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A novel stretched-compressed exponential low-pass filter and its application to electrocardiogram signal denoising

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ABSTRACT

The study investigates a novel stretched-compressed exponential low-pass (SCELP) filter to denoise electrocardiogram (ECG) signals. As an extension of Gaussian filter and unlike other denoising filters, the SCELP filter utilizes the stretched-compressed exponential function (SCEF) in the convolution kernel, being the Gaussian function its particular case. A MATLAB implementation is provided with a single parameter (β), which allows to modify the filter strength, to increase the signal-to-noise ratio (SNR) and reduce the mean squared error (MSE). The SCELP filter's advantages over traditional denoising filters (*i.e.*, Gaussian, Mittag–Leffler, and Savitzky-Golay filters) were assessed on 100 ECG signals, 50 normal and 50 abnormal (affected by sleep apnea), provided by the PhysioNet dataset. The SCELP filter's efficacy in rejecting noise was evaluated as the β parameter varies, quantifying the filters' performance in terms of mean SNR and MSE to determine the optimal β value. The obtained results showed that the SCELP filter's best performances are achieved for β equal to ≈ 1.6 (i.e., 16.9508 dB and 13.7574 dB SNR values, and 0.01025 and 0.01178 MSE values for normal and abnormal ECGs, respectively). Furthermore, the SCELP filter was tested on ECG signals with added white noise; compared to Gaussian, Mittag–Leffler, and Savitzky-Golay filters, the SCELP filter yields better performance regarding SNR (16.495 and 14.940 dB) and MSE (0.0106 and 0.0114) values, for normal and abnormal ECGs, respectively, suggesting its applicability for ECG signals' denoising.

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1. INTRODUCTION

Filtering a signal involves suppressing some of its characteristics or deleting unwanted components. Usually, it means eliminating specific frequencies or frequency ranges from a noisy signal's intended use. However, specific frequency components can be selectively removed by sparingly filtering in the frequency domain without causing interference. Filters are widely used in many applications, including computer graphic, radar, image processing, control system sensors, signal processing, electronics, and tele-communications. Many types of filters are commonly employed, such as the Laplacian filter [1], Bayesian one [2], and the Gaussian one [3]. A significant area of applications is filtering data related to biomedical signals, such as electroencephalogram (EEG), electrocardiogram (ECG), and electromyogram (EMG). Numerous denoising approaches are proposed in the literature using different types of filters [4]–[6].

Heart-related anomalies can be identified and detected using ECG signals, commonly employed as a diagnostic tool for associated illnesses [7], [8]. For precise categorization or decision-making, the ECG signal's quality is essential [9], [10]. Denoising is essential in ECG processing because high-quality, noise-free signals are required for reliable diagnosis and detection of cardiac diseases [11]. ECG signals are often contaminated by various types of noise, such as baseline wander, power line interference, and muscle artifacts, which can distort the original ECG signal [12]. The noise makes it difficult to identify important features, such as the P-wave, QRS complex, and T-wave, essential for diagnosing conditions like arrhythmias or ischemia. Several types of filters for biomedical signal processing have been used: Gaussian, Mittag Leffler, Golley, adaptive, and median filters. They were analyzed in terms of effectiveness in suppressing fluctuations due to artifacts, power line noise and other noise sources in order to improve the original ECG signal's quality [8], [13]–[15]. In signal filtering, a key advancement is the development of generalized filters such as the Mittag-Leffler one, which provides additional user-adjustable parameters, ensuring higher flexibility than traditional exponential or Gaussian filters. This filter type allows for a better balance between noise reduction and signal retention by tuning the parameters, making it particularly effective for noisy ECG signals. This approach enhances the filter performance by addressing challenges such as filtering stochastic components while reducing computational complexity and outperforming conventional filters [16]. For instance, in [17], the authors introduced a Mittag-Leffler filter, an extension of the Gaussian one, using the Mittag-Leffler function in its probability-density function and convolution kernel. The filter has three adjustable parameters influenced by a forgetting factor, offering advantages over classical Gaussian filtering in tasks such as ECG signals denoising. The implementation details and the developed MATLAB function are provided.

In [18], the authors proposed the Alexander fractional differential window (AFDW) filter for ECG signal denoising based on the generalized Alexander polynomial and fractional calculus; it uses a forward and backward filtering approach, averaging the coefficients of both filters. Based on morphological power preservation measure (MPPM), the results demonstrated that the filter preserves signal power and QRS morphology. The study in [19] proposed fractional-order wavelet filters for ECG signal denoising, replacing traditional low- and high-pass filters. The fractional wavelets were compared using appropriate thresholding and wavelet decomposition. Results showed superior performance of fractional wavelets compared to traditional wavelets, especially in removing high-frequency noise without requiring prior frequency knowledge. Furthermore, a new algorithm for denoising ECG data contaminated by wide-band noise was proposed, where the ECG signal is segmented into components with disjoint time and overlapping frequency domains [20]. Each segment is denoised using ideal filters designed by minimizing a penalized least-squares function; the method outperforms existing techniques. Also, Savitzky-Golay filters were useful for efficiently denoising the ECG signals; in [21], a low-distortion adaptive Savitzky-Golay (LDASG) filtering method for ECG denoising was proposed, which, unlike standard Savitzky-Golay filter, uses discrete curvature estimation to adjust for signal variations, reducing distortion while maintaining effective data smoothing.

Based on the traditional exponential filter, this article investigates a novel stretched-compressed exponential filter for denoising the ECG signals. Given the effectiveness of the exponential filter, which can lead to the Gaussian one, especially in ECG signals' processing, a novel idea emerged for a stretched-compressed exponential low-pass (SCELP) filter. The SCELP filter retains the basic structure of the exponential filter, but it has superior performance by adapting its shape, making it more effective in handling ECG signals while maintaining simplicity and efficiency. The SCELP filter employs an exponential function as a kernel to the standard exponential Gaussian filter that mathematically modifies the input signal through convolution. Like the exponential filter, featured by an exponential function as impulse response, the SCELP filter has the advantage of not overshooting the signal, minimizing rise and fall times [22]–[27].

A MATLAB implementation of the SCELP filter has been developed and tested, denoising publicly available ECG samples (50 normal and 50 abnormal) from the PhysioNet database by varying the β parameter. The characterization results demonstrated that a β value ranging from 1 to 2 provides optimal performance in terms of mean SNR and MSE values. Furthermore, the capability of SCELP filter in treating signals affected by additive white noise was tested. Compared with other filter typologies (*i.e.*, Gaussian, Mittag–Leffler, and Savitzky-Golay), the proposed SCELP filter provides better performance, as detailed in section 3. The main contributions to the proposed research article are the following:

- A novel stretched-compressed exponential low-pass filter is presented, whose impulse response is optimized as a function of the β parameter to maximize the SNR and minimize MSE, outperforming traditional filters typically used in denoising ECG signals.
- The proposed SCELP filter has been characterized, proving its effectiveness in denoising the ECG signals by varying the hidden exponential parameter (β) to determine the optimal filter setting.

- A comparative analysis of the SCELP filter, in terms of mean SNR and MSE, with different filter typologies (*i.e.*, Gaussian, Mittag–Leffler, and Savitzky-Golay filters) is presented, demonstrating the superiority in denoising the ECG signals. In addition, the SCELP filter's performance was evaluated on ECG signals affected by additive white noise and compared with the other filters' typologies, demonstrating its effectiveness and slight superiority in terms of mean SNR and MSE.

The research article is organized as follows: in section 2, the filter's mathematical representation is proposed, and the ECG signals' dataset used to evaluate the filter performance is described. The results related to the SCELP filter characterization are reported in section 3, demonstrating the best performance of the proposed filter compared to other filter typologies. Section 3 also reports on the discussion on the SCELP filter's performance and a comparative analysis with other common ECG-denoising filters. Finally, section 4 summarizes the main results presented in the research work and future perspectives.

2. PROPOSED METHOD

This section introduces the fundamentals and definitions of the exponential filters and their main properties; then, the stretched-compressed exponential function and distribution are presented, which, integrated into the kernel, are used for defining the novel stretched-compressed exponential filter. Finally, the MATLAB implementation of the proposed customizable exponential filter and metrics to evaluate its performance in denoising the ECG signals are reported.

2.1. Exponential filter: mathematical formulation and step response

The exponential filter is one of the simplest forms of low-pass filter, with high frequencies attenuated and low frequencies passed unchanged. The sampling interval is the only other tuning parameter available, and the previous output is the only variable that needs to be stored. It is an infinite impulse response (IIR) auto-regressive filter, meaning that the effects of an input change on the filter output diminish exponentially, taking into account the computational limits of the processing and display devices. This filter is also known as exponential smoothing in some academic fields. The exponential filter is referred to as an exponentially weighted moving average (EWMA) or simply an exponential moving average (EMA) filter in certain fields, such as investment research; for this filter typology is improperly used the term "moving average" referred to the time-series analysis, in the classic autoregressive moving average (ARMA) model since a moving average filter just considers the current input rather than the input history. The exponential filter is the analog counterpart of the first-order lag frequently employed in the analog modeling of continuous-time control systems for discrete time. An RC filter, constituted by a single resistor and capacitor, is a first-order lag in electrical circuits. When highlighting the parallelism with analog circuits, the time constant, represented by the Greek symbol tau (τ), is the only tuning parameter; given the same time constant, the values at the discrete sampling periods perfectly match the corresponding continuous-time lag.

The following equations illustrate the link between the digital implementation (i.e., a smoothing constant) and time constant (τ). To ensure that the output and input signals are identical under steady-state conditions, the exponential filter combines the most recent input data with a weighted combination of the prior estimate (output), with the total weights set to 1 as shown in Figure 1. The output $y(k)$ vs input $x(k)$ relationship of an exponential filter is expressed as (1):

$$y(k) = a * y(k-1) + (1-a) * x(k) \quad (1)$$

where $x(k)$ is the raw input at time step k, $y(k)$ the filtered output at time step k, and a a parameter (called smoothing constant) between 0 and 1 (typical values are chosen in the range 0.8 ÷ 0.99).

Figure 1. Block diagram of a generic discrete-time filter

The smoothing constant is computed and saved for convenience only in cases when the application developer specifies a new value for the desired time-constant τ in systems with a fixed sampling period T:

$$a = e^{\left(-\frac{T}{\tau}\right)} \quad (2)$$

where τ is the filter time constant in the same time units as T. The exponential function in (2) must be applied to each time step in systems where data sampling occurs at irregular intervals (namely, T is not constant). Usually, the first input is used to initialize the filter output. There is no filtering if the smoothing constant (a) goes to zero, namely when the time constant approaches zero, and consequently, the output $(y(k))$ equals the new input $(x(k))$. Otherwise, if the smoothing constant (a) is close to 1, and therefore τ increases, the new input is essentially ignored, resulting in extremely strong filtering.

The filter equation can be rearranged into the following equivalent predictor-corrector relationship:

$$y(k) = y(k-1) + (1-a) * \big(x(k) - y(k-1)\big) \tag{3}$$

Therefore, the filter output is anticipated to remain constant from the prior estimate $y(k-1)$ plus a corrective term based on unexpected contribution, given by the difference between the new input $x(k)$ and forecast $y(k-1)$; the (3) makes this prediction clearer. This form may also be obtained by considering the exponential filter as a straightforward special case of a Kalman filter, which is the best solution for an estimation problem under specific presumptions. Figure 2 shows the unit-step response of an exponential filter, obtained by abruptly changing the input value to 1 from zero initial value.

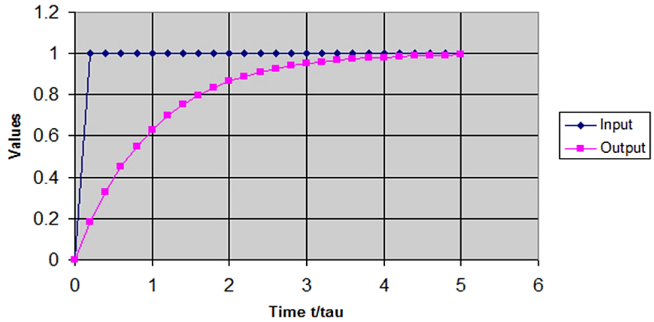


Figure 2. Step response of the exponential filter: input (blue trace) and output (purple trace) plots as a function of t/τ

The exponential filter's step response enables more readily predicting the outcomes for any time interval and value of the filter time constant (τ) by dividing the time by τ. The filter output climbs to 63.21% of its final value after a single τ and increases to 86.47% after two-time constants; after 3, 4, and 5τ the output reaches 95.02%, 98.17%, and 99.33% of the final value, respectively. These percentages are unchanged for any input step's amplitude, because the filter is linear. The filter output transient can be considered exhausted after a time equal to 4 or 5 time constants, even if the step response, in theory, takes unlimited time. Other settings include a=0.90 and a=0.998, respectively, corresponding to τ=9.49 and 499.5 minutes for a sampling period T of one minute. One way to arrange the exponential filters is in series, resulting in greater attenuation of high-frequency noise but also causing a greater output delay, often excessive for control loops or diagnostic applications. A non-linear exponential filter is an exponential filter's variant that responds more quickly to greater input changes and is designed to filter out noise substantially within a specific amplitude range [28]. In essence, the exponential filters operate by assuming that the signal is a random walk or Brownian motion pattern, with random process noise as the only variation source. Then, before displaying more recent data, the previous value represents the best estimate of the subsequent value. All that remains of the final estimate is a weighted average of the new observed value and the expected one.

Creating a representative dataset of the system is helpful for some applications, such as sophisticated control systems as well as fault detection and isolation algorithms, because it serves as the basis for understanding the system's behavior, identifying anomalies, and designing effective control strategies. The dataset captures the relationships between the system's inputs, outputs, and internal states under normal and abnormal conditions, making it an invaluable resource for analysis and decision-making. Some control techniques interact directly with that dataset without the need for an explicit model of the system, including the model-free binary decision and action control (BDAC) approach and some fault isolation and detection strategies. These model-free methods are particularly useful in systems where developing an accurate mathematical model is challenging due to complexity, nonlinearity, or uncertainty. In other cases, the dataset serves as a training set for creating models that are subsequently used for control or diagnosis. This approach is particularly relevant in model-based approaches, where a mathematical or data-driven model of the system is constructed and utilized for decision-making. Neural network (NN) models, model-based control (like model predictive control (MPC)), and traditional regression models fall into the category of the model-based approach. A complete multi-variable method known as real time exponential filter clustering (RTEFC) has been created that combines exponential filtering with real-time clustering [29].

2.2. Stretched-compressed exponential function and distribution

The stretched exponential function can be expressed as (4):

$$I(t) = I_0\, e^{-\left(\frac{t}{\tau}\right)^{\beta}} \quad (4)$$

It is achieved by fitting the exponential function with a fractional power law, which is only significant for argument t between 0 and +∞ in most applications [30]. The standard exponential function is obtained when $\beta = 1$. The function gets its name from the characteristic stretching of the log I against t graph, which has a stretching exponent β between 0 and 1. Another practical significance is attached to the compressed exponential function (with β >1), with the notable exception of β=2, which yields to the normal distribution. Figure 3 plots the $I(t)$ function reported in (4) as a function of the t/τ ratio, varying the β parameter.

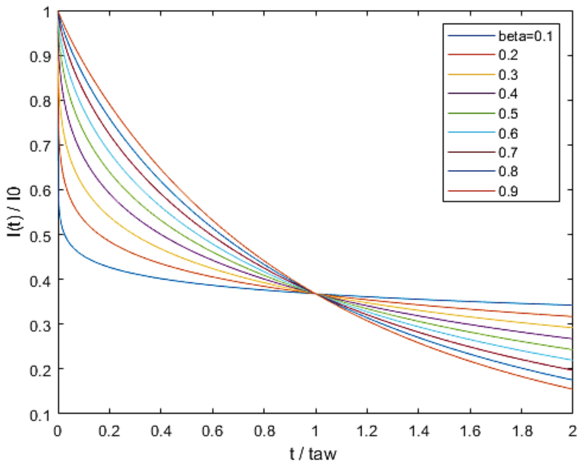


Figure 3. Plot of I(t)/I_0 vs. (t⁄τ), showing the stretched exponential function for several β values

In mathematics, the complementary cumulative Weibull distribution is another name for the stretched exponential function [31]. The characteristic function of the Lévy symmetric alpha-stable distribution is also the stretched exponential one, or in simpler terms, the Fourier transform [30]. The stretched exponential function is a phenomenological explanation of relaxation in disordered systems frequently utilized in physics. The Kohlrausch function was proposed from R. Kohlrausch to explain how a capacitor discharges [32]. The Fourier transform of the stretched exponential function is also known as the Kohlrausch–Williams–Watts (KWW) function; it was first applied in 1970 by Williams and Watts to characterize the dielectric spectra of polymers [33]. For small-time arguments, the Cole-Cole and Cole-Davidson equations, and the Havriliak-Negami relaxation are examples of the primary dielectric models whose time-domain charge response is correlated with the KWW function [34]. In phenomenological applications, it is often unclear whether the stretched exponential function should describe the integral distribution function, the differential one, or neither. The asymptotic decay is the same in all cases, but the power law pre-factor varies, leading to a more ambiguous fit than simple exponentials. The asymptotic decay has been shown to be a stretched exponential [30], [31], although the pre-factor is typically an unrelated power.

There have been attempts to explain the stretched exponential behavior using a linear superposition of simple exponential decay, as seen in the distribution function for the stretched exponential function. Therefore, a nontrivial relaxation time distribution, $\rho(x;\beta)$, is required, which is implicitly defined by (5):

$$e^{-t^{\beta}} = \int_0^{\infty} e^{\frac{-t}{x}}\, \rho(x;\beta) dx \quad (5)$$

Alternatively, a distribution related to the parameter β is given:

$$G(x;\beta) = x\rho(x;\beta) \quad (6)$$

where $\rho(x;\beta)$ can be expressed as (7):

$$\rho(x;\beta) = -\frac{1}{\pi x}\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}\sin(\pi\beta k)\,\Gamma(\beta k+1)\,x^{(\beta k)} \quad (7)$$

where Γ is the Gamma function. For rational values of β, $\rho(x;\beta)$ can be calculated in terms of elementary functions. But the expression is, in general, too complex to be useful except for the case β=1/2, where the distribution will be as (8):

$$G(x;1/2) = x\rho(x;1/2) = \frac{1}{2\sqrt{\pi}}\sqrt{x}e^{\frac{-x}{4}} \quad (8)$$

Figure 4 shows the function in (8) for different values of the β.

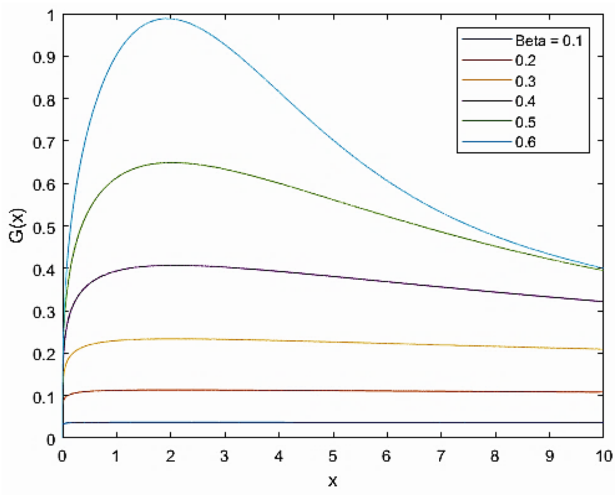


Figure 4. Plot of the stretched exponential distribution function $G(x;\beta)$ vs. (x) for different β values

The Fourier transform of the stretched exponential function has to be computed using a series expansion or numerical integration; the Havriliak–Negami function can be used to approximate the Fourier transform [13]. However, modern numerical computation is so efficient that the Kohlrausch–Williams–Watts function should always be used in the frequency domain [35].

In most cases, the filter aims to separate the actual signal from the noisy measured signal:

$$y(t) = y_d(t) + y_s(t) \quad (9)$$

where $y(t)$ is the observed (measured) signal at the time t, $y_d(t)$ the true, deterministic part of the signal, and $y_s(t)$ a stationary noise, a stochastic (random) part in the signal, which is assumed with zero mean value.

An exponential low-pass filter is featured by an impulse response equal to the exponential function $\rho(x;\sigma)$; thus, the output of the exponential filter ($y_{EF}(t)$) is defined as the convolution of the measured (observed) signal $y(t)$and the impulse response $\rho(x;\sigma)$:

$$y_{EF}(t) = y(t) * \rho(t;\sigma) = \int_{-\infty}^{\infty} y(t-\tau)\,\rho(\tau;\sigma)d\tau \quad (10)$$

2.3. Proposed compressed exponential filter and performance metrics

The novel exponential filter, hereinafter indicated as the stretched exponential filter, is defined by extending the exponential function to the stretched exponential function with a single parameter. Therefore, the output of the stretched exponential filter $y_{SEF}(t)$ is given by:

$$y_{SEF}(t) = y(t) * \rho(t;\beta) = \int_{-\infty}^{\infty} y(t-\tau)\, \rho(\tau;\beta) d\tau \tag{11}$$

where:

$$\rho(t;\beta) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\left(\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2\right)^\beta} \tag{12}$$

Therefore, the filter output is expressed as (13):

$$y_{SEF}(t) = y(t) * \rho(t;\beta) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} y(t-\tau)\, e^{-\left(\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2\right)^\beta} d\tau \tag{13}$$

When $\beta = 1$, a classical Gaussian filter is obtained, additionally, various filter typologies are achieved by tuning the β hidden factor, allowing to modify the curve of the probability-density function. Conversely, different distribution shapes necessitate the usage of distinct distribution functions, such as the normal distribution, the Cauchy one, and others of this family. Compared to a classical one-parameter filter, the proposed one is more versatile given the extra tuning parameters β, enabling modifying the distribution curve and the exponential stretch and, thus, providing more degrees of freedom.

The signal-to-noise ratio (SNR) and mean square error (MSE) are usually used in the literature to evaluate the filter's ability to reduce the noise [36]. This article uses the following defined evaluation markers to compare the proposed algorithm with the existing methods and assess its effectiveness in reducing the noise. The definition of the output signal-to-noise ratio is as (14):

$$SNR = 10 * log_{10}\left(\frac{\sum_{i=1}^{i=N}[x(i)]^2}{\sum_{i=1}^{i=N}\left[x(i)-\widehat{x(i)}\right]^2}\right) \tag{14}$$

where $x(i)$ are the original (ideal) signal's samples, while $\hat{x}(i)$ are the samples of the filtered ECG signal [37]. The mean square error is defined as in [15]; both, the mean SRN and MSE values calculated on all ECG signals in the dataset were considered for testing the effectiveness of the proposed SCELP filter.

$$MSE = \frac{1}{N}\sum_{i=1}^{i=N}[x(i) - \hat{x}(i)]^2 \tag{15}$$

A technique to implement the Gaussian filter in the discrete-time domain is to follow the instructions in [38], [39]. Since the Gaussian filter is not causative, the time-domain filter window is symmetric. Because the Gaussian function for x ∈(-∞,∞) would theoretically require an infinite window length, the Gaussian filter is physically unfeasible. In practice, it makes sense to reduce the filter window's length and apply it straight to narrow windows; however, occasionally, this shortening can result in serious mistakes. The filter cannot be applied to the signal being processed until the incoming samples occupy the filter window, resulting in a latency in real-time systems. In convolution, the Gaussian filter kernel is continuous, but it is commonly approximated by a discrete sampled Gaussian kernel created by sampling points from the continuous kernel. This discrete version is the most widely used substitute for the continuous Gaussian kernel. The summing process across all samples can be used in place of an integration operation in convolution [40].

It is also commonly recognized that traditional moving average filters, or weighted moving average filters with stretched exponential, are not always appropriate for allocating weights to preceding filtered signal samples [41]. A common requirement is that samples with a high proportion of stochastic (noisy) components should be assigned lower weights rather than simply assigning lower weights to older samples. By prioritizing more recent samples, such filters can respond faster to changes in deterministic or stochastic components. To be used even in digital controllers with limited computing power, the filtering algorithm must meet two requirements: reasonably simple to implement and effective even when the measured waveforms contain significant stochastic noise. Similar challenges to those discussed earlier may arise when implementing the stretched exponential filter. Specifically, while flexible in its weighting of past samples, the stretched exponential function can be computationally demanding and sensitive to parameter choices. In real-time applications, evaluating the function over extended time ranges can pose practical difficulties, particularly in systems with limited computational resources. Additionally, the absence of a natural cutoff in the stretched exponential function necessitates careful truncation to balance accuracy and computational efficiency, further complicating its use in real-time scenarios. A MATLAB function for the stretched exponential function is proposed in this research work; its header is given below:

```
function [y]=SEF (beta, x)
% SEF (beta, x) is the stretched exponential function
% for each element of x and beta are scalars,
% array. The output is of the same size as x.
```

The filter output expression reported in the (13) can be implemented by a MATLAB function [y]=SEF (beta, x). The MATLAB function of developed stretched exponential filter has the following header:

```
function [y_filt]=SE_filter (t, y, beta, sigma)
% function [y_filt]=SE_filter (t, y, beta, sigma)
% Stretched Exponential filter
%   Inputs:  t=independent variable
%            y=noisy data to be filtered at the points t
%            beta=parameters of the Stretched Exponential function
%   Output:  y_filt=filtered data given in variable y
```

2.4. ECG signals' dataset

The ECG signals were taken from the public Physionet archive, available at the website https://archive.physionet.org/physiobank/database/apnea-ecg/; the used dataset consists of 100 ECG records in total, 50 labeled as normal and 50 as abnormal (related to patients affected by sleep apnea), each featured by 30 min duration and 360 Hz sampling rate; thus, each record contains a total of 648.000 samples. For testing the proposed stretched filter, three QRS complexes were considered for normal ECG signals and two QRS complexes for abnormal ECGs with 2- and 1-second durations, respectively. Table 1 summarizes the features of ECG signals used for testing the proposed stretched exponential filter.

Table 1. Normal and abnormal ECG signal parameters selected for testing the stretched exponential filter

	Normal ECG	Abnormal ECG (sleep apnea)
Number of ECG Records	50	50
ECG Time Length	30 min	30 min
Sampling Rate	360 Hz	360 Hz
Processed ECG signals' duration	2 sec	1 sec

3. RESULTS AND DISCUSSION

Figures 5(a)-5(d) and 6(a)-6(d) show the effect of the designed SCELP filter applied to abnormal and normal ECG noisy signals, respectively, for different β parameter values. The purpose of filtering process is to denoise the raw ECG signal (input, red plots) and provide a filtered output (blue plots) suitable for further processing.

Table 2 and 3 present the obtained mean SNR and MSE values after applying the SCELP filter for different β values in the range [0.2 ÷ 10.0] to the dataset's abnormal and normal ECG signals. The SNR and MSE values were calculated from all ECG signals for each β value; then, the mean values have been reported in Table 2 for abnormal ECG signals and in Table 3 for the normal ones. For both signal types, the filter outperforms for β between 1.2 and 2, providing the highest SNR and lowest MSE values for $\beta \approx 1.6$. Table 4 shows the overall performance in terms of average SNR and MSE for β higher than 2, between 1.2 and 2, and equal to or smaller than 1. As explained in the method section, based on (13), the filter will operate as a compressed exponential filter if $\beta > 2$, a stretched-compressed exponential filter if $1 < \beta \leq 2$, a Gaussian filter if $\beta=1$, and as a stretched exponential filter when $\beta < 1$. Finally, Figures 7(a) and 7(b) shows the obtained results as histograms relating to SNR and MSE values, as already reported in Table 4, for the different β ranges.

In addition, to further investigate the effectiveness of developed filter, a normal white noise has been added to the ECG signals constituting the dataset described in section 2 in order to verify the SCELP filter's efficacy and compare its performance with different denoising filters' typologies like the Gaussian, Mittag–Leffler, and Savitzky-Golay ones. In more detail, nine seconds were extracted from each ECG signal, and then a normal white noise ($N(t)$) was added. This process is used to test the capability of filters to elucidate the inherent signal ($y_T(t)$) from the contaminated, noise-ridden signal ($y_{noisy}(t)$). Thus, a white noise array was generated through MATLAB and then added to the ECG signal by point-by-point addition of the ECG and noise values (16); for this purpose, a Matlab code has been implemented, reported below.

$$y_{noisy}(t) = y_T(t) + N(t) \tag{16}$$

```
load('AbnormalECGSignalone.mat')
%  load('Normal ECG_Filter.mat')
noiseSignal=randn(size(X2));
newSignal=noiseSignal + X2;
y=newSignal
```

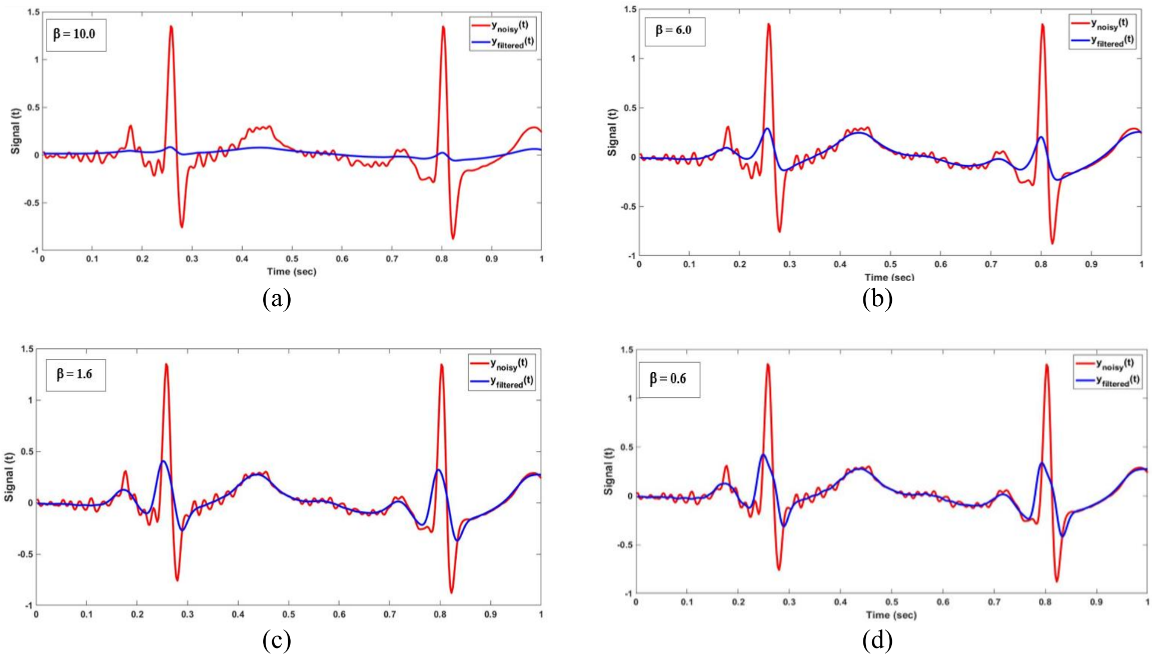


Figure 5. Filtered abnormal ECG signals by compressed and stretched exponential filters for different β values: β=10.0 (a) β=6.0, (b) β=1.6, (c) β=0.6, and (d) all tested filters have σ=$\frac{\sigma_{ECG}}{10}$

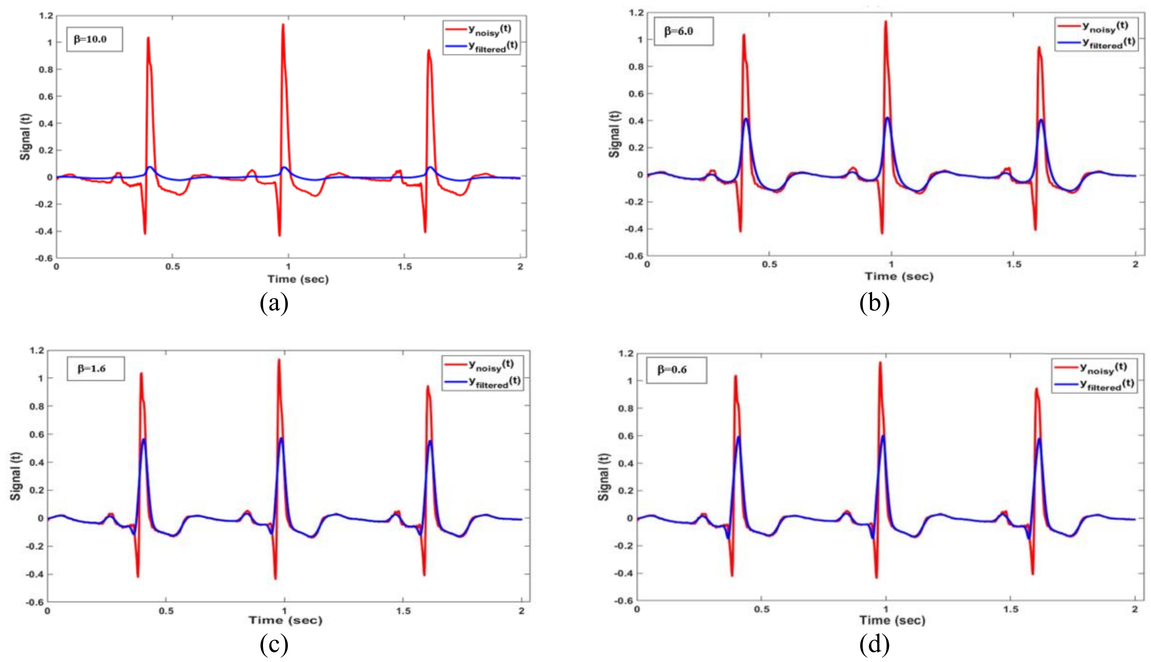


Figure 6. Filtered normal ECG signals by compressed and stretched exponential filters for different β values: β=10 (a) β=6, (b) β=1.6, (c) β=0.6, and (d) all tested filters have σ=$\frac{\sigma_{ECG}}{10}$

Table 2. Performance comparison of proposed compressed-stretched exponential filter for different β values applied to normal ECG signals (totally 50) from the Physionet dataset

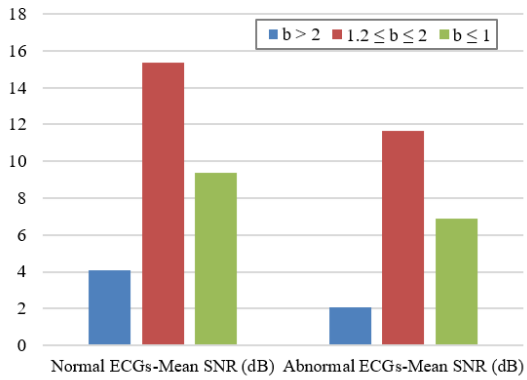
Filter type	Filter parameter	Mean SNR (dB)	Mean MSE
Compressed exponential filter	β=10	1.7778	0.04508
Compressed exponential filter	β=6.0	6.4092	0.01487
Stretched-compressed exponential filter	β=2.0	14.4321	0.01157
Stretched-compressed exponential filter	β=1.6	16.9508	0.01025
Stretched-compressed exponential filter	β=1.2	14.6729	0.01140
Gaussian filter	β=1.0	11.2580	0.01328
Stretched exponential filter	β=0.6	9.6678	0.01395
Stretched Exponential filter	β=0.2	7.2595	0.01598

Table 3. Performance comparison of proposed compressed-stretched exponential filter for different β values applied to abnormal ECG signals (totally 50) from the Physionet dataset

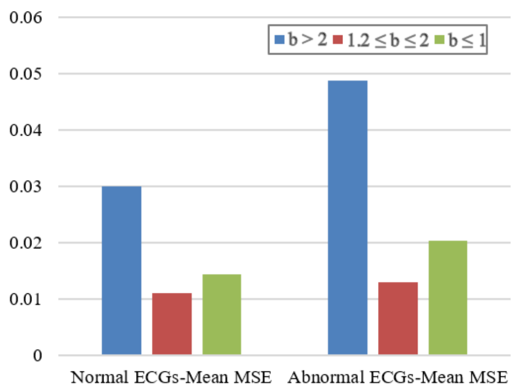
Filter type	Filter parameter	Mean SNR (dB)	Mean MSE
Compressed exponential filter	β=10	0.9444	0.05980
Compressed exponential filter	β=6.0	3.2498	0.03756
Stretched-compressed exponential filter	β=2.0	9.6594	0.01365
Stretched-compressed exponential filter	β=1.6	13.7574	0.01178
Stretched-compressed exponential filter	β=1.2	11.5354	0.01348
Gaussian filter	β=1.0	8.6939	0.01685
Stretched exponential filter	β=0.6	6.2048	0.01893
Stretched exponential filter	β=0.2	5.7240	0.02486

Table 4. Average SNR and MSE values related to normal and abnormal ECGs (from the Physionet dataset) denoised by the developed compressed/stretched exponential filter for different β ranges

	Normal ECGs (50 signals)		Abnormal ECGs (50 signals)	
Exponential parameter (β)	Average SNR (dB)	Average MSE	Average SNR (dB)	Average MSE
$\beta > 2$	4,0935	0,02997	2.0971	0,04868
$1.2 \leq \beta \leq 2$	15,3519	0,01107	11,6507	0,01297
$\beta \leq 1$	9,3951	0,01440	6,8742	0,02021



(a)



(b)

Figure 7. Histograms with the obtained average SNR (a) and MSE (b) values from normal and abnormal ECGs processed by the developed compressed/stretched exponential filter for different β ranges

Finally, the denoising performances of the different filter types have been examined and reported in Table 5. For a Gaussian filter, the most critical parameter is the standard deviation (σ) of the Gaussian kernel, as it determines the smoothing extent; a larger σ results in greater smoothing but may blur the fine details, while a smaller σ preserves finer details but might not remove the noise effectively. The kernel size is typically chosen as a function of σ, often using a size of (6σ+1) to ensure the filter encompasses most of the Gaussian distribution [39]. Additionally, the boundary conditions such as "reflect," "constant," or "wrap" must be selected based on the input data's nature to avoid edge artifacts. The main parameters for Mittag-Leffler filter are the scaling factor α and fractional order β, which govern the weight of the Mittag-Leffler function in modeling memory or smoothing effects [42]; a smaller β emphasizes long-term memory effects, while a larger β provides more localized smoothing. The parameters should be tuned based on the desired level of smoothing and signal's noise characteristics. Finally, the key parameters for Savitzky-Golay filters include the polynomial order (M) and the window size (nl and nr) [43], [44]. The window size should be large enough to smooth noise effectively but small enough to avoid over-smoothing the signal. The polynomial order determines the degree of the fitted curve and should be chosen based on the signal's complexity; for example, a higher order captures more intricate trends but risks amplifying noise. To maintain stability and accuracy in the filter's output, it is essential to ensure the window size exceeds the polynomial order.

Table 5 shows the average SNR and MSE values obtained from ECG signals of the dataset described in section 2 with the added noise and processed by the stretched-compressed exponential low-pass, Gaussian, Mittag-Leffler, and Savitzky-Golay filters. Referring to scientific literature (specifically, Ref. [25] for Gaussian filter, Ref. [16] for the Mittag-Leffler filter, and Refs. [21], [43], [45] for the Savitzky-Golay filter), the values of different filters' parameters were optimized for noise removal from the ECG signals; the selected values for each filter type are reported in Table 5. For the SCELP filter, based on previously reported results (Tables 2 and 3), the β parameter's value was chosen equal to 1.6 to get higher SNR and lower MSE values. Figure 8 shows the applied input signals (with the added noise) and the filtered output ones from the different types of filters tested to allow the performance comparison. The results reported in Table 5 demonstrated that the proposed SCELP filter provides slightly better SNR and MSE values than other types of filters, demonstrating the effectiveness of the proposed solution in removing the noise of the ECG signal.

This study aims to validate the use of a novel SCELP filter for noise reduction in ECG signals; the results demonstrated that it has better performance, in terms of higher SNR and lower MSE values, for β parameter between 1.2 and 2. Different filters used in ECG denoising [16], [21], [25], [43], [45] were compared to SCELP filter; the results in Table 5 regarding normal ECG signals with added noise showed that the SCELP filter performs slightly better than others, certifying its ability in removing noise affecting ECG signal.

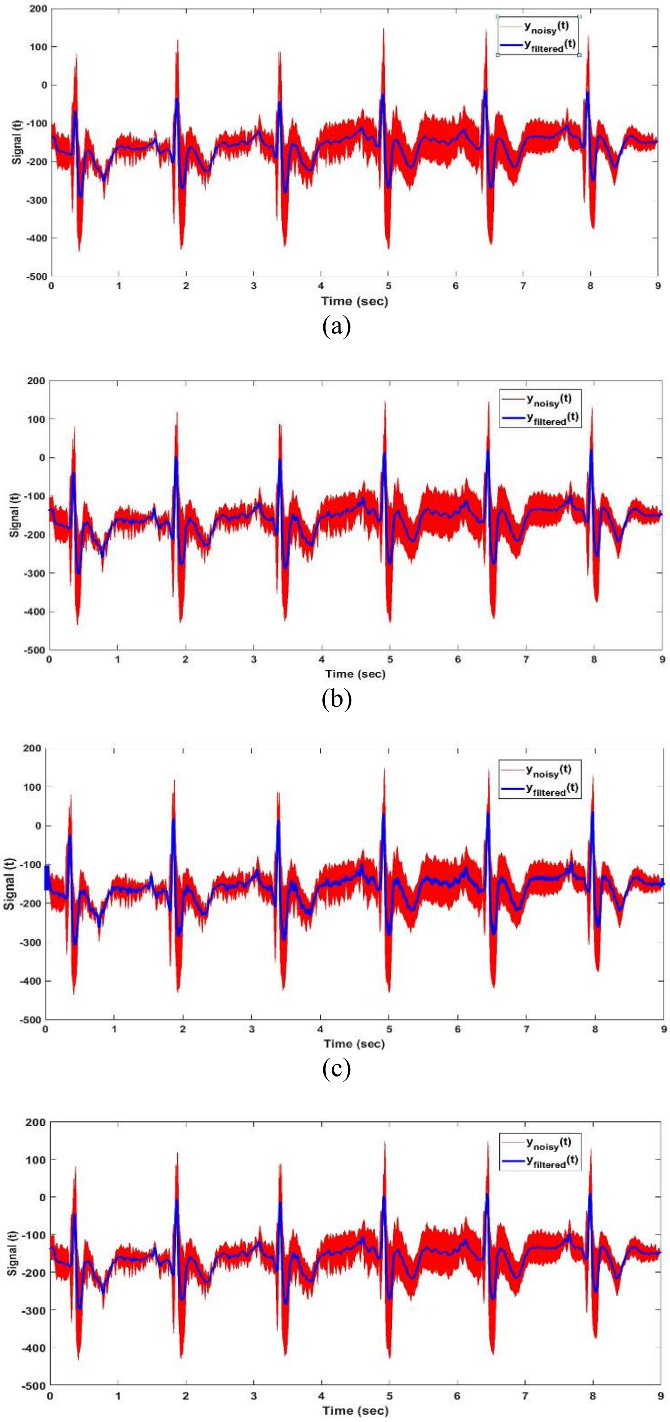
Table 5. Average SNR and MSE values related to the Physionet ECG signals with added white noise for different denoising filters compared to the stretched-compressed exponential low-pass filter's performance

		Gaussian filter $\sigma=0.015=\frac{\sigma_{ECG}}{10}$ (Figure 8(a))	Mittag-Leffler filter σ=0.01, α=0.95, β=0.9 (Figure 8(b))	Savitzky-Golay filter (nl=16, nr=16, M=4) (Figure 8(c))	SCELP filter (β=1.6) (Figure 8(d))
Normal ECG signals + white noise (n=50)	Average SNR (dB)	14.233	11.988	12.864	16.495
	Average MSE	0.0119	0.0135	0.0126	0.0106
Abnormal ECG signals + white noise (n=50)	Average SNR (dB)	11.219	10.625	10.957	14.940
	Average MSE	0.0154	0.0184	0.0167	0.0114

A Gaussian filter has a typical Gaussian impulse response, with no overshoot in response to a step input, reduced rise and fall times, and minimal group delay. As for uncertainty principle in signal processing, which is the suppression of high frequencies while minimizing spatial dispersion, the Gaussian function is the best trade-off between time (spatial) and frequency localization, as it reduces the uncertainty effect. The Weierstrass transform represents how a Gaussian filter modifies the input signal by convolving it with the Gaussian function. The Gaussian-exponential filter shows good group delay, but the attenuation slope above the cutoff frequency is not ideal; tables were created to get the desired Gaussian group-delay response at low and mid frequencies and a Chebyshev-type transition with steeper slope at high frequency.

A Savitzky-Golay filter is used to smooth out a signal and improve its accuracy without changing its trend [25], [45], [46]. This task is accomplished using the convolution technique, which involves using linear least squares to fit successive subsets of nearby data with a low-degree polynomial. An analytical solution to the least-squares equations can be found when data points are evenly spaced; it takes the form of a single set of convolution coefficients to be applied to all data subsets to estimate the smoothed signal or its derivatives at each subset's central point. Savitzky and Golay popularized the mathematical approach [46], releasing tables of convolution coefficients for a range of polynomials and sub-set sizes [43]. The technique was expanded to handle two- and three-dimensional data. Savitzky-Golay filters have a stronger cutoff in the frequency domain, an initially flatter response, and greater signal-to-noise ratio for bandwidth-limited signals, but have two drawbacks: the generation of artifacts when polynomial fits are used for first-final points and relatively poor suppression of some high frequencies [45].

The experimental results demonstrated that the SCELP filter outperforms other denoising filters, requiring only one customizable parameter according to which the distribution function can be adapted, making the filter ideal for ECG signal denoising, in alternative to Butterworth or Savitzky-Golay filters [47]. Further research will allow to validate its effectiveness for different types and levels of noise. Based on the presented results, the designed SCELP filter offers new perspectives for denoising medical signals, as well as EEG and EMG in addition to ECG signals, and for image-processing medical applications [48], [49].



(a)

(b)

(c)

(d)

Figure 8. Comparison of input ECG signals (labeled as normal) with added noise (red trace) and output filtered signals (blue), processed by different filters: (a) Gaussian ($\sigma=\frac{\sigma_{ECG}}{10}$), (b) Mittag– Leffler (σ=0.01, α=0.95, β=0.9), and (c) Savitzky-Golay (nl=16, nr=16, M=4), and (d) the proposed SCELP filter (β=1.6)

4. CONCLUSION

This study presents a novel low-pass filter based on the stretched-compressed exponential function for signal processing applications. Compared to traditional exponential and Gaussian filters, the proposed filter is more versatile with only one adjustable parameter. A MATLAB code for the SCELP filter has been developed, to be tested on 100 ECG signals from the PhysioNet dataset, verifying the denoising capabilities by varying the β parameter; the best performance was obtained for $1.2<\beta<2$ (exactly for $\beta=1.6$). Then, additive noise was added to ECG signals to compare its performance with common filters for ECG signals' denoising (i.e., Gaussian, Mittag–Leffler, and Savitzky-Golay). The obtained results showed that the SCELP filter provides higher average SNR (16.495 and 14.940 dB) and lower MSE (0.0106 and 0.0114) values, than other filter typologies, for normal and abnormal ECG signals, respectively. Finally, the results validated the SCELP filter's use for new application fields, namely for bio-signals' denoising and medical image processing.

AUTHOR CONTRIBUTIONS STATEMENT

This journal uses the Contributor Roles Taxonomy (CRediT) to recognize individual author contributions, reduce authorship disputes, and facilitate collaboration.

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C : **C**onceptualization
M : **M**ethodology
So : **So**ftware
Va : **Va**lidation
Fo : **Fo**rmal analysis
I : **I**nvestigation
R : **R**esources
D : **D**ata Curation
O : Writing - **O**riginal Draft
E : Writing - Review & **E**diting
Vi : **Vi**sualization
Su : **Su**pervision
P : **P**roject administration
Fu : **Fu**nding acquisition

CONFLICT OF INTEREST STATEMENT

Authors state no conflict of interest.

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author [PV], upon reasonable request.

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