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Computational modelling under uncertainty: statistical mean approach to optimize fuzzy multi-objective linear programming problem with trapezoidal numbers

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ABSTRACT

This study presents a comprehensive approach to solving fuzzy multi-objective linear programming problems (FMOLPP) under uncertainty using trapezoidal fuzzy numbers. The authors propose a novel integration of Yager's ranking method, the Big-M optimization technique, and Chandra Sen's statistical mean methods to effectively convert fuzzy objectives into crisp values and optimize them. The methodology allows for managing multiple fuzzy objectives by ranking and aggregating them using various statistical means such as arithmetic, geometric, quadratic, harmonic, and Heronian averages. The model is implemented using TORA software and demonstrated through a detailed numerical example. The results validate the robustness and practicality of the proposed approach, showcasing consistent optimal solutions across all statistical methods. This research significantly enhances decision-making processes in uncertain environments by offering a structured, computationally efficient solution strategy for complex real-world optimization problems.

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1. INTRODUCTION

In real-life scenarios, outcomes are not always predictable due to various factors, and we especially observe that this issue has different types of areas *i.e.* industry-related issues (production and supply chain model), our social issues, and many types of economic systems. These undefined problems can be classified into two main categories: the first one is identified as stochastic uncertainty and the second one is fuzziness [1]. Fuzzy optimization techniques are used to handle unpredictable systems with the help of fuzzy numbers and parameters because fuzzy numbers easily handle the condition of vagueness. The fuzzy linear programming method is one of the most important and usable techniques from methods of fuzzy decision-making because this model easily handles many types of industry-related issues. Many authors developed various types of methods to solve fuzzy linear programming models in the past years but the First literature is presented by Bellman and Zadeh for decision-making to handle vagueness conditions with the help of fuzzy numbers in [2]. Behera and Nayak [3] solve the related problem by using Zimmermann's method. Das *et al.* [4] introduced the new algorithm with a grouping Charnes-Cooper scheme to obtain the optimal solution of fully fuzzy linear programming problems for real-life problems. Nahar *et al.* [5] are converting multi-objective modal into a single objective by using two types of function namely the ranking function and the weighted function to handle both trapezoidal and triangular fuzzy numbers. Sen [6] introduced an intermediate method for planning for the development of rural areas with multi-objective optimization models. Ackert get literature [7] projected a method namely fuzzy evaluation for natural disaster evaluation.

The intriguing idea of the Spherical fuzzy linear programming problem was introduced by Ahmad and Adhami [8]. It may be divided into three categories and transformed into clear problems by applying a ranking function. With the use of t-norm and t-conorm from Archimedean fuzzy sets, Ashraf and Abdullah [9] opened up a new area of study for fuzzy sets: spherical fuzzy sets. Expanding on this work, Ashraf *et al.* [10] introduced spherical fuzzy t-norms and t-conorms to further enhance the concept. Ashraf *et al.* [11] developed several aggregation operators for spherical fuzzy dombi (SD) averaging, and ordered averaging. hybrid averaging, geometric, and hybrid geometric averaging to efficiently tackle decision-making problems. These operators in the context of fuzzy sets are revolutionary. As part of an automated storage and retrieval systems technology selection challenge.

Garg *et al.* [12] developed and enhanced immersive aggregation procedures for T-spherical fuzzy sets in multi-attribute decision-making. Giuleria and Bajaj [13] introduced the innovative concept of T-spherical fuzzy graphs, including their arithmetic operations, and applied them to business logistics management decision-making and service resto assessment problems Furthering their efforts. Kutlu Gundogdu and Kahraman [14] pioneered the MULTIMOORA methodology to solve personnel selection problems. To make decision-making even more accessible. This extension allowed for more precise and nuanced decision-making. Gündoğdu and Kahraman [15] pushed the limits of the VIKOR method by introducing the spherical fuzzy VIKOR (SF-VIKOR) approach and successfully applying it to select a warehouse placement, demonstrating its superior performance

Jin *et al.* [16], [17] introduced spherical fuzzy entropy to identify unknown criterion weight information and proposed new logarithmic operations on spherical fuzzy sets. Liu *et al.* [18], [19] introduced the Lt-SFNs operator, which evaluates language value understanding among the public. They then developed the Lt-SF weighted averaging operator, integrating language evaluation knowledge. Building on these concepts, the authors enhanced the TODIM approach and established an MABAC methodology based on L1-SFNa, a generalization of picture fuzzy sets. Ullah *et al.* [20], [21] proposed novel similarity metrics, such as cosine similarity measurements, grey similarity measures, and set-theoretical similarity measures applied to a construction material identification problem in the context of spherical fuzzy sets and T-spherical fuzzy sets. Zeng *et al.* [22] devised a novel approach for hybrid spherical fuzzy sets using rough set concepts by implementing a covering-based spherical fuzzy rough set (CSFRS) model within the TOPSIS framework. Zheug *et al.* [23] proposed an analysis for optimizing the ceramic fibers using the differential method. The multi Goal Fuzzy problems were discussed using elementary Transformation by Shrivastava [24]. Profit maximization in the small mechanical industry due to the application of Linear Programming was explored by Jain *et al.* [25].

The primary objective of the research is to address the challenges in optimizing multi-objective linear problems when the data is not deterministic, a common scenario in industrial, economic, and engineering applications. Existing approaches either lack effective defuzzification or oversimplify fuzzy parameters, leading to inaccurate or suboptimal solutions. The following gap was found and it has been addressed in this work: i) Lack of robust defuzzification methods that capture the nuance of trapezoidal fuzzy parameters; ii) Inadequate integration of statistical tools in the change multiple objectives into the single objective in optimization process; and iii) Absence of a unified framework that combines uncertainty modeling, defuzzification, and multi-objective optimization.

This study considers the statistical mean approaches to handle the fuzzy multi-objective linear programming problem. The core methodology involves as follows:

a. Fuzzy representation: Objective and constraints of the linear programming problem are prepared by using Trapezoidal fuzzy numbers because of its recognized format that more successfully represents the vagueness and imprecision present in the real-world data compared to crisp or triangular fuzzy numbers.
b. Statistical mean approach: The novelty of this approach lies in using statistical mean to change the multi-objective functions into the single objective function in place of weighted sum method.
c. Optimization technique: The classical Big-M method is employed with the help of Tora Software to solve this transformed problem and obtained an optimal solution.

2. METHOD

This research distinguishes itself through several innovative contributions, the first integration of the statistical mean approach with Yager's ranking function in the context of FMOLPPs involving trapezoidal fuzzy numbers. Theoretical refinement of how fuzzy data is translated into crisp values-maintaining the informational integrity of fuzzy parameters throughout the transformation. A unified computational framework that systematically handles uncertainty, fuzzification, and multi-objective optimization, which can be generalized across various domains. The methodology allows for more realistic modeling of uncertainties that arise in economic planning, production scheduling, and logistics, where vague human judgments often shape decision variables.

2.1. Fuzzy set

Let X be a non-empty set. A fuzzy set $\tilde{A}$ in X is characterized by its membership function $\mu_{\tilde{A}}:X \rightarrow [0,1]$ and $\mu_{\tilde{A}}(x)$ is interpreted as the degree of membership of element x in fuzzy set A for each $x \in X$.

2.2. Multi-objective fuzzy linear programming problem

If all the parameters of a linear programming problem (LPP) are presented in terms of vagueness *i.e.* fuzzy numbers then our LP problem is identified as a fuzzy linear programming problem (FLPP) and if FLP problem consists of more than one objective for a particular modal then it is modal namely know as a multi-objective fuzzy linear programming problem. In our study, fuzzy numbers are assigned as $\tilde{c}_r, \tilde{a}_{ij}$. Here we consider MOFLPP as

$$Max\ or\ Min\ P_m = \sum_{r=1}^{n} \tilde{c}_r x_r \qquad \forall\, m \in N$$

Subject to

$$\sum_{r=1}^{n} \tilde{a}_{ir} x_i \leq \tilde{b}_i$$
$$1 \leq i \leq m \ \exists\ x_i > 0$$

2.3. Fuzzy trapezoidal number

Let m, n, α, β are real numbers then if these numbers can be arranged in the following manner

$$\tilde{A} = (m, n, \alpha, \beta)$$

Then $\tilde{A} = (m, n, \alpha, \beta)$ this fuzzy number is known as fuzzy trapezoidal numbers if its membership function can be represented by the function

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-(m-n)}{\alpha} & m-\alpha \leq x \leq m \\ 1 & m \leq x \leq n \\ \frac{(n+\beta)-x}{\beta} & n \leq x \leq n+\beta \\ 0 & else \end{cases}$$

Now here we present the arithmetic operation for fuzzy trapezoidal numbers as follows. Let $\tilde{A}(m_1, n_1, \alpha_1, \beta_1)$ and $\tilde{B} = (m_2, n_2, \alpha_2, \beta_2)$ are representing two fuzzy trapezoidal numbers define

$$x > 0, x \in R: x\tilde{A} = (xm_1, xn_1, x\alpha_1, x\beta_1)$$
$$x < 0, x \in R: x\tilde{A} = (xm_1, xn_1, -x\alpha_1, -x\beta_1)$$
$$\tilde{A} + \tilde{B} = (m_1 + m_2, n_1 + n_2, \alpha_1 + \alpha_2, \beta_1 + \beta_2)$$

2.4. Ranking function

Let $\mathcal{R}$ is a function which math every fuzzy number $G(\mathcal{R})$ in the real line *i.e.* $\mathcal{R}: G(\mathcal{R}) \rightarrow \mathcal{R}$. Now here we presenting the order on $G(\mathcal{R})$ as follows

$$\tilde{A} \leq \tilde{B} \ \Leftrightarrow \mathcal{R}(\tilde{A}) \leq \mathcal{R}(\tilde{B}), \tilde{A} \geq \tilde{B} \ \Leftrightarrow \mathcal{R}(\tilde{A}) > \mathcal{R}(\tilde{B})$$
$$\tilde{A} \simeq \tilde{B} \ \Leftrightarrow \mathcal{R}(\tilde{A}) = \mathcal{R}(\tilde{B}), \tilde{A} \leq \tilde{B} \ \Leftrightarrow \tilde{B} \geq \tilde{A}$$

Where $\tilde{A}$ and $\tilde{B}$ belong in $G(\mathcal{R})$. Here we specially focused on only about linear ranking function *i.e.* a ranking function $\mathcal{R}$ define such that

$$\mathcal{R}(K\tilde{A} + \tilde{B}) = K\mathcal{R}(\tilde{A}) + \mathcal{R}(\tilde{B})\ \forall\, \tilde{A}, \tilde{B}\ \in G(\mathcal{R})$$

Now taking the $G(R)$ as linear ranking function as follows:

$$\mathcal{R}(\tilde{A}) = \frac{1}{2}\int_0^1 \left(inf\tilde{A}_\lambda + \sup \tilde{A}_\lambda\right)d\lambda$$

Which reduce to

$$\mathcal{R}(\tilde{A}) = \frac{1}{2}(m+n) + \frac{1}{4}(\beta - \alpha)$$

Then for fuzzy trapezoidal number $\tilde{A} = (m_1, n_1, \alpha_1, \beta_1)$ and $\tilde{B} = (m_2, n_2, \alpha_2, \beta_2)$
We have $\tilde{A} \geq \tilde{B} \iff \frac{1}{2}(m_1 + n_1) + \frac{1}{4}(\beta_1 - \alpha_1) \geq \frac{1}{2}(m_2 + n_2) + \frac{1}{4}(\beta_2 - \alpha_2)$

2.5. Arithmetic operation

In this sub-section, we are going to present the operation procedures for the addition and multiplication of two fuzzy trapezoidal numbers.
Let $\tilde{A} = (m_1, n_1, \alpha_1, \beta_1)$ and $\tilde{B} = (m_2, n_2, \alpha_2, \beta_2)$ be two trapezoidal fuzzy numbers then

$$\tilde{A} + \tilde{B} = (m_1, n_1, \alpha_1, \beta_1) + (m_2, n_2, \alpha_2, \beta_2) = (m_1 + m_2, n_1 + n_2, \alpha_1 + \alpha_2, \beta_1 + \beta_2)$$
$$\tilde{A} = -(m_1, n_1, \alpha_1, \beta_1) = \left(-n_1, -m_1, \beta_{1,}\alpha_1\right)$$

If $\tilde{A} \geq 0$ and $\tilde{B} \geq 0$ then, $\tilde{A} \times \tilde{B} = (m_1, n_1, \alpha_1, \beta_1) + (m_2, n_2, \alpha_2, \beta_2) = (m_1m_2, n_1n_2, \alpha_1\alpha_2, \beta_1\beta_2)$

2.6. Chandra Sen's method

The steps involved in the algorithm [6] are:

– Apply the Big M Method and determine the optimum solution for every objective function.
– Let Max $P_n = \chi_k$, where k=1, 2, 3, …, g, $P_n = \chi_k$, where $K = g+1, g+2 \ldots h$.
– Calculate B_1 and B_2, where B_1=max ($|\chi_k|$, where k=1, 2, 3, …, g, B_2=min $|\chi_k|$, where K=g+1, g+2, …, h.
– Calculate the value of the mean by different mean methods.

a. Arithmetic mean method

Arithmetic mean $Max\ P = \frac{(\sum_{n=1}^{g} P_n - \sum_{n=g+1}^{h} P_n)}{A.M.}$, Arithmetic mean by average $A.M. = \frac{B_1+B_2}{2}$

b. Quadratic mean method

Quadratic mean $Max\ P = \frac{(\sum_{n=1}^{g} P_n - \sum_{n=g+1}^{h} P_n)}{Q.M.}$, Quadratic mean by average $QM = \sqrt{\frac{(B_1^2+B_2^2)}{2}}$

c. Geometric mean method

Geometric mean $Max\ P = \frac{(\sum_{n=1}^{g} P_n - \sum_{n=g+1}^{h} P_n)}{G.M.}$, Geometric mean by average $GM = \sqrt{B_{1\times}B_2}$

d. Harmonic mean method

Harmonic mean $Max\ P = \frac{(\sum_{n=1}^{g} P_n - \sum_{n=g+1}^{h} P_n)}{H.M.}$, Harmonic mean by average $HM = \frac{2}{\frac{1}{B_1}+\frac{1}{B_2}}$

e. Heronian mean method

Heronian mean $Max\ P = \frac{(\sum_{n=1}^{g} P_n - \sum_{n=g+1}^{h} P_n)}{H_e.M.}$,
Heronian mean by average $H_e.M = \frac{1}{3}\left(B_1 + \sqrt{B_{1\times}B_2} + B_2\right)$

3. RESULTS AND DISCUSSION

In this study, we consider a multi-objective fuzzy linear programming problem with four objectives and six constraints with trapezoidal fuzzy numbers. Our problems are as follows:

Multiple objective

$$Max\ P1 = (3,8,11,13)x_1 + (3,6,8,10)x_2 + (3,8,11,13)x_3 + (3,4,5,7)x_4$$
$$Max\ P2 = (8,9,12,14)x_1 + (7,8,10,12)x_2 + (3,8,11,13)x_3 + (3,4,5,7)x_4$$
$$Min\ P3 = (4,8,12,16)x_1 + (9,13,17,21)x_2 + (7,8,10,12)x_3 + (8,9,12,14)x_4$$
$$Min\ P4 = (9,11,12,28)x_1 + (13,15,16,32)x_2 + (10,12,13,27)x_3 + (7,9,10,26)x_4$$

Subject to constraint

$$(3,5,6,22)x_1 + (5,7,8,24)x_2 + (4,6,7,23)x_3 + (6,8,9,25)x_4 \le 271.75$$
$$(5,7,8,24)x_1 + (6,8,9,25)x_2 + (7,9,10,26)x_3 + (10,12,13,27)x_4 \le 411.75$$
$$(8,10,11,27)x_1 + (8,10,11,27)x_2 + (8,10,11,27)x_3 + (12,14,15,31)x_4 \le 573.75$$
$$(6,8,9,25)x_1 + (9,11,12,28)x_2 + (9,11,12,28)x_3 + (8,10,11,27)x_4 \le 385.5$$
$$(9,11,12,28)x_1 + (13,15,16,32)x_2 + (12,14,15,31)x_3 + (9,11,12,28)x_4 \le 539.5$$
$$(11,13,14,30)x_1 + (14,16,17,33)x_2 + (15,17,18,34)x_3 + (13,15,16,32)x_4 \le 759.5$$
$$x_1, x_2, x_3, x_4 \ge 0$$

To solve the objective function the ranking function of the trapezoidal no.

Let $\tilde{A} = (m_1, n_1, \alpha_1, \beta_1)$ and $\tilde{B} = (m_2, n_2, \alpha_2, \beta_2)$.

Now the ranking of the Trapezoidal no. is

$$R(\tilde{A}) = \frac{1}{2}(m+n) + \frac{1}{4}(\beta - \alpha),$$

Then $R\ (8, 9, 12, 14) = \frac{1}{2}(8+9) + \frac{1}{4}(14-12) = 6.$

Now fully fuzzy linear programming problem reduces in this form
Objective functions:

$$Max\ P1 = 6x_1 + 5x_2 + 6x_3 + 4x_4$$
$$Max\ P2 = 9x_1 + 8x_2 + 6x_3 + 4x_4$$
$$Min\ P3 = 7x_1 + 12x_2 + 8x_3 + 9x_4$$
$$Min\ P4 = 14x_1 + 17x_2 + 15x_3 + 12x_4$$

Subject to:

$$8x_1 + 10x_2 + 9x_3 + 11x_4 \le 271.75$$
$$10x_1 + 11x_2 + 12x_3 + 15x_4 \le 411.75$$
$$13x_1 + 13x_2 + 13x_3 + 17x_4 \le 573.75$$
$$11x_1 + 14x_2 + 14x_3 + 13x_4 = 385.5$$
$$14x_1 + 18x_2 + 17x_3 + 14x_4 = 539.5$$
$$16x_1 + 19x_2 + 20x_3 + 18x_4 = 759.5$$
$$x_1, x_2, x_3, x_4 \ge 0$$

a. First fuzzy objective function $Max\ P_1 = 6x_1 + 5x_2 + 6x_3 + 4x_4$
Subject to:

$$8x_1 + 10x_2 + 9x_3 + 11x_4 \le 271.75$$
$$10x_1 + 11x_2 + 12x_3 + 15x_4 \le 411.75$$
$$13x_1 + 13x_2 + 13x_3 + 17x_4 \le 573.75$$
$$11x_1 + 14x_2 + 14x_3 + 13x_4 = 385.5$$
$$14x_1 + 18x_2 + 17x_3 + 14x_4 = 539.5$$
$$16x_1 + 19x_2 + 20x_3 + 18x_4 = 759.5$$
$$x_1, x_2, x_3, x_4 \ge 0$$

The optimized value is $Max\ P_1 = -25426.7$.

b. Second fuzzy objective function $Max\ P_2 = 9x_1 + 8x_2 + 6x_3 + 4x_4$

Subject to:

$8x_1 + 10x_2 + 9x_3 + 11x_4 \leq 271.75$
$10x_1 + 11x_2 + 12x_3 + 15x_4 \leq 411.75$
$13x_1 + 13x_2 + 13x_3 + 17x_4 \leq 573.75$
$11x_1 + 14x_2 + 14x_3 + 13x_4 = 385.5$
$14x_1 + 18x_2 + 17x_3 + 14x_4 = 539.5$
$16x_1 + 19x_2 + 20x_3 + 18x_4 = 759.5$
$x_1, x_2, x_3, x_4 \geq 0$
The optimized value is $Max\ P_2 = -25349.4$

c. Third fuzzy objective function $Min\ P_3 = 7x_1 + 12x_2 + 8x_3 + 9x_4$
Subject to:

$8x_1 + 10x_2 + 9x_3 + 11x_4 \leq 271.75$
$10x_1 + 11x_2 + 12x_3 + 15x_4 \leq 411.75$
$13x_1 + 13x_2 + 13x_3 + 17x_4 \leq 573.75$
$11x_1 + 14x_2 + 14x_3 + 13x_4 = 385.5$
$14x_1 + 18x_2 + 17x_3 + 14x_4 = 539.5$
$16x_1 + 19x_2 + 20x_3 + 18x_4 = 759.5$
$x_1, x_2, x_3, x_4 \geq 0$
The optimized value is $Min\ P_3 = 25863.7$

d. Fourth fuzzy objective function $Min\ P_4 = 14x_1 + 17x_2 + 15x_3 + 12x_4$
Subject to:

$8x_1 + 10x_2 + 9x_3 + 11x_4 \leq 271.75$
$10x_1 + 11x_2 + 12x_3 + 15x_4 \leq 411.75$
$13x_1 + 13x_2 + 13x_3 + 17x_4 \leq 573.75$
$11x_1 + 14x_2 + 14x_3 + 13x_4 = 385.5$
$14x_1 + 18x_2 + 17x_3 + 14x_4 = 539.5$
$16x_1 + 19x_2 + 20x_3 + 18x_4 = 759.5$
$x_1, x_2, x_3, x_4 \geq 0$
The optimized value is $Min\ P_4 = 26095.1$

Table 1 shows the initial table:

Table 1. Initial table

Objectives	χ_k	$\lvert\chi_k\rvert$	Value of B_1 and B_2
1	-25426.7	25426.7	B_1= 25426.7
2	-25349.4	25349.4	
3	25863.7	25863.7	B_2=25863.7
4	26095.1	26095.1	

- Arithmetic Mean $AM = \frac{B_1+B_2}{2} = \frac{25426.7+25863.7}{2} = 25645.2$
- Quadratic Mean $QM = \sqrt{\frac{(B_1^2+B_2^2)}{2}} = \sqrt{\frac{(25426.7^2+25863.7^2)}{2}} = 25646.1$
- Geometric Mean $GM = \sqrt{B_{1\times}B_2} = \sqrt{25426.7 \times 25863.7} = 25644.3$
- Harmonic Mean $HM = \frac{2}{\frac{1}{B_1}+\frac{1}{B_2}} = \frac{2}{\frac{1}{25426.7}+\frac{1}{25863.7}} = 25643.4$
- Heronian Mean $HeM = \frac{1}{3}\left(B_1 + \sqrt{B_{1\times}B_2} + B_2\right) = 25644.9$
- Mean Deviation $Max\ P = (P_1 + P_2) - (P_3 + P_4)$

$$Max\ P = \{(6x_1 + 5x_2 + 6x_3 + 4x_4) + (9x_1 + 8x_2 + 6x_3 + 4x_4)\} \\ -\{(7x_1 + 12x_2 + 8x_3 + 9x_4) + (14x_1 + 17x_2 + 15x_3 + 12x_4)\}$$

Now the objective function is converting in this form.
Max P by $Arithmetic\ mean = arithmetic\ mean$

$$Max\ P = \frac{(\sum_{n=1}^{g} P_n - \sum_{n=g+1}^{h} P_n)}{A.M.}$$
$$Max\ P = (-6x_1 - 16x_2 - 11x_3 - 13x_4)/25645.2$$
$$Max\ P = (-0.000234x_1 - 0.000624x_2 - 0.000429x_3 - 0.000507x_4)$$

Objective function: $Max\ P = (-0.000234x_1 - 0.000624x_2 - 0.000429x_3 - 0.000507x_4)$
Subject to:

$$8x_1 + 10x_2 + 9x_3 + 11x_4 \leq 271.75$$
$$10x_1 + 11x_2 + 12x_3 + 15x_4 \leq 411.75$$
$$13x_1 + 13x_2 + 13x_3 + 17x_4 \leq 573.75$$
$$11x_1 + 14x_2 + 14x_3 + 13x_4 = 385.5$$
$$14x_1 + 18x_2 + 17x_3 + 14x_4 = 539.5$$
$$16x_1 + 19x_2 + 20x_3 + 18x_4 = 759.5$$
$$x_1, x_2, x_3, x_4 \geq 0$$

Max P by $Arithmetic\ Mean = -25625.01$.
Similarly, Quadratic Mean: $Max\ P = \frac{(\sum_{n=1}^{g} P_n - \sum_{n=g+1}^{h} P_n)}{Q.M.}$
– Max P by $Quadratic\ Mean = (-6x_1 - 16x_2 - 11x_3 - 13x_4)/25646.1$

$$Q.M. = (-0.000234x_1 - 0.000624x_2 - 0.000429x_3 - 0.000507x_4)$$

Geometric Mean: $Max\ P = \frac{(\sum_{n=1}^{g} P_n - \sum_{n=g+1}^{h} P_n)}{G.M.}$
– Max P by $Geometric\ Mean = (-6x_1 - 16x_2 - 11x_3 - 13x_4)/25644.3$

$$G.M = (-0.000234x_1 - 0.000624x_2 - 0.000429x_3 - 0.000507x_4)$$

Harmonic Mean: $Max\ P = \frac{(\sum_{n=1}^{g} P_n - \sum_{n=g+1}^{h} P_n)}{H.M.}$
– Max P by $Harmonic\ Mean = \frac{-6x_1 - 16x_2 - 11x_3 - 13x_4}{25643}$

$$H.M. = (-0.000234x_1 - 0.000624x_2 - 0.000429x_3 - 0.000507x_4)$$

Heronian Mean: $Max\ P = \frac{(\sum_{n=1}^{g} P_n - \sum_{n=g+1}^{h} P_n)}{H_e.M.}$
– Max P by $Heronian\ Mean = (-6x_1 - 16x_2 - 11x_3 - 13x_4)/\ 25644.9$

$$H_e.M = (-0.000234x_1 - 0.000624x_2 - 0.000429x_3 - 0.000507x_4)$$

With same constraints.
Optimized value of FFMOLPP by mean approaches is
– Max P by $Arithmetic\ Mean = -25625.01$, Max P by $Quadratic\ Mean = -25625.01$
– Max P by $Geometric\ Mean = -25625.01$, Max P by $Harmonic\ Mean = -25625.01$
– Max P by $Heronian\ Mean = -25625.01$

4. CONCLUSION

Fuzzy multi-objective linear programming problems with trapezoidal numbers present remarkable improvement in optimizing results when we are using the proposed statistical mean approach methods. Here we found that from obtained results are the same form all the applicable conditions, with this observation ability of decision-making has been enhanced, especially in ambiguous conditions.

The proposed framework demonstrates consistent optimal results across different statistical measures, indicating its robustness and reliability in handling vagueness and imprecision inherent in real-world decision-making scenarios. The use of TORA software for implementation adds to the practicality of the approach, making it suitable for a wide range of applications in operations research, engineering, and management science. Overall, this study contributes a structured and computationally viable solution to FMOLPPs and enhances decision-making capabilities in uncertain environments.

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