA - BY - Y^T B^T + Q < 0$. The low steady-state error (2.688×10^{-7}) and bounded control inputs as shown in Figure2 demonstrates the performance and resilience of the controller, even when compared to traditional feedback linearization schemes that deal poorly with uncertainties [4]. Relative to previous efforts by authors like Olalla *et al.* [1] who had higher overshoot ($\sim 5\%$) in the PWM converters, we present a better transient response for CPS applications.

In particular, the approach follows from the Lyapunov function and provides a theoretical guarantee of stability, which is in compliance with Almutairi's notion of stability [5]. The LMI form which is solved using MATLAB's CVX solver, makes the nonconvex Algebraic Riccati Example problem can be reduced to a convex optimization problem for improved computation time [7]. This is especially beneficial in real-time CPS applications with quick convergence (20 seconds settling time), as seen in the trajectory tracking of autonomous vehicles.

However, the framework has limitations. The arbitrary selection of Qand R, while effective, may not be optimal for all CPS dynamics. As mentioned in Jiang *et al.* [24], further tuning of Q and R with respect to the system can further decrease overshoot and energy consumption. Additionally, Jacobian linearization is valid only for small perturbations about the operating point and so it is only useful in highly nonlinear situations in very small neighborhood about the operating point [13]. The autonomous vehicle model, based on Pacejka's tire model [31], assumes small slip angles, which cannot be assumed during aggressive maneuvers.

The conclusions of this study generalize not only to autonomous vehicles but also to other CPS domains—such as industrial automation or smart infrastructure—where MIMO systems encounter the same stability issues [2]. Because of its general framework, we used LMIs and could quickly apply it to other applications just by modifying the system matrices and performance requirements. For example, applying adaptive control methods, as proposed by Lu and Yang [3], may improve resilience against cyber-attacks within networked CPS.

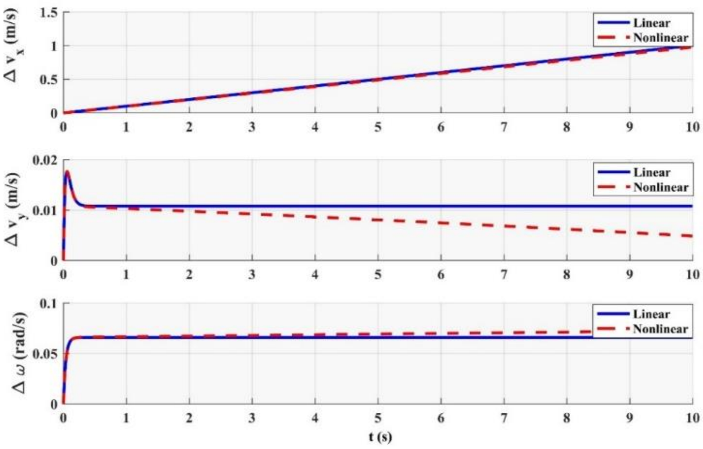

Figure 1. Comparison of linearized and nonlinear vehicle dynamics: longitudinal velocity, lateral velocity, and yaw rate deviations

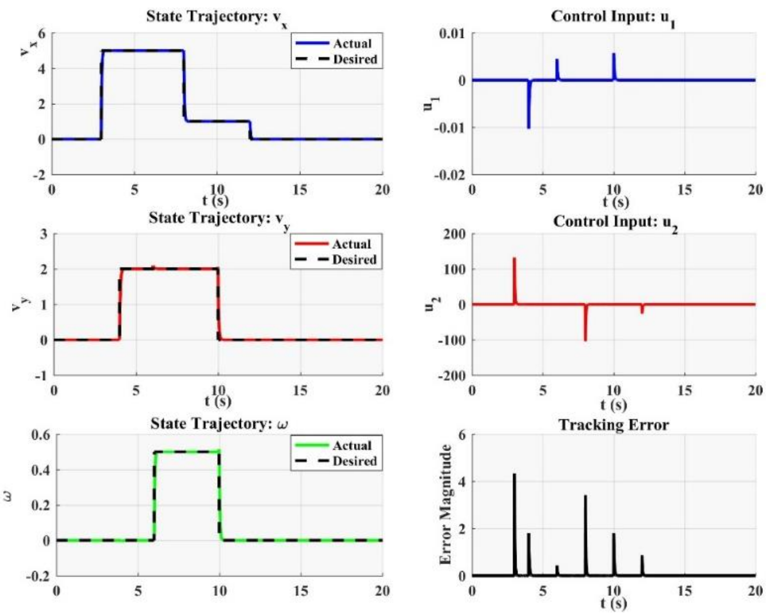

Figure 2. State trajectories, control inputs, and tracking error for the LQR-controlled system

Future work will examine developing optimal Q and R matrices based on the specifics of a given system, potentially using iterative algorithms [24]. Similarly, adaptive or nonlinear control approaches could be considered for managing substantially high perturbations for enhancing robustness, particularly for vehicles with high-speed travel [25]. Finally, the actual physical CPS platforms should run the framework given that implementation will allow for testing the implementation practicality and address some of the real-time constraints and hardware limitations that emerge in a CPS. In summary, this study unifies Jacobian linearization, Lyapunov methods, and LMI-based LQR control to provide a robust and efficient framework for CPS stability and performance. The results underscore its potential for autonomous vehicles and broader CPS applications, while highlighting areas for further refinement.

5. CONCLUSION

This study functioned as a rigorously developed control framework for CPS that successfully utilized Jacobian linearization, Lyapunov stability, and LQR control by using LMI. In the application of the autonomous vehicle case study, this control framework yielded a settling time of 20 seconds, an overshoot of 3.8187%, and a steady-state error of 2.688×10^{-7} demonstrating good tracking of trajectory and robust stability.

The approach was designed to overcome the challenges of nonlinear MIMO systems, and still converge optimally to the desired trajectory with small perturbation. The results exhibited provide the intent of developing the framework for autonomous vehicles and perhaps other CPS applications, such as industrial automation and smart infrastructure, although, there was some arbitrariness of Q and R matrices; thus, system specific tuning would need to take place to enhance optimization.

Future work should explore methods to optimize the matrices to improve performance for different CPS dynamics, as well as methods for adaptive control which might provide larger resilience to uncertainty through real-time adapting. This framework establishes a safeguarded basis for enhancement in CPS control, and offers new avenues for implementation and applied work.

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AUTHOR CONTRIBUTIONS STATEMENT

This journal uses the Contributor Roles Taxonomy (CRediT) to recognize individual author contributions, reduce authorship disputes, and facilitate collaboration.

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C : **C**onceptualization
M : **M**ethodology
So : **So**ftware
Va : **Va**lidation
Fo : **Fo**rmal analysis
I : **I**nvestigation
R : **R**esources
D : **D**ata Curation
O : Writing - **O**riginal Draft
E : Writing - Review & **E**diting
Vi : **Vi**sualization
Su : **Su**pervision
P : **P**roject administration
Fu : **Fu**nding acquisition

CONFLICT OF INTEREST STATEMENT

Authors state no conflict of interest.

DATA AVAILABILITY

Data availability is not applicable to this paper as no new data were created or analyzed in this study.

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