Generalization of reactive power definition for periodical waveforms

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ABSTRACT

The article presents a selection of reactive power definitions, which are applicable for implementation in energy meters. For sinusoidal current and voltage waveforms, all provided dependencies yield equivalent reactive power values. However, in the presence of distorted current and voltage, the power values are determined by the applied method (algorithm). Standardization requirements for reactive energy meters stipulate metrological verification under sinusoidal conditions. The selection of an optimal reactive power definition remains a problematic and ongoing subject of debate within the field. The paper shows that a generalized unique definition of additive reactive power derives from the definition of active power. Unlike active power, reactive power must be independent of the conversion of electric energy into work and heat. This independence is achieved if one of the waveforms – the current in the scalar voltage and current product (specifying active power) – is replaced by a special orthogonal waveform. An orthogonal waveform can be derived through either differentiation or integration. Reactive power obtained by this method is an additive within the system. When differentiation is employed, the reactive power for a nonlinear resistive load with a unique, time-invariant current-voltage characteristic will be zero. Some other properties of reactive power defined in this way are presented. This method is straightforward to implement in reactive energy meters.

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1. INTRODUCTION

The distribution system operator (DSO) outlines the method for reactive energy consumption charges within its tariff, based on national regulations. Consumers supplied from medium, high, and extra-high voltage networks are subject to these reactive energy billing arrangements. Furthermore, consumers supplied from networks with a nominal voltage not exceeding 1 kV may also be included in such billing.

Reactive energy measurements are performed using meters that comply with the requirements of relevant standardization regulations [1]–[3]. The reactive energy, E_q, is determined from formula (1):

$$E_q = \sum_{k=1}^{m} Q(k) \cdot T \quad (1)$$

where Q represents the reactive power within the measurement window of duration T, and m denotes the sequential number of the window. Reactive energy forms the basis for financial settlements, and as such, its measurement accuracy and its measurement methodology should be beyond doubt.

The average electrical energy over a voltage or current waveform period, is called active power, is converted into work and heat.

$$P = \frac{1}{T}\int_{t_0}^{T+t_0} pdt = \frac{1}{T}\int_{t_0}^{T+t_0} uidt \tag{2}$$

where p – instantaneous electrical power; u – periodically varying voltage and current with period T, t_0 – any moment, it can be zero.

Reactive power for sinusoidal voltage $u = \sqrt{2}Usin\,\omega t$ and current $i = \sqrt{2}Isin(\omega t + \varphi)$ waveforms is derived from active power.

$$P = \frac{2UI}{T}\int_{0}^{T} sin\omega t\; sin(\omega t + \varphi) = UI\cos\varphi \tag{3}$$

by replacing one of the functions with the orthogonal function

$$Q = \frac{2UI}{T}\int_{0}^{T} sin\omega t\; cos(\omega t + \varphi) = UI\sin\varphi. \tag{4}$$

It follows from (3) and (4) that.

$$P^2 + Q^2 = S^2 \tag{5}$$

where S=UI – apparent power.

It seems that the problem with defining reactive power for non-sinusoidal voltage and current waveforms stems from the fact not the physical principles, but formal equation (5) is used as the basis for this purpose.

For nearly a hundred years orthogonal power components for non-sinusoidal waveforms have been so defined as having their geometric sum equal to the apparent power [4]–[6] as in the case of sinusoidal waveforms (4). An overview of the development of power theory can be found in review papers [7]–[12]. But apparent power is not an additive quantity. Also, the newly formed power components are most often nonadditive. They are suitable for describing the energy properties of an object only in unusual cases.

By analogy to the formula for active power calculated from Fourier series

$$P = \sum_{n=1}^{\infty} U_n I_n\, cos\,\varphi_n. \tag{6}$$

C.I. Budeanu [13] defined the reactive power of distorted waveforms.

$$Q_B = \sum_{n=1}^{\infty} U_n I_n\, sin\,\varphi_n \tag{7}$$

where: n indicates the harmonic number, φ_n is the phase shift between the n-th harmonic of the current and voltage, and U_n and I_n are the RMS values of the harmonics.

Iliovici's idea [14] of measuring reactive power through the scalar product of voltage and orthogonal functions to current has not gained acceptance since the results obtained in this way may exceed the apparent power value [15], [16]. However, this method - replacing one of the voltage or current quantities in the active power formula (2) with an orthogonal function was proposed in the IEEE 1459-2010 standard [17] when calculating power for sinusoidal waveforms. This standard is an attempt to standardize and systematize the method of defining and measuring reactive power - dedicated to engineering applications.

2. INSTANTANEOUS POWER AND ACTIVE POWER

If electric energy is transferred between an object and the rest of the electric power system by currents i_k in $n+1$ conductors, see Figure 1, then from the energy conservation law it follows that the instantaneous power of the object is,

$$p = \sum_{k=1}^{n} u_k i_k. \tag{8}$$

Voltages u_k in (8) are defined by the necessary condition.

$$\sum_{k=1}^{n} i_k = 0 \tag{9}$$

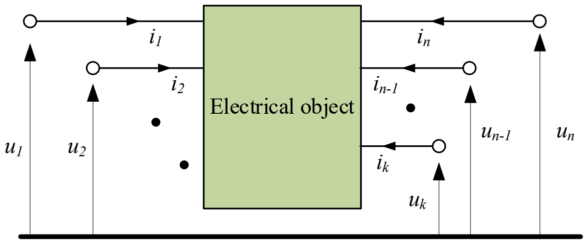


Figure 1. An electrical object

If from (9) one calculates, for instance, current in and substitutes it into equation (8), then one will get,

$$p = \sum_{k=1}^{n-1} (u_k - u_n) i_k, \tag{10}$$

According to which voltages u_k in (8) are not unique, i.e., they can be the difference between the potentials of the particular conductors and the potential of the n-th or any other conductor. Also, one can add one more conductor (in which the current is always equal to zero) to the n conductors and starting from this conductor measure all voltages u_k. Thus, each common point with any potential can be reference, but usually a point with zero potential is adopted for this purpose [14].

Depending on the potential of the adopted common point, the object can be variously divided into parts associated with the particular conductors and the total power will be the sum of the powers of all the parts. A quantity having this property is additive and satisfies the principle of energy conservation. The same procedure is followed for the remaining conductors.

Since instantaneous power is an additive quantity, it is enough to analyze only one arbitrarily selected part of the object, assigned to one (the k-th) current conducting wire. When electric energy is converted into work and heat, the average instantaneous power of the electrical object, in energy conversion period T is not equal to zero since this instantaneous power parameter (called active power) is also the scalar product of voltage and current, referred to the period, i.e. it specifies the electric current work in a unit time (1).

Being defined by the linear operation on the additive quantity (instantaneous power), active power is an additive quantity. Time T is a period if the current or voltage waveform is periodic. Generally, it is an interval of continuous functions u, i which have the same values at the interval's ends.

Referring to Faraday's law $u(t) = d\psi/dt$ or the definition of current intensity $i(t) = dq/dt$, the instantaneous power of the system can be written as:

$$p(t) = \frac{d\psi}{dt} i(t) \text{, or } p(t) = u(t) \frac{dq}{dt}. \tag{11}$$

By changing the integration limits in formula (2) we obtain that active power can also be geometrically defined as follows

$$P = \frac{1}{T} \oint i \, d\psi \tag{12}$$

or in the second form

$$P = \frac{1}{T} \oint u \, dq \text{ ,} \tag{13}$$

where: ψ - a magnetic flux, q - an electric charge

$$\psi = \int u \, dt \text{ , } q = \int idt. \tag{14}$$

According to (12) and (13), the surface areas of the closed loops formed by the characteristics of the object's components in coordinates i, ψ or in coordinates u, q represent the geometric picture of active power (the amount of energy delivered or received during one period T). For sinusoidal current and voltage waveforms - mutually shifted by an angle different from 0 and 180 degrees - the loop has the shape of an ellipse.

3. REACTIVE POWER

Reactive power is to describe the energy processes arising from the existence of electric energy in the form of electric, magnetic, and electromagnetic fields. They are elementary processes which may run irrespective of the conversion of electric energy into work and heat. Since the general definition of active power is based on the scalar product of two functions: u and i, (2), a generalized reactive power definition is obtained by replacing one of the functions, *i.e.*, current function i_k, with an orthogonal function. Reactive power defined in this way is an additive quantity.

A function orthogonal to periodic current i is each of its odd-order time derivatives and each multiple integrals with odd multiplicity (analogously, it can be done with voltage). Both Illovici [14] and the IEEE 1459-2010 [17] standard mention equivalently a first-order function: derivative or integral. From all the functions only one – the first current derivative – forms with voltage u a scalar product always equal to zero when the electric energy is completely dissipated in the object [15], [16], [18].

Geometrically, the scalar product of the voltage and the current derivative is equal to the scalar product of the current and the voltage derivative.

$$Q_d = \frac{1}{T\omega}\int_0^T u\frac{di}{dt}dt = -\frac{1}{T\omega}\int_0^T i\frac{du}{dt}dt = \frac{1}{2\pi}\oint u\, di = -\frac{1}{2\pi}\oint i\, du \tag{15}$$

is the area of the loop formed by the characteristic of the object in coordinates i, u.

Also, first order integrals (14) are a function orthogonal to current i and to voltage u. The integral and voltage u form the scalar product.

$$Q_i = \frac{\omega}{T}\int_0^T uq dt = -\frac{\omega}{T}\int_0^T i\psi dt = \frac{\omega}{T}\oint q\, d\psi = -\frac{\omega}{T}\oint \psi\, dq \tag{16}$$

whose geometric picture is the area of the loop in coordinates q, ψ.

For sinusoidal current and voltage waveforms, both reactive power formulas (15) and (16) give the same results. Reactive powers calculated from (15) and (16) give different values only for both non-sinusoidal waveforms. In order to distinguish reactive powers resulting from formula (2) - by inserting the orthogonal waveform obtained by applying differentiation, the index d was added (Q_d), while in the case of integration the index i was added (Q_i). In the presence of non-sinusoidal voltage and current waveforms, the reactive powers Q_d and Q_i exhibit differing values. In [14], the geometric mean of these powers is employed, denoted as equivalent reactive power. Relations (15) and (16) in the frequency domain take the form of series.

$$Q_d = \sum_{n=1}^{\infty} nU_n I_n \sin\varphi_n \tag{17}$$

$$Q_i = \sum_{n=1}^{\infty} \frac{1}{n} U_n I_n \sin\varphi_n. \tag{18}$$

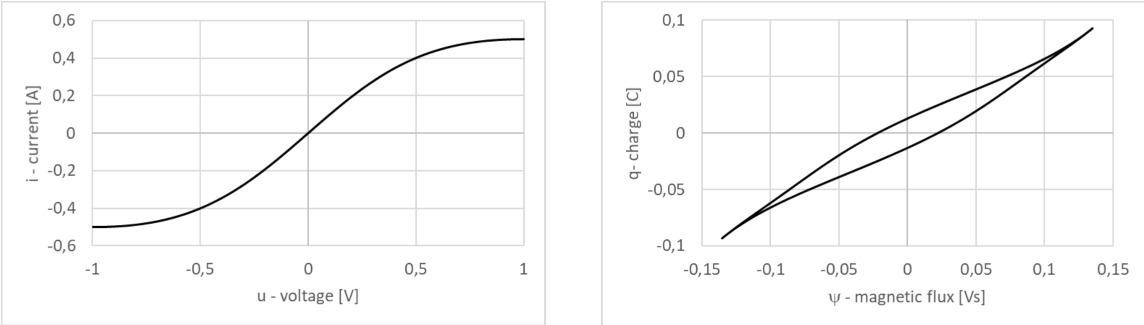
Power Q_i - formulas (16) and (18) - are not recommended for calculating and cannot be the basis for a reactive power definition since Q_i is not always equal to zero when the electric energy in the object is completely dissipated. For a voltage having the first and third harmonic.

$$u = U_1 \sin\omega t + U_3 \cos 3\omega t, \tag{19}$$

when the factor of voltage and current proportionality uniquely depends on voltage in accordance with the equation.

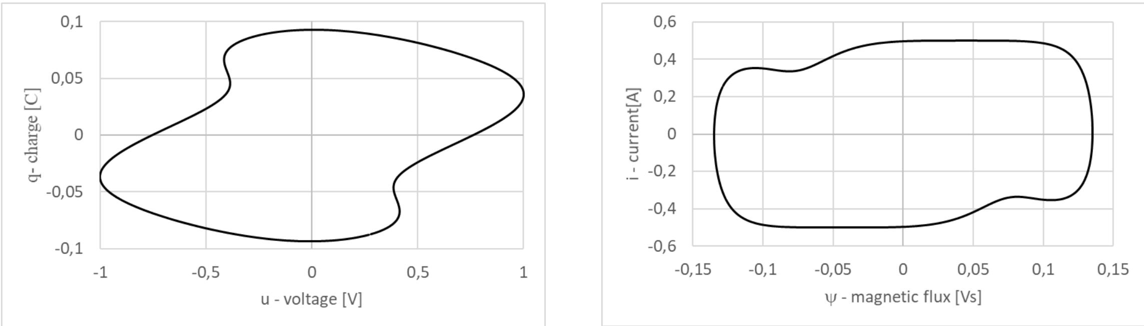
$$R(u) = a_0 + a_2 u^2, \tag{20}$$

Characteristics in coordinates i, u and in coordinates q, ψ, with waveforms as in Figure 2 are obtained. The characteristic in coordinates i, u Figures 2(a) is a line segment, and it correctly indicates the absence of reactive power while the characteristic in coordinates q, ψ Figures 2(b) forms a loop whose area is not equal to zero. The energy dissipated during one period is equal to the area of the loop in coordinates u,q - Figure 3(a) and ψ,i - Figure 3(b).



(a) (b)

Figure 2. Reactive power loops. Tahe characteristics of the nonlinear object completely dissipating electric energy, (a) in current-voltage coordinates and (b) in electric charge- magnetic flux coordinates. The characteristics were obtained from (19) and (20) for: U_1=1V, U_3=1/3 V, $\omega = 2\pi$, a_0=1 **Ω**, a_2 = 1 **Ω**/V2



(a) (b)

Figure 3. Active power loops of a nonlinear resistor in (a) electric charge - voltage coordinates and (b) current - magnetic flux coordinates

The dissipation of electric energy and the accumulation of electric energy in the form of an electric field and a magnetic field in the object can be approximately modelled by an electric circuit as shown in Figure 4. The current drawn by this circuit depends on the applied voltage, according to:

$$i = \frac{u}{R} + \frac{1}{L}\psi + C\frac{du}{dt}. \quad (21)$$

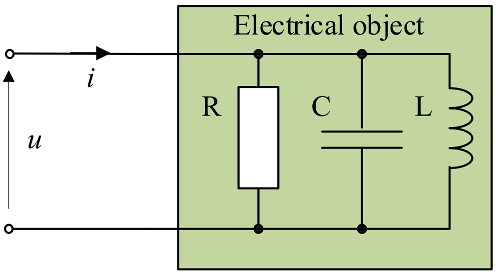


Figure 4. An equivalent circuit of the object

Considering that the resistance which models energy dissipation must uniquely depend on voltage, the reactive power of an equivalent circuit part with resistance *R* is equal to zero. If the object includes ferromagnetic circuits, then inductance *L* of the equivalent circuit should be treated as a time-dependent quantity. Thus, the reactive power Q_d of a component with inductance *L* is equal to,

$$Q_d = \frac{1}{2\pi}\int_0^T \frac{d}{dt}\left(\frac{1}{L}\right) u\psi dt + \frac{1}{2\pi}\int_0^T \frac{1}{L}u^2 \ dt. \tag{22}$$

Inductance L of the equivalent circuit component modelling the occurrence of electric energy in the form a magnetic field must uniquely depend on equivalent flux ψ. Thus, the first integer in (22) can be written as a loop area in coordinates $\psi, f(\psi)$.

$$\frac{1}{2\pi}\int_0^T \frac{d}{dt}\left(\frac{1}{L}\right) u\psi dt = \oint \frac{d}{dt}\left(\frac{1}{L}\right) \psi d\psi = \oint f(\psi)\, d\psi = 0. \tag{23}$$

Since the graph of periodic function $f(\psi)$ is a line segment, the loop area is equal to zero. Thus, reactive power Q_d of the inductance component can be determined through averaged inductance L.

$$Q_d = \frac{1}{2\pi}\int_0^T \frac{1}{L}u^2 \ dt = \frac{T}{2\pi}\frac{1}{L}U^2, \tag{24}$$

where U – a rms voltage.
The reactive power of a component with constant capacitance C is

$$Q_d = \frac{C}{2\pi}\int_0^T u\frac{d^2u}{dt^2}dt = -\frac{C}{2\pi}\int_0^T \left[\frac{du}{dt}\right]^2 dt = -\frac{TC}{2\pi}\left(\dot{U}\right)^2, \tag{25}$$

where $\dot{U}$ – a rms value of the derivative of voltage u.

The reactive power of the whole equivalent circuit as shown in Figure 4 is the sum of (24) and (25).

$$Q_d = \frac{T}{2\pi}\left[\frac{1}{L}U^2 - C\left(\dot{U}\right)^2\right]. \tag{26}$$

It follows from (26) that the reactive power of an inductive object (*e.g.*, an electric motor) can be compensated to zero by means of a capacitor with a proper capacitance. The optimum capacitance $C_{(opt)}$ is calculated from the reactive power (Q'_d) zeroing condition.

$$C_{(\text{opt.})} = \frac{2\pi}{T}\frac{Q_d'}{(\dot{U})^2} \tag{27}$$

Generalized reactive power Q_d' is measured before a capacitor with capacitance $C_{(\text{opt.})}$ is connected to the object or before this capacitance is changed.

Result (27) is exactly equivalent to the optimum capacitance, obtained under different assumptions in [11], [19]–[22], at which the minimum of rms current occurs. An example calculation for minimizing the current of an induction motor using a capacitor is provided in [23]. Thus, if the generalized reactive power of a given conductor becomes zero, the rms current reaches a minimum which does not depend on the resistance in the equivalent circuit. This property can be formally proved if the equivalent resistance uniquely depends on voltage and when the inductance uniquely depends on the magnetic flux ψ. Measurements show that when the generalized reactive power Q_d becomes zero [22], [23], the minimum rms current occurs also when the above relations are non-unique. Reactive powers Q_d and Q_i can be used to determine the constant parameters L, C of a parallel or series equivalent circuit of the receiver [10]. A necessary condition is that the values of Q_d and Q_i are different, which occurs when the voltage and current are non-sinusoidal.

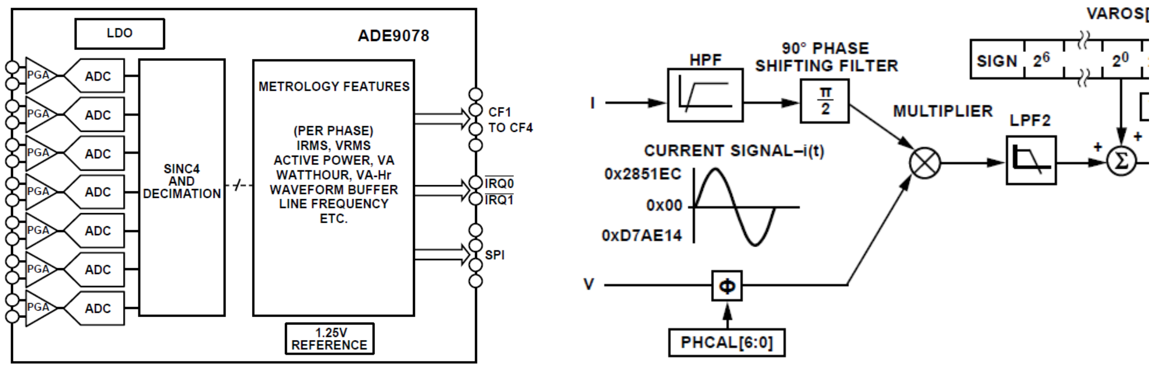
4. METHODS FOR MEASURING REACTIVE POWER IN DIGITAL METERS

Modern electricity meters are composed of analogue-to-digital converters (ADCs) for measuring instantaneous voltage and current values, and a signal processing unit. This unit calculates power, energy, power factors, and other parameters, including those related to power quality. A meter can be implemented as a specialized integrated circuit (as shown in Figures 5(a)) or as a combination of a microprocessor with ADCs. The microprocessor or computational unit can be programmed to determine reactive power (energy) according to user requirements.

For sinusoidal current and voltage waveforms, all dependencies presented in the preceding section yield the identical value of reactive power. The meter manufacturer is not constrained to a specific method for measuring reactive power. They can choose any dependency or implement the algorithm that is simplest to implement, provided it satisfies the established design criteria (*e.g.*, utilization of a cost-effective microprocessor) [24]–[26].

One such straightforward algorithm involves shifting the voltage samples relative to the current samples by an amount corresponding to one-quarter of the period of the measured waveform, see Figure 5(b). This 90-degree phase shift between current and voltage occurs exclusively at the fundamental frequency. For harmonics, the phase shift is multiplied proportionally, which results in a final value that is the sum of the fundamental harmonic reactive power and the active or reactive power of the harmonics (27).

$$Q_{\frac{T}{4}} = \frac{1}{T}\int_0^T u(t)i\left(t-\frac{T}{4}\right)dt = Q_1 - P_2 - Q_3 + P_4 + Q_5 - \quad (27)$$



(a) (b)

Figure. 5. Reactive energy (a) IC Functional block diagram of energy metering device ADE9078 – [27] and (b) reactive power calculation from 90° phase shift (27), ADE7758 [28]

The measurement block of the integrated meter measuring reactive energy from the "power triangle" in accordance with equation (5) is implemented in many IC like [29]. Differences in the results of measuring the reactive energy of objects, resulting from the calculation algorithm used in the meter, are discussed many times in scientific studies, *e.g.*, [30]–[33]. Standardization requirements for reactive energy meters stipulate metrological verification under sinusoidal conditions [1], [2]. Meters incorporating higher-harmonic filters effectively function as fundamental-component reactive energy meters, distinct from active energy meters. Other approaches to measuring power and reactive energy are being undertaken [34]–[36]. They have not yet been implemented in commercial reactive energy meters. Similarly, the recommendations of the IEEE 1459-2010 standard [17] have not been implemented.

5. CONCLUSION

There is no universally accepted theory of reactive power for non-sinusoidal current and voltage waveforms. Fundamental reactive energy meters are used to account for reactive energy. The meters should use a reactive power measurement algorithm that considers distorted waveforms. A unique definition of additive reactive power, covering non-sinusoidal, periodical waveforms, is obtained by replacing the current in the equation (the scalar product of voltage and current) defining active power with a special orthogonal function – a derivative of current or voltage.

Geometrically, the scalar product of voltage and current derivative is the surface area of the loop formed by the object characteristic in current-voltage coordinates (*i, u*). If the loop area in coordinates *i, u*, divided by 2π is adopted as the generalized definition of reactive power for periodical waveforms, then the previously defined reactive power for sinusoidal voltage and current waveforms will be its special case. The zero value of the reactive power Q_d of the object indicates that the rms current in this object has reached its minimum. This state can be achieved by connecting a capacitor – a simple passive compensator.

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AUTHOR CONTRIBUTIONS STATEMENT

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C : **C**onceptualization
M : **M**ethodology
So : **So**ftware
Va : **Va**lidation
Fo : **Fo**rmal analysis
I : **I**nvestigation
R : **R**esources
D : **D**ata Curation
O : Writing - **O**riginal Draft
E : Writing - Review & **E**diting
Vi : **Vi**sualization
Su : **Su**pervision
P : **P**roject administration
Fu : **Fu**nding acquisition

CONFLICT OF INTEREST STATEMENT

Authors state no conflict of interest.

DATA AVAILABILITY

Data availability is not applicable to this paper as no new data were created or analyzed in this study.

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