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Geometrical determination of the focal point of parabolic solar concentrators

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ABSTRACT

Parabolic solar concentrators play a crucial role in harnessing solar energy by focusing sunlight onto a single focal point, enhancing efficiency in solar thermal applications. However, accurately determining the focal point remains a significant challenge, affecting energy efficiency, stability, and operational costs. This study presents a novel approach to determining the focal point of parabolic solar concentrators using two distinct geometric and mathematical methods. The first method applies standard parabolic equations to derive the focal point, while the second method introduces a geometric approach based on the properties of straight-line tangents and angular measurements. Experimental validation was conducted by comparing the proposed method against laser-based focal point determination. The results demonstrate that the proposed method enhances heat collection efficiency and stability, leading to improved energy output. The findings of this study contribute to optimizing solar concentrator designs, reducing energy losses, and promoting sustainable energy applications.

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1. INTRODUCTION

Parabolic solar concentrators are devices designed to collect and concentrate solar energy at a central focal point. These systems are mainly used in solar thermal power plants, heating supply systems, industrial heat applications, and high-temperature processes. In the case of industrial heat applications, using them in drying processes provides energy efficiency, energy resource savings, and several other advantages.

The use, manufacturing, and efficient operation of solar concentrators come with various complexities and challenges. The most critical of these is determining and accurately identifying the focal point of the concentrator. This is a key factor that defines the main performance indicators of a solar concentrator. Extensive research has been conducted by various scholars on methods for determining the focal point of these concentrators.

Beltagy [1] emphasizes that accurately identifying the focal point of a solar concentrator significantly improves its energy efficiency. Accurate identification can increase energy production by 20%

to 30%. For instance, if a concentrator generates approximately 1,000 watts per hour at an estimated focal point, a precise determination can raise this figure to 1,200–1,300 watts. Moreover, proper focal point identification can improve heat collection efficiency by up to 90%, while at an estimated focal point, this efficiency may drop to as low as 60%.

From an adaptability standpoint, correctly determining the focal point ensures energy production stability of up to 95%, whereas an estimated focal point may reduce this stability to 70%. When the focal point is incorrectly identified, material waste increases by up to 25% to achieve full capacity, leading to higher production costs. Additionally, an improperly chosen focal point increases the risk of system failure. When the concentrator's efficiency declines, reliance on fuel-based energy sources increases, leading to higher carbon emissions, fuel shortages, and other related issues. Additionally, incorrect focal point determination raises maintenance costs due to the need for frequent servicing. Therefore, precise focal point identification is crucial.

In related research, the secondary reflector geometry of a Fresnel-type solar concentrator was optimized. It resulted in a new design called the double parabolic concentrator (DPC). This innovation improved prototype efficiency by 10% to 13%, achieving 100% optical efficiency at the receiver.

Tsekouras *et al.* [2] performed optical and thermal analyses on a linear Fresnel collector with a trapezoidal cavity. Their findings confirmed that concentrated solar collectors effectively generate heat between 150 °C and 400 °C. Using ray-tracing models, they demonstrated solar radiation distribution across the absorber's perimeter. Computational fluid dynamics (CFD) simulations further analyzed temperature distribution in receiver components.

Al-Arab [3] developed optical designs for spherical and parabolic dish concentrators of varying diameters. Simulations using ZEMAX software revealed that both concentrator types can exhibit spherical and coma aberrations under normal conditions. Li and Dubowsky [4], [5] noted that parabolic dish concentrators offer the highest thermal and optical efficiency among existing designs. Their research introduced a novel method for manufacturing large-scale parabolic dishes by optimizing segmented mirrors via particle swarm optimization-genetic algorithm (PSO-GA). The design involved cable-adjusted petal-like segments and an analytical model for stiffness optimization using perforated holes. Kalidasan *et al.* [6] improved linear cavity receiver efficiency, achieving 77% thermal efficiency and operating temperatures exceeding 627 °C. Touaref *et al.* [7] enhanced a solar distiller's performance using an automatic tracker, producing 160 liters of water in 10 hours with 70.3% thermal efficiency.

Jung *et al.* [8] developed a solar-energy-plant-cultivation hybrid module, reaching 54% solar conversion efficiency. Foulaadvand *et al.* [9] analyzed heat flux distribution in solar collectors, comparing spherical and parabolic reflector efficiency using novel analytical methods. Hafez *et al.* [10] examined design parameters, mathematical models, and simulations for parabolic trough collectors (PTCs), assessing their feasibility in different regions. Tawfik *et al.* [11] studied a solar cooker with a bottom parabolic reflector (TBPR), applying 4E (energy, exergy, economic, environmental) analysis. The design improved efficiency by 32.69% and reduced CO_2 emissions. Khaledi *et al.* [12] developed a nanofluid-based thermal model for compound parabolic concentrators (CPCs), validating it with SiO_2/ethylene glycol-water nanofluid (4.61% average error). Drira [13] proposed semi-parabolic linear Fresnel reflectors (SPLFRs) as cost-effective alternatives to traditional PTCs for low-income regions. Li *et al.* [14] enhanced parabolic trough collector (PTC) performance using a homogenizer and spiral structure (RHS design), reducing thermal deformation by 96% and improving efficiency by 0.63%–1.2%. Bharti *et al.* [15] optimized secondary reflectors for PTCs, finding parabolic reflectors increased temperature by 10.9 °C compared to triangular (9.6 °C) and no-reflector (7.4 °C) setups. Madadi *et al.* [16] evaluated glass-covered PTCs under different tracking modes, noting a 10% efficiency drop with a 6° tilt error in full-tracking mode.

Maatoug *et al.* [17] modeled parabolic cylindrical collectors mathematically. The research investigated the conversion of solar energy into thermal energy using a parabolic cylindrical concentrator. The energy balance equation for the absorber tube was solved using the finite difference method, and a MATLAB-based computer simulation was developed to analyze thermal efficiency, absorber tube temperature, working fluid temperature, glass temperature, and heat loss coefficients.

Kumar *et al.* [18] conducted an optical analysis of a cylindrical semi-spherical receiver integrated with a 3-meter-diameter parabolic dish concentrator. The study utilized SolTrace software to analyze the effect of varying parameters such as receiver aperture diameter (Da: 0.125 to 0.162 m), concentrator surface error (1.7453 to 34.907 mrad), and surface absorptivity (α: 75% to 95%) at different receiver distances (H: 1.7 to 1.95 m). The findings concluded that the optimal optical efficiency was achieved when the receiver (diameter: 0.150 m) was positioned 1.85 m away from the concentrator. Additionally, increasing slope errors from 1.7453 to 17.453 mrad reduced average optical efficiency by nearly 50% for all receiver diameters.

Chen *et al.* [19] improved secondary solar applications using hybrid parabolic/spherical mirrors, validated via Zemax simulations. Cuce *et al.* [20] integrated thermoelectric generators (TEGs) with PTCs, achieving 70% higher efficiency than standalone systems. Latrache *et al.* [21] developed geometric parameters for conical concentrators, deriving a generalized focal distance equation. Donga *et al.* [22] compared rhombic vs. circular absorbers in PTCs, showing 2.88% higher thermal efficiency with rhombic designs. Mallayya *et al.* [23] analyzed parabolic dish concentrators (PDCs) for micro gas turbines (18.3% electrical efficiency) and steam plants. Nandanwar *et al.* [24] reviewed PDC advancements, including receiver geometry, nanofluids, and thermal storage. Terrón-Hernández *et al.* [25] designed a north-facing CPC solar water heater with PID/FOPID control algorithms. Qu *et al.* [26] reduced cosine losses in PTCs using rotating-axis tracking, boosting winter efficiency from 43% to 48%. This research underscores the importance of geometry, materials, and optimization in focal point determination, enhancing solar energy efficiency.

2. METHOD

Parabolic solar concentrators come in several distinct configurations, each tailored for specific applications:

a. Parabolic dish concentrators: high-efficiency systems typically used for concentrated photovoltaic (CPV) or thermal applications;
b. Parabolic dish-Stirling systems: combine parabolic dishes with Stirling engines for electricity generation;
c. Parabolic trough water heating systems: designed primarily for industrial and residential water heating applications;
d. Parabolic trough concentrators: widely used in utility-scale solar thermal power plants.

Each variant exhibits unique performance characteristics, operational efficiencies, and engineering specifications that make them suitable for particular use cases. The research investigation included comprehensive analysis of focal point identification methods for parabolic trough systems. Two principal methodologies emerge for this critical alignment process.

2.1. Mathematical function of the parabolic solar concentrator

In this method, the mathematical function of the parabolic solar concentrator must be known. For example, let us determine the focal point of the function $y = \frac{x^2}{n}$. To find the focal point of the function $y = \frac{x^2}{n}$, it must first be rewritten in the general form of a parabolic equation. The equation of a parabola is closely related to its mathematical and geometrical properties. A parabola is a set of points in a plane where each point is equidistant from a fixed point (focus) and a fixed line (directrix). This fundamental geometric condition defines the shape and focal properties of the parabola, see Figure 1.

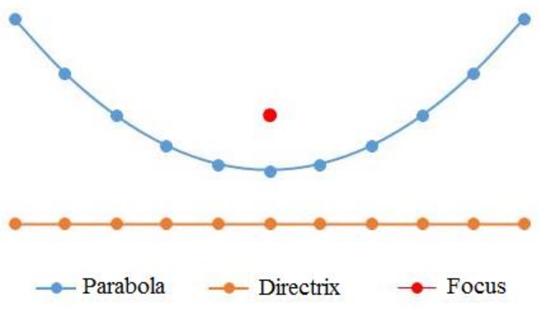

Figure 1. Geometric representation of the function $y = \frac{x^2}{n}$

Using these properties of a parabola, we can transform the function $y = \frac{x^2}{n}$ into the general form of a parabolic equation. To analyze the general equation of a parabola, we must first consider its geometric characteristics. If the vertex of the parabola is at (0, 0), its equation satisfies the following conditions: The directrix is given by $y = -p$. The focus is at (0, p), where p represents the distance from the vertex to the directrix or the focus, see Figure 2. According to the definition of a parabola, the perpendicular distance from any point (x, y) on the parabola to the directrix is always equal to its distance from the focus, see Figure 3.

Based on these conditions, we perform the following steps and thereby transform our function $y = \frac{x^2}{n}$ into a general parabolic equation:

The distance between an arbitrary point (x, y) and the focus (0, p) on a parabola is:

$$\sqrt{x^2 + (y-p)^2} \quad (1)$$

The distance between the point (x, y) and the directrix on the parabola is:

$$(y+p) \quad (2)$$

According to Figure 3, These two distances must be equal, therefore:

$$\sqrt{x^2 + (y-p)^2} = (y+p) \quad (3)$$

Let's simplify this expression. We create the following function:

$$x^2 + (y-p)^2 = (y+p)^2 \quad (4)$$

$$x^2 + y^2 - 2yp + p^2 = y^2 + 2yp + p^2 \quad (5)$$

$$x^2 - 2yp = 2yp \quad (6)$$

$$x^2 = 4yp \quad (7)$$

$$y = \frac{x^2}{4p} \quad (8)$$

This function $y = \frac{x^2}{4p}$ is a general function of parabola. We satisfy the given condition (finding the focus) by comparing the function $y = \frac{x^2}{n}$ to the general function of a parabola $y = \frac{x^2}{4p}$.

$$y = \frac{x^2}{n} = \frac{x^2}{4p} \quad (9)$$

$$\frac{1}{n} = \frac{1}{4p} \quad (10)$$

$$n = 4p \quad (11)$$

$$p = \frac{n}{4} \quad (12)$$

Therefore, the value of the focus of the parabola p is equal to (12). From this, the focal point of the parabolic function $y = \frac{x^2}{n}$ given to us is located on the vertical axis, and its coordinates are (0, $\frac{n}{4}$). Thus, it was determined by method 1 that the focal point of the function $y = \frac{x^2}{n}$ is (0, $\frac{n}{4}$).

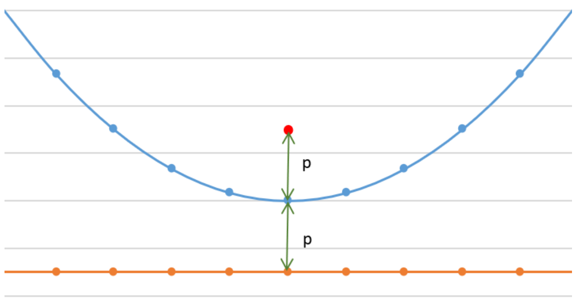

Figure 2. Equality of the distance (p) from the origin of the parabola to the directrix and the focus

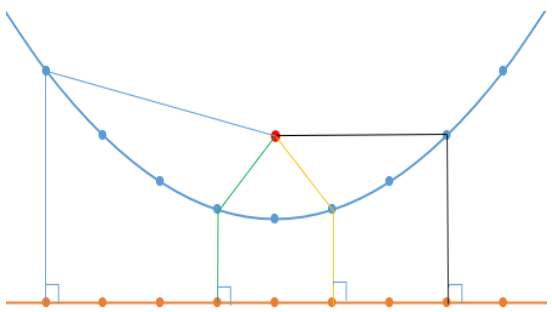

Figure 3. The distance from an arbitrary point of a parabola perpendicular to the directrix is equal to the distance to the focus

2.2. Determination of the focus of a parabola given by the equation of a straight line converted to a function as an attempt

This method determines the focus of the given parabola through the equation of a straight line transformed into the function $y = \frac{x^2}{n}$. Therefore, the following must be done to construct the equation of a straight line transformed into the function $y = \frac{x^2}{n}$.

To write the equation of a straight line, we write the following using the explanations in Figure 4.

$$k = \frac{(y-y_0)}{(x-x_0)} = \frac{(y-y_0)}{(x-x_0)} \quad (13)$$

Here $(x_0;\ y_0)$ is a point on both the straight line and the parabola. k is the slope of the straight line $(tan\,\alpha)$. This is also known to us from mathematical rules, see Figure 4.

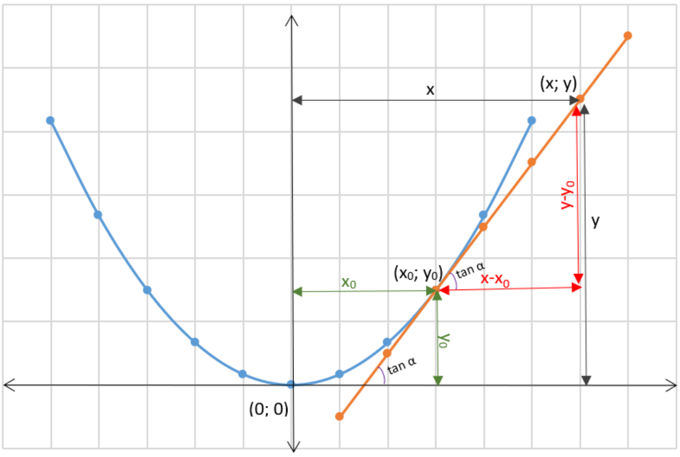

Figure 4. The function $y = \frac{x^2}{n}$ and the equation of the straight line touching it at the point $(x_0;\ y_0)$

From mathematical rules, we can also derive that the first derivative of a parabola function is equal to the slope of the straight line drawn to it from the point $(x_0;\ y_0)$, that is, k. Function $y = \frac{x^2}{n}$; its derivative $\frac{dy}{dx} = \frac{2x}{n}$.

$$k = \frac{2x}{n} = (tan\,\alpha) \quad (14)$$

Since the point $(x_0;\ y_0)$ belongs to both the parabola and the straight line:

$$k = \frac{2x_0}{n} \quad (15)$$

$$y_0 = \frac{x_0^2}{n} \quad (16)$$

Considering the above:

$$(y - y_0) = k(x - x_0) \quad (17)$$

$$(y - \frac{x_0^2}{n}) = \frac{2x_0}{n}(x - x_0) \quad (18)$$

By operating, we get (19) and (20):

$$y = \frac{2x_0}{n}x - \frac{2x_0^2}{n} + \frac{x_0^2}{n} \quad (19)$$

$$y = \frac{2x_0}{n}x - \frac{x_0^2}{n} \quad (20)$$

So, the equation of the straight line drawn through the point $(x_0;\ y_0)$ as a function $y = \frac{x^2}{n}$ has the form $y = \frac{2x_0}{n}x - \frac{x_0^2}{n}$. This was necessary for us to better understand the numerical value of $x_0;\ y_0$, what $\tan\alpha$ is, and its location here. Because, in method 2, these are of great importance. For example, let us consider the problem of determining the focus of a parabola with an unknown function. Such a situation is encountered in energy and engineering work. Technical measurements of the values of $x_0;\ y_0$, the angular value of α can be carried out using an engineering ruler.

We rely on the fact that *"the distance from an arbitrary point of a parabola perpendicular to the directrix is equal to the distance to the focus"*. This can also be seen below, see Figure 5. A ray incident on a parabola at point $x_0;\ y_0$ is directed towards the focus. The length of the ray from point $x_0;\ y_0$ to the focus is equal to $p + y_0$, see Figure 6. A ray incident on a parabola at point $x_0;\ y_0$ returns to the focus at the same angle as the ray incident on point $x_0;\ y_0$. The value of this angle is equal to $90 - \alpha$, see Figure 7. Using Figure 7, we can see that the angle between the intercepts $p + y_0$ and y_0 is $180 - 2\alpha$. We draw a line (c) from the focus to the point $(x_0;\ 0)$ to form a triangle, see Figure 8. We find the length of line (c) using the law of cosines.

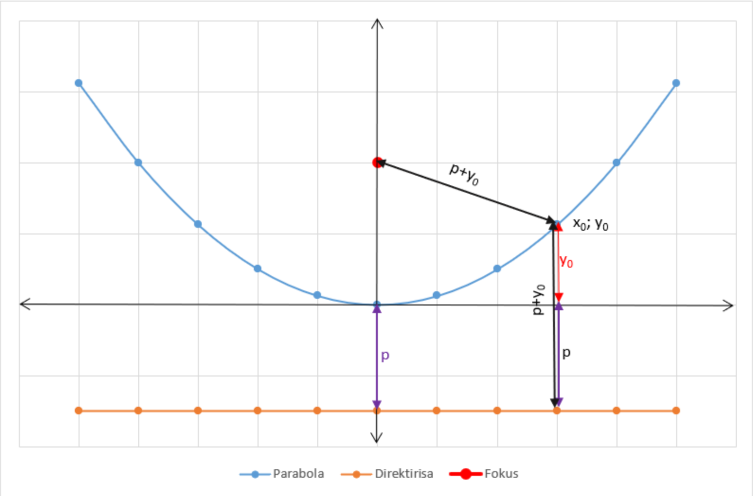

Figure 5. A drawing of a geometric property of a parabola

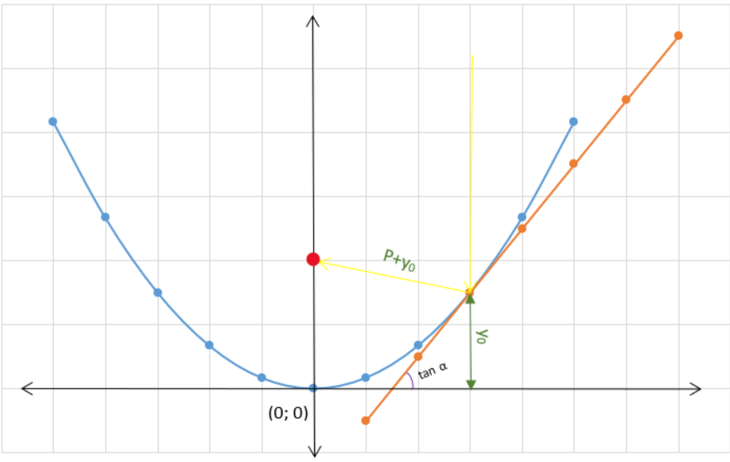

Figure 6. Refraction of ray incident on a parabola towards the focus

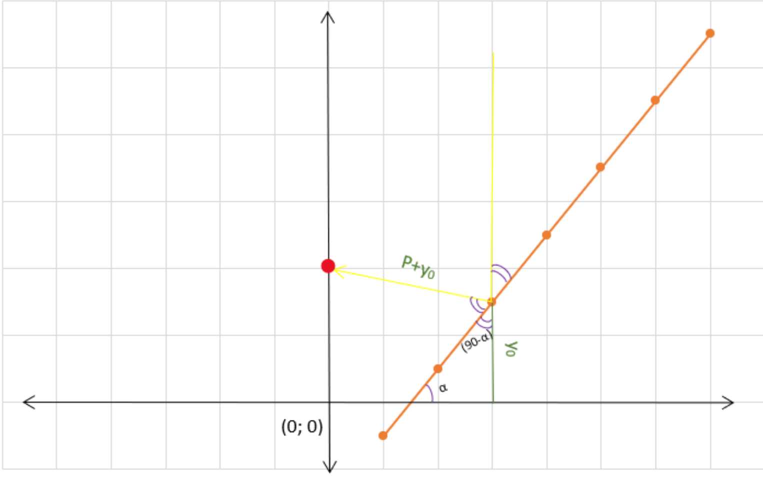

Figure 7. The angle of incidence and return of a ray to a point $x_0;\ y_0$ of a parabola

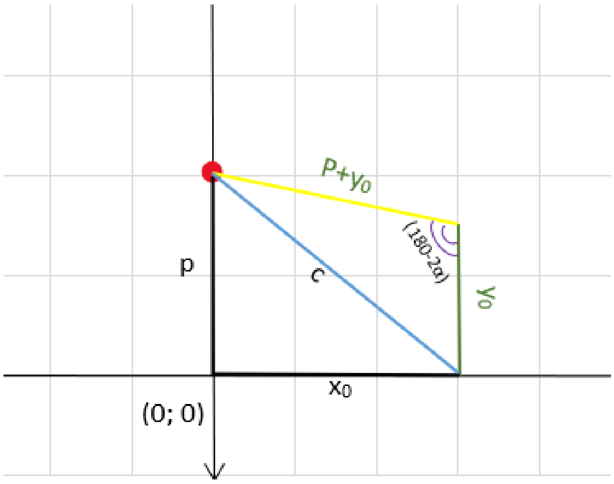

Figure 8. A right triangle is shown to determine the focus of a parabola with an unknown function

According to the theorem of cosines:

$$c^2 = (p + y_0)^2 + y_0^2 - 2 \cdot (p + y_0) \cdot y_0 \cdot \cos(180 - 2\alpha) \quad (21)$$

According to the Pythagorean theorem:

$$c^2 = p^2 + x_0^2 \ (22)$$

From this, we can write the following:

$$p^2 + x_0^2 = (p + y_0)^2 + y_0^2 - 2 \cdot (p + y_0) \cdot y_0 \cdot \cos(180 - 2\alpha) \quad (23)$$

We simplify this equation and determine the value of p:

$$cos(180 - 2\alpha) = -cos(2\alpha) \quad (24)$$

$$p^2 + x_0^2 = (p + y_0)^2 + y_0^2 + 2 \cdot (p + y_0) \cdot y_0 \cdot \cos(2\alpha) \quad (25)$$

$$p^2 + x_0^2 = p^2 + 2py_0 + 2y_0^2 + 2 \cdot (p + y_0) \cdot y_0 \cdot \cos(2\alpha) \quad (26)$$

$$x_0^2 = 2py_0 + 2y_0^2 + 2 \cdot (p + y_0) \cdot y_0 \cdot \cos(2\alpha) \quad (27)$$

$$x_0^2 = 2py_0 + 2y_0^2 + 2py_0 \cos(2\alpha) + 2y_0^2 \cos(2\alpha) \quad (28)$$

$$x_0^2 = 2py_0(1 + \cos(2\alpha)) + 2y_0^2 (1 + \cos(2\alpha)) \quad (29)$$

$$2py_0(1 + \cos(2\alpha)) = x_0^2 - 2y_0^2 (1 + \cos(2\alpha)) \quad (30)$$

$$p = \frac{x_0^2 - 2y_0^2 (1+\cos(2\alpha))}{2y_0(1+\cos(2\alpha))} \quad (31)$$

Therefore, the presented method can be used to determine the focus of a parabolic solar concentrator with an unknown function. That is, the focal point of a parabolic solar concentrator with an unknown function can be determined (by knowing the values of $x_0;\ y_0$, and α using a rectangular engineering ruler).

3. RESULTS AND DISCUSSION

Let's compare the exact results of the two methods above. If the focal point of a parabolic function solar concentrator is the same in both methods, then the methods are consistent, compatible, and provide accurate results. For example, let's determine the focus of the parabolic function $y = \frac{x^2}{8}$, see Figure 9.

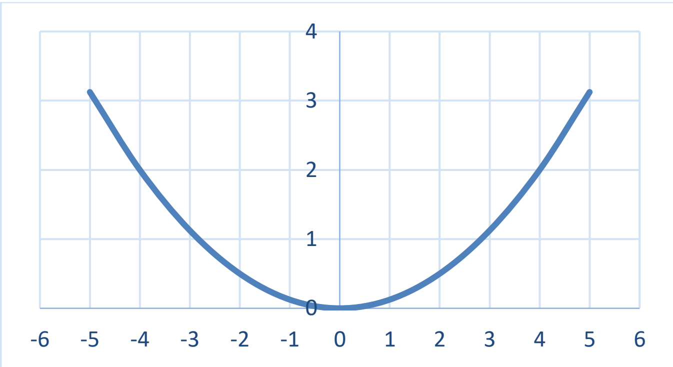

Figure 9. Parabolic function $y = \frac{x^2}{8}$

From mathematical laws, we know that the parabola's focus with the function $y = ax^2 + bx + c$ equals $\frac{1}{4\,a}$. Here $a = \frac{1}{8}, b = 0, c = 0$. The focus of the parabolic function $y = \frac{x^2}{8}$ is $p = \frac{1}{4*\frac{1}{8}} = \frac{8}{4} = 2$. If the origin of the parabolic function $y = \frac{x^2}{8}$ is at point (0; 0), then since the focus is at (0; p), the focus is located at the point (0; 2).

According to method 1, based on formula (12), $p = \frac{n}{4}$, that is $\frac{8}{4} = 2$. So, in this method, the focus is also located at the point (0; 2). According to method 2, we write the equation of a straight line of the parabolic function $y = \frac{x^2}{8}$, for example, passing through the points (4; 2). According to (20), $y_{\text{straight line}} = \frac{2x_0}{n} x_{\text{straight line}} - \frac{x_0^2}{n}$; from this we obtain the following.

$$y_{\text{straight line}} = \frac{2*4}{8} x_{\text{straight line}} - \frac{4^2}{8} \quad (32)$$

$$y_{\text{straight line}} = x_{straight\text{ line}} - 2 \quad (33)$$

The arbitrary point (4; 2) corresponds to both the parabolic function $y = \frac{x^2}{8}$ and the equation of the straight line $y_{\text{straight line}} = x_{\text{straight line}} - 2$, see Figure 10.

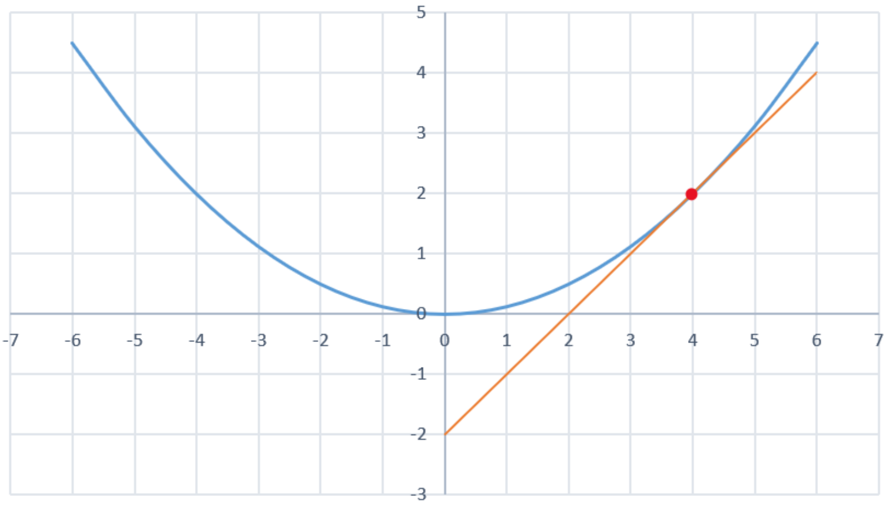

Figure 10. The graph of the straight-line equation $y_{\text{straight line}} = x_{\text{straight line}} - 2$, which is an attempt to convert the parabolic function $y = \frac{x^2}{8}$

According to (14) and (15), the value of α is $k = (tan\,\alpha) = \frac{2x}{n} = \frac{2x_0}{n}$. Therefore, $tan\,\alpha = \frac{2*4}{8} = 1$. It follows that $\alpha = 45^{\circ}$. As we have chosen arbitrarily, the point (4; 2) is x_0; y_0. Therefore, $x_0 = 4$; $y_0 = 2$; $\alpha = 45^{\circ}$. Accordingly, we check method 2 using (31).

$$p = \frac{4^2 - 2 \cdot 2^2\,(1+\cos(90°))}{2 \cdot 2(1+\cos(90°))} = \frac{16-8(1+0)}{4(1+0)} = \frac{8}{4} = 2 \tag{34}$$

As can be seen from (34) the focus is located at the point (0; 2). It follows that, like the previous methods, the new method, which was invented using geometric shapes, rules, theorems, and mathematical concepts, is also correct. It can be said that it has justified itself.

In one of the two experiments conducted, the focus of the concentrator was determined experimentally by focusing the laser beam. In the second method, it was determined using the method we propose using (31). In both experiments, one concentrator was used. Table 1 shows the parameters of the concentrators.

Table 1. The parameters of the concentrators

	Surface	Light reflectance	Copper pipe diameter	Air drive fan
Concentrator	2 m^2	91%	20mm	OWB-4011-24

The experimental results of the first method on a laser-focused concentrator are presented in Table 2. The experimental results of the second method on a concentrator with a focus determined according to (31) are presented in Table 3. As can be seen from the results in Tables 2 and 3, the temperatures of the air extracted from the concentrator with the focal point determined by the proposed method have increased. This can be seen in the diagram in Figure 11.

Table 2. The experimental results of the first method

Times of day	Outdoor temperature °C	Solar intensity w/m^2	Heating air temperature °C
9:00	21	730	60
10:00	23	738	70
11:00	26	755	74
12:00	30	770	76
13:00	33	790	79
14:00	31	775	77
15:00	28	765	75

Table 3. The experimental results of the second method

Times of day	Outdoor temperature °C	Solar intensity w/m^2	Heating air temperature °C
9:00	20	730	61
10:00	23	738	75
11:00	27	755	79
12:00	30	770	81
13:00	33	790	86
14:00	32	775	81
15:00	29	765	78

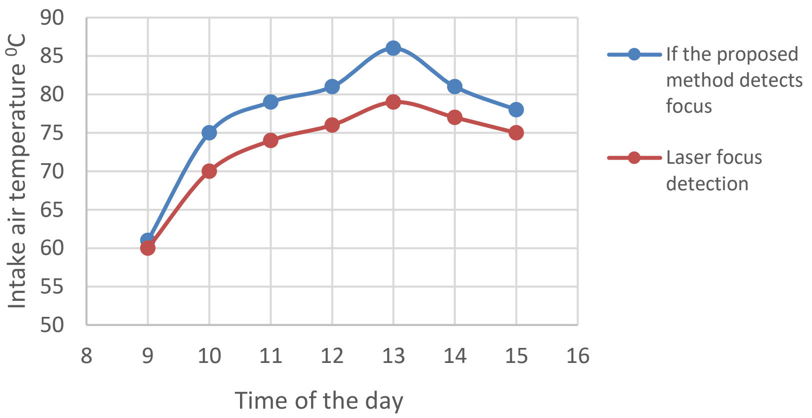

Figure 11. Graphical representation of the advantages of the proposed method

If there is a solar concentrator with an unknown parabolic function, the angle α formed by an arbitrary point x_0; y_0 of the parabola (concentrator) with the abscissa axis of the equation of the straight line drawn from this point as an attempt to the parabola can be determined using a rectangular engineering ruler as mentioned above. By using the results of these measurements and the proposed method 2, which is explained in this research work, it is possible to determine its focal point. This, in turn, serves to increase the useful efficiency of the solar concentrator. This is because the light collected in the solar concentrators is always concentrated at the focal point. From this point of view, the importance of knowing the focal point of parabolic solar concentrators is very important.

4. CONCLUSION

The study successfully developed and validated a new method for accurately determining the focal point of parabolic solar concentrators. By applying both mathematical and geometric approaches, the research demonstrated that the proposed technique improves the accuracy of focal point identification, leading to higher energy efficiency and thermal performance. Experimental comparisons confirmed that the concentrator with a focus determined using the new method exhibited increased heat collection efficiency. The findings suggest that integrating this approach into solar concentrator design and manufacturing processes can significantly enhance the reliability and performance of solar energy systems. Future research can explore the integration of this method with advanced tracking systems and adaptive optical designs to further optimize solar energy capture and utilization.

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