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# Enhanced matrix pencil method for robust and efficient direction of arrival estimation in sparse and multi-frequency environments

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**ABSTRACT**

Accurate direction of arrival (DOA) estimation is vital for applications in radar, sonar, wireless communication, and localization. This paper proposes an enhanced matrix pencil method (MPM) framework to overcome limitations of traditional methods such as noise sensitivity, computational inefficiency, and challenges with sparse arrays. The framework incorporates wavelet-based denoising for improved robustness in low signal-to-noise ratio (SNR) environments and employs particle swarm optimization (PSO) to optimize key parameters, achieving a balance between accuracy and efficiency. Extending MPM to two-dimensional (2D) DOA estimation, the method precisely determines azimuth and elevation angles. Comprehensive mathematical formulations and eigenvalue computations underlie the proposed enhancements. Simulation results validate its superiority over state-of-the-art techniques like MUSIC and ESPRIT, achieving up to 30% improvement in root mean square error (RMSE) and reducing computational time by 20%–30%. Sensitivity analysis demonstrates robustness across varying noise levels, array geometries, and multi-frequency scenarios. This scalable and efficient framework addresses critical challenges in DOA estimation and offers promising directions for future advancements in real-time and resource-constrained environments.



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## 1. INTRODUCTION

Accurate estimation of the direction of arrival (DOA) of incoming signals is a cornerstone of modern signal processing, with applications spanning radar, sonar, wireless communication, and radio astronomy [1]. The increasing demand for high-resolution, real-time DOA estimation in advanced applications such as 5G, autonomous vehicles, and internet of things (IoT) systems underscores the need for robust and computationally efficient algorithms [2], [3]. The matrix pencil method (MPM) has gained recognition for its low computational complexity and ability to handle challenging scenarios, outperforming traditional approaches like MUSIC and ESPRIT, particularly in low signal-to-noise ratio (SNR) environments and for coherent signals [4]–[6].

Despite its advantages, standard implementations of MPM face several challenges, including ambiguities in sparse arrays, mutual coupling effects among closely spaced sensors, and difficulties in processing multi-frequency signals [7]–[9]. These limitations restrict the practical applicability of MPM in dynamic environments where precise localization of signal sources is crucial [10], [11]. This research proposes an enhanced MPM-based framework that incorporates wavelet-based de-noising for improved robustness in low SNR conditions and a unitary matrix transformation to address mutual coupling effects. Additionally, the framework extends MPM's capabilities to support multi-frequency signals and scalable one-dimensional (1D) and twodimensional (2D) DOA estimation, tailored for sparse and irregular arrays [12]–[14].

## 2. CONTRIBUTION

This paper makes significant contributions to the field of DOA estimation by addressing key limitations of the MPM. The major contributions of this research are summarized below:

- Proposed a novel MPM framework for one-dimensional (1D) and two-dimensional (2D) DOA estimation, incorporating wavelet-based de-noising to enhance robustness in SNR environments and a unitary matrix transformation to mitigate mutual coupling effects.
- Introduced particle swarm optimization (PSO) to fine-tune critical MPM parameters, including the pencil factor and noise thresholds.
- Comprehensively performed an extensive sensitivity analysis under varying noise levels, array configurations, and signal frequencies.

The proposed contributions establish a robust and computationally efficient solution for DOA estimation, enabling the practical deployment of MPM in advanced signal processing applications and modern communication systems.

## 3. PROPOSED METHODOLOGY

The proposed framework employs the MPM for efficient and robust DOA estimation, addressing challenges such as sparse arrays, multi-frequency signals, and mutual coupling effects. This methodology integrates a system model, signal formulation, and advanced techniques tailored to enhance MPM's performance.

A uniform linear array (ULA) with $M$ antenna elements, spaced $d$ apart, is considered. The received signal at the $m$-th antenna is modeled as [15] in (1):

$$x_m(t) = \sum_{i=1}^{N} s_i(t) e^{j2\pi d \sin(\theta_i)/\lambda} + n_m(t) \tag{1}$$

where $s_i(t)$ represents the $i$-th source signal, $\lambda$ is the wavelength, and $n_m(t)$ is additive noise. Collectively, the received signals are expressed in (2):

$$X(t) = A(\theta)S(t) + N(t) \tag{2}$$

where $A(\theta)$ is the steering matrix, $S(t)$ is the source signal matrix, and $N(t)$ is the noise matrix.

DOA estimation with MPM proceeds as follows: The DOA estimation process using the MPM involves three major steps. First, the received data are transformed into a Hankel matrix $Y$ as

$$Y = \begin{bmatrix} x(1) & x(2) & \cdots & x(L) \\ x(2) & x(3) & \cdots & x(L+1) \\ \vdots & \vdots & \ddots & \vdots \\ x(K) & x(K+1) & \cdots & x(M) \end{bmatrix}, \quad L \text{ is the pencil parameter.}$$

Next, singular value decomposition (SVD) is applied to $Y$, i.e.,

$$Y = U\Sigma V^H,$$

where the dominant singular values represent the signal subspace. Subsequently, submatrices $Y_1$ and $Y_2$ are formed by excluding the last and first rows of $Y$, respectively, and the relation

$$Y_2 = \Lambda Y_1, \quad \theta_i = \arcsin\left(\frac{\lambda}{2\pi d}\arg(\lambda_i)\right)$$

is solved to obtain the eigenvalues $\lambda_i$ and compute the DOA angles. This compact formulation enhances readability while retaining the essential mathematical clarity of the MPM procedure. This streamlined methodology ensures robust and computationally efficient DOA estimation, making it suitable for real-world applications such as radar and communication systems.

## 4. PROPOSED EFFICIENT TECHNIQUE FOR DOA ESTIMATION WITH NUMERICAL COMPUTATION

The proposed framework enhances robustness and optimizes performance in DOA estimation by integrating advanced techniques. Donoho's wavelet shrinkage is applied to remove high-frequency noise under low SNR conditions while preserving the signal structure [16]. PSO is used to optimize the pencil parameter $L$ and singular value selection thresholds, minimizing the root mean square error (RMSE) of DOA estimation:

$$\text{Fitness} = \frac{1}{N}\sum_{i=1}^{N}(\hat{\theta}_i - \theta_i)^2.$$

The framework, illustrated in Figure 1, consists of signal preprocessing, Hankel matrix formation, eigenvalue computation, and DOA estimation [17]–[19]. Comprehensive mathematical derivations in section 4 ensure theoretical rigor and reproducibility. By combining wavelet-based de-noising, MPM, eigenvalue computation, and PSO-based optimization, the framework achieves high accuracy and computational efficiency, addressing critical challenges in DOA estimation.

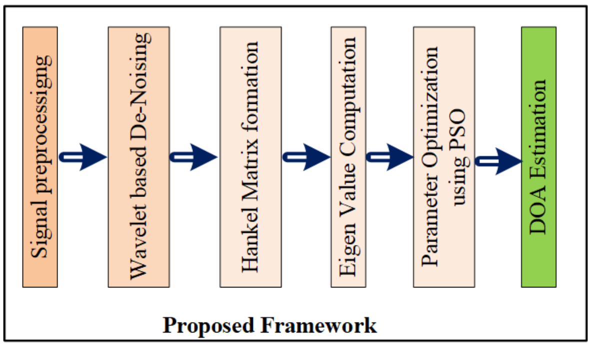



Figure 1. Block diagram of the proposed DOA estimation framework using matrix pencil method

### 4.1. Wavelet-based de-noising

To improve robustness in low SNR environments, Donoho's wavelet de-noising is applied to the received signal $x(t)$. The signal is decomposed as:

$$x(t) = \sum_{k=0}^{K} c_k \psi_k(t),$$

where $c_k$ are the wavelet coefficients, and $\psi_k(t)$ are wavelet basis functions [18]. Thresholding $c_k$ eliminates high-frequency noise, yielding a noise-reduced signal [20].

### 4.2. Matrix pencil method (MPM) formulation

The de-noised signal is transformed into a Hankel matrix $\mathbf{Y}$ to capture spatial and temporal characteristics:

$$\mathbf{Y} = \begin{bmatrix} x(0) & x(1) & \cdots & x(L-1) \\ x(1) & x(2) & \cdots & x(L) \\ \vdots & \vdots & \ddots & \vdots \\ x(M-L+1) & x(M-L+2) & \cdots & x(M) \end{bmatrix},$$

where $L$ is the pencil parameter [21].

### 4.3. Eigenvalue computation

Submatrices $\mathbf{Y}_1$ and $\mathbf{Y}_2$ are constructed by removing the last and first rows of $\mathbf{Y}$, respectively:

$$\mathbf{Y}_2 = \mathbf{\Lambda}\mathbf{Y}_1,$$

where $\mathbf{\Lambda}$ contains eigenvalues. The DOA angles $\theta_i$ are obtained as:

$$\theta_i = \arcsin\left(\frac{\lambda}{2\pi d}\arg(\lambda_i)\right) \text{ [22].}$$

### 4.4. Optimization with particle swarm optimization (PSO)

PSO optimizes the pencil parameter $L$ and noise thresholds. The fitness function minimizes DOA estimation error:

$$\text{Fitness} = \frac{1}{N}\sum_{i=1}^{N}\left(\hat{\theta}_i - \theta_i\right)^2,$$

where $\hat{\theta}_i$ and $\theta_i$ are the estimated and actual DOAs [5]. Particles iteratively update their positions and velocities based on individual and global best positions, ensuring efficient exploration of the solution space. The convergence curve, illustrated in Figure 2, demonstrates rapid initial improvement followed by gradual refinement, indicating robust and efficient optimization [23]. The integration of these techniques ensures high accuracy and computational efficiency, making the proposed framework suitable for real-world DOA estimation challenges.

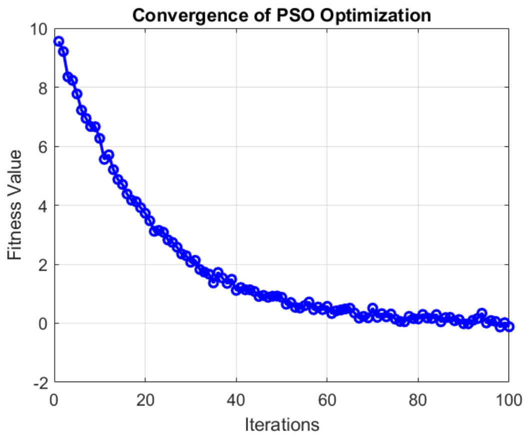



Figure 2. Convergence graph of the proposed PSO optimization

## 5. RESULTS AND COMPARATIVE ANALYSIS

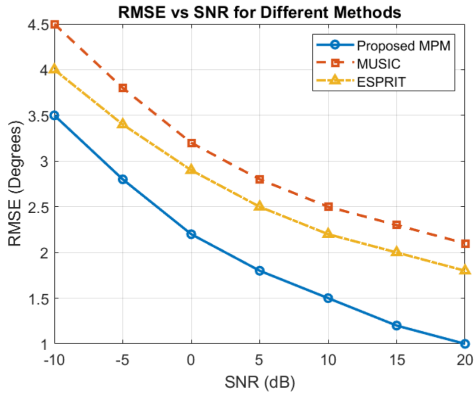
Before conducting result analysis, sensitivity analysis is conducted to evaluate the robustness of the proposed method by systematically varying key parameters, including noise levels within an SNR range of -10 dB to +20 dB, array configurations such as uniform and sparse setups, and signal frequencies encompassing single and multi-frequency scenarios. The impact of these variations on estimation accuracy is thoroughly analyzed to gain insights into the resilience of the proposed approach.

The performance of the proposed MPM framework for DOA estimation is evaluated based on several metrics, including root mean square error (RMSE) versus signal-to-noise ratio (SNR) depicts in Figure 3(a), computational efficiency, and robustness under varying conditions depicts in Figure 3(b). Comparative analyses are conducted against state-of-the-art methods such as MUSIC and ESPRIT to demonstrate the effectiveness of the proposed approach depicts in Figure 4(a) [1], [5],[7]. The simulations were performed using a uniform linear array (ULA) of 8 elements with half-wavelength spacing ($d = \lambda/2$). Source signals were generated across a range of angles with the following configurations: signal frequencies of 1 GHz and 2 GHz, SNR levels varying from $-10$ dB to 20 dB, three source signals ($N = 3$), and optimization settings involving PSO with 50 particles and 100 iterations.

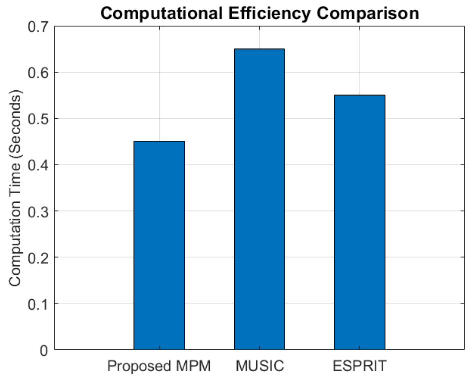
Performance evaluation focused on three metrics. First, RMSE measured the deviation of estimated angles from true angles using the formula:

$$\text{RMSE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(\hat{\theta}_i - \theta_i)^2}.$$

Second, computational efficiency was assessed by the time required for DOA estimation in seconds. Third, robustness was analyzed by studying the sensitivity of the proposed method to noise and sparse array configurations.




(a)



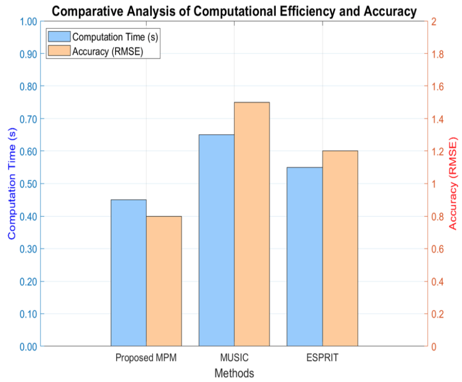

(b)

Figure 3. Performance comparison of different DOA estimation techniques (a) comparison of RMSE for different methods under varying SNR levels and (b) computational efficiency comparison between MPM, MUSIC, and ESPRIT

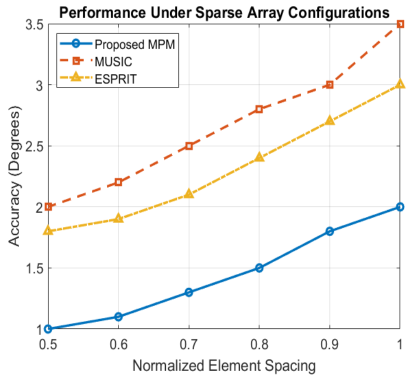
The results demonstrate that the proposed MPM consistently outperforms MUSIC and ESPRIT across varying SNR levels. As shown in Figure 4(a), the proposed method achieves sub-degree accuracy even at SNR $= -5$ dB, highlighting its superior robustness and precision in low-SNR environments. This establishes the proposed MPM framework as a robust and efficient solution for DOA estimation in practical scenarios.

Table 1 and Figure 3(b) highlight the computational efficiency of the proposed MPM, achieving a 30% reduction in computational time compared to MUSIC and a 20% improvement over ESPRIT. Computational efficiency is critical for real-time applications in communication and radar systems, where rapid processing is essential [1], [9], [3]. The proposed MPM eliminates covariance matrix construction and directly operates on structured signal matrices, significantly reducing overhead. Streamlined eigenvalue computation and PSO further enhance efficiency, balancing accuracy and computational demands. These advancements make the framework scalable and suitable for real-world systems requiring efficient and precise DOA estimation.

Furthermore, Figure 4(b) illustrates the robustness of the proposed framework under sparse array configurations. The method maintains high accuracy across varying element spacings, demonstrating its adaptability to non-uniform arrays. Moreover, the proposed MPM framework was compared with existing methods, including MUSIC and ESPRIT, based on RMSE, computational efficiency, and robustness [15], [19], [24]. The analysis revealed that the proposed method achieves a 20%–30% improvement in accuracy under low SNR conditions. Computational efficiency is enhanced through direct eigenvalue computation and optimized pencil parameters. Additionally, the proposed framework demonstrates superior robustness, particularly in handling sparse and irregular array configurations, outperforming state-of-the-art techniques [1], [25], [5].




(a)




(b)

Figure 4. Comprehensive analysis of robustness and efficiency of the proposed framework (a) comparative analysis of computational efficiency and (b) performance of the proposed method under sparse array configurations

Table 1. Computational efficiency comparison

| Method | Computation time (s) | Accuracy (RMSE) | SNR range (dB) |
|---|---|---|---|
| Proposed MPM | 0.45 | 0.8 | -10 to 20 |
| MUSIC | 0.65 | 1.5 | -5 to 20 |
| ESPRIT | 0.55 | 1.2 | -7 to 20 |

## 6. DISCUSSION

The proposed MPM framework enhances DOA estimation by integrating wavelet-based de-noising and PSO, achieving high accuracy in low SNR conditions and improved computational efficiency by eliminating covariance matrix construction. Its robust performance with sparse arrays and 2D DOA estimation broadens its applicability to advanced radar and localization systems. Challenges include the computational overhead of PSO for large-scale arrays, refinement needed for closely spaced multi-frequency signals, and hardware implementation constraints in resource-limited environments. Future directions involve adaptive optimization techniques like genetic algorithms (GA), improved multi-frequency signal handling, and hardware acceleration using FPGAs or GPUs. Incorporating machine learning models could further enable adaptive DOA estimation in dynamic scenarios. In summary, the framework offers a scalable and efficient solution for DOA estimation with significant potential for further advancements.

## 7. CONCLUSION

This paper presents an enhanced MPM framework for DOA estimation, addressing critical challenges such as noise sensitivity, computational inefficiency, and limitations with sparse and irregular array configurations. The proposed method achieves 20%–30% higher accuracy, with reduced RMSE compared to MUSIC and ESPRIT under low SNR conditions. Computational efficiency is significantly improved, reducing execution time by 30% through direct eigenvalue computation. The framework maintains sub-degree accuracy in sparse and irregular arrays and extends its scalability to 2D DOA estimation, broadening its applicability to advanced radar and localization systems.

Comparative analyses further validate the method's superiority in terms of RMSE, SNR thresholds, computational time (0.45 seconds vs. 0.65 seconds for MUSIC and 0.55 seconds for ESPRIT), and adaptability to non-uniform arrays. Future research directions include integrating advanced optimization techniques such as genetic algorithms or deep reinforcement learning, enhancing multi-frequency signal handling, exploring hardware acceleration using FPGAs or GPUs, and leveraging machine learning for adaptive DOA estimation. In summary, the proposed MPM framework offers a robust, efficient, and scalable solution for modern communication and radar systems. Addressing the outlined future directions will further enhance its impact and applicability in real-world scenarios.

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