International Journal of Electrical and Computer Engineering (IJECE)
Vol. 15, No. 5, October 2025, pp. 4642~4652
ISSN: 2088-8708, DOI: 10.11591/ijece.v15i5.pp4642-4652

On design of a small-sized arrays for direction-of-arrival-estimation taking into account antenna gains

Ilia Peshkov, Natalia Fortunova, Irina Zaitseva
Department of Physics, Radio engineering and Electronics, Bunin Yelets State University, Yelets, Russia

Article Info

Article history:

Received Aug 24, 2024
Revised Apr 14, 2025
Accepted Jul 3, 2025

Keywords:

Cramer-Rao lower bound
Direction-of-arrival estimation
Hybrid antenna array
Smart antenna
Super-resolution

ABSTRACT

In the paper a technique for designing antenna arrays composed of directional elements for direction-of-arrival (DOA) estimation is proposed. Especially this approach is applied for developing hybrid antenna arrays with increased accuracy which features digital spatial spectral estimation after preliminary analog beamforming. The earlier obtained explicit formula for calculating the Cramér–Rao lower bound (CRLB) which determines the relationship between the variance of the DOA-estimation and antenna elements' radiation patterns, array geometry, has been used. Main idea of the proposed technique is that it takes into account spatial pattern and gain of the antenna elements. The high gain unlike the number of the antenna elements or interelement distance is the most important factor which allows reducing the value of the DOA-estimation errors. A couple of the examples of calculating radiation patterns of antenna elements improving accuracy of DOA-estimation with super-resolution are provided in the paper. Proposed antenna arrays are modeled according to the method of moments (MoM). The values of the root mean square error after the DOA-estimation are obtained. It is shown that the resulting hybrid systems can reduce the error value in DOA-estimation with super-resolution.

Corresponding Author:

Ilia Peshkov
Department of Physics, Radio engineering and Electronics, Institute of Mathematics, Nature Science and Technique, Bunin Yelets State University
39 Kommunarov street, Yelets 399770, Russia
Email: ilvpeshkov@gmail.com

1. INTRODUCTION

The state-of-art technologies actively use multiantenna wireless systems. The analysis of the spatial spectrum of signals underlies these devices [1]. DOA-estimation is critical in applications requiring spatial awareness, minimal delay and exceptional reliability. For example, in 5G direction-of-arrival estimation contributes in beamforming by identifying user directions for optimal signal transmission [2]. Detecting angles of incoming signals from obstacles/vehicles is utilized in automotive radar (self-driving cars) [3].

Low accuracy and resolution are the most serious problems of these systems using methods and algorithms for estimating spatial coordinates, which are limited by array aperture [4]. Thus, the primary goal of the paper is how to improve the accuracy of DOA-estimation algorithms. However, the accuracy of the estimates cannot be increased in a easy way and has a lot of challenges. Today, there are several basic approaches regarding the accuracy of DOA-estimation.

The most common way the researchers apply to improve accuracy is to enlarge the aperture with additional elements [5]. The attitude is widespread in the millimeter range applications or military radars [6]. Nevertheless, the complexity of the multi-antenna system architecture increases which results in a dramatic escalation of overall device costs [7].

Another way improving the accuracy which has gained particular popularity is related to the design of an innovative DOA-estimation algorithm. Usually, It is tailored to particular use-case requirements (for instance, a multipath propagation channel), a certain geometry of antenna array (such as linear antenna array and estimation of signal parameters via rotational invariance technique (ESPRIT) [8]), or for a signal waveform (so-called blind methods) [9]. The main disadvantage of this approach is that new algorithms possess high computational complexity. For example, the well-known MUSIC algorithm has $O(N^2)$, at the same time ESPRIT and others $O(N^3)$ depending on the number of antennas [10]. In other words, the load increases a lot. This fact makes it difficult to implement for real-time applications.

A similarly widespread technique increasing accuracy is based on the optimal arrangement of antenna elements [11]. Several different criteria have been introduced such as random distribution of antenna coordinates [12] or complex coordinate expression of antennas [13], or using arrays composed of omnidirectional radiators [14], [15]. The main disadvantage of the mentioned approaches is the consideration of ideal array patterns because omnidirectional antennas are absent in real applications. In other words, they are hardly implemented.

The main idea of the proposed methodology for enhancing DOA estimation precision is the optimal placement of directional antennas, as well as their spatial patterns. The novelty of the proposed approach arises because that the directional spatial patterns of antenna elements are taken into account. So that the procedure for designing a dual-element array configuration for estimating spatial parameter estimation by means of manually forming the beam patterns and coordinates of the array elements to greatly reduce the variance. In this way, the proposed approach highlights the gain of antenna elements enabling the sensor array to be developed for 360° range scanning by means of synthesizing the radiation patterns.

Microstrip antennas can be designed using the method of moments by demonstrating the practical implementation of the presented method. Furthermore, the paper describes an example of creating an antenna array prototype for super-resolution DOA-estimation, which is computed using the closed-form analytical formulations of Cramér–Rao lower bound (CRLB) comparing to the optimization or neural networks which are black box solutions. The elaborated array consists of an analog beamforming scheme, which allows reducing the number of digital processing channels without decreasing the accuracy of DOA-estimation [16]. Thus, the paper discusses the step-by-step process of antenna array design, starting from a concept based on analytical expressions and ending with the practical implementation on the basis of patch antennas. A completed device is used to demonstrate the feasibility of the proposed approach. Especially, there is no unified methodology for designing that kind of devices which are built on sub-array preprocessing networks [17] or beam selection schemes [18].

2. SYSTEM MODEL

2.1. DOA-estimation formulation

In this section the model of antenna systems widespread in direction finding is described. The arrays can be configured in different ways influencing on such properties as accuracy and resolution. Figure 1 shows such an array, which consists of a certain number of antenna elements.

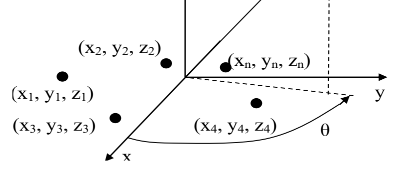


Figure 1. Antenna array view in the Cartesian system

In the paper it is assumed that the signal is narrow-band and has the following spatial coordinates on azimuth θ and elevation φ relative to the *x*, *y*, and *z* axes in the Cartesian system, respectively. Thus, the coordinates θ and φ has to be estimated with maximum accuracy. The analytical model of the antenna array is expressed in the following manner [19]:

$$a(\theta, \varphi) = \left[g_1(\theta, \varphi)e^{jkr_1^T} \quad \cdots \quad g_N(\theta, \varphi)e^{jkr_N^T}\right] \tag{1}$$

where $k = \frac{2\pi}{\lambda}(k_x, k_y, k_z) = (\sin\varphi\cos\theta, \sin\varphi \sin\theta, \cos\varphi)$ is the spatial frequency specifying the oscillating of the phase of the propagating wave in the x, y, z directions, $r_n^T = (x_n, y_n, z_n)^T$ is the position vector indicated to the n^{th} antenna element and $g_n(\theta, \varphi)$ is the pattern of the n^{th} antenna element.

The antenna element output signals are represented by the complex vector [19]:

$$\vec{x}(t) = A(\theta_m) \cdot \vec{s}(t) + \vec{n}(t), \tag{2}$$

where $\vec{x}(t) = [x_1(t),\dots,x_N(t)]^T$ is the vector of the dimension $1\times N$, which describes the antenna array output signals; $\vec{s}(t) = [s_1(t),\dots,s_M(t)]^T$ is the N-dimensional of the signals; $\vec{n}(t) = [n_1(t),\dots,n_N(t)]^T$ is the noise vector; $A(\theta_m)$ is the $N\times M$ dimensional matrix of the steering vectors $[\vec{a}(\theta_1),\dots,\vec{a}(\theta_M)]$.

In practical applications, the spatial covariance matrix is estimated from a collection of K time samples [19]:

$$\hat{R} = \frac{1}{K}\sum_{k=1}^{K} \vec{x}(k)^H \vec{x}(k) = \hat{E}_s \hat{\Lambda}_s \hat{E}_s^H + \hat{E}_N \hat{\Lambda}_N \hat{E}_N^H, \tag{3}$$

where $\vec{x}(k)$ is the N-dimensional vector at the k^{-th} time sample at the digital antenna array output, the symbol "^" describes the averaging over K samples, $\hat{E}_S$ is the signal eigenvector matrix, $\hat{E}_N$ is the noise eigenvector matrix, Λ_S and Λ_N is the eigenvalues matrices.

Spatial spectrum by the method MUSIC is calculated as [20]:

$$P_{MUSIC}(\varphi,\theta) = \frac{1}{\vec{a}^H(\theta) E_N E_N^H \vec{a}(\theta)}. \tag{4}$$

2.2. Reduction of DOA-estimation variance

The variance of the bearings θ and φ is assessed using the Cramer-Rao lower bound criterion, which is determined by the amount of noise, the positioning of the antenna elements in space and their radiation patterns. In this case, an arbitrary direction-of-arrival estimation algorithm cannot have values of variance below this limit, but only approaching it. Covariance error matrix for estimating the angular coordinates of signals with super-resolution in both azimuthal and elevation scanning planes can be written as (5) [19]:

$$B_{STO} = \frac{\sigma^2}{2K} \Re\left[Tr\left\{\begin{bmatrix} \Lambda_1 & \Lambda_2 \\ \Lambda_3 & \Lambda_4 \end{bmatrix} \circ \begin{bmatrix} \Xi & \Xi \\ \Xi & \Xi \end{bmatrix}^T\right\}\right]^{-1} \tag{5}$$

here $\Lambda_1 = D_\theta^H P_A^\perp D_\theta$, $\Lambda_2 = D_\theta^H P_A^\perp D_\phi$, $\Lambda_3 = D_\phi^H P_A^\perp D_\theta$, $\Lambda_4 = D_\phi^H P_A^\perp D_\phi$, $\Xi = SA^H R^{-1} AS$, D_θ и D_φ are the matrices of steering vector differentiation $a(\theta, \varphi)$ along the corresponding planes, K is the number of the samples. Consequently, it is possible to reduce the values B_{STO} by influencing the coordinates of the antennas and the shape of their radiation patterns of DOA-estimator.

Therefore, let us examine the CRLB expression given in (5) in greater detail. Previously, an explicit general formula of the CRLB was obtained for estimating the coordinates of the radiation source via Multi-sensor antenna arrays with directional sensors, also oriented in space in an arbitrary manner [21]:

$$var(\theta,\phi) \approx \frac{\sigma^2}{2K} \Re\left\{\left(\sum_{ij} g_i^2 g_j^2 (a_i' - a_j')^2 + \sum_{ij} (g_i' g_j - g_j' g_i)^2\right)^{-1}\right\} \tag{6}$$

where ' denotes a derivative along θ or φ depending on the scanning direction, i and j are the antenna elements indexes. The antenna array is supposed consisting of two antenna elements, then the equation (6) can be rewritten in the following form [22]:

$$var(\theta,\varphi) = \frac{\sigma^2}{2K}\{(g_1^2 g_2^2 (a_1' - a_2')^2 + (g_2' g_1 - g_1' g_2)^2)^{-1}\} \tag{7}$$

As demonstrated in the equation's left-hand side (7) in curly brackets, the main physical factors determining the variance are the square of the gains of the sensor elements, just like the area occupied by the employed antenna array. Therefore, it is possible to increase the accuracy of DOA-estimates by changing one or both of these factors. On the contrary, the smallest possible number of antenna elements can be compensated by a larger gain, or by being located on a larger area. For example, if the gain of each antenna is increased twice, then the variance of DOA-estimates can be reduced radically.

Therefore, the proposed methodology for designing antenna arrays for DOA-estimation consists of synthesizing the directional patterns of individual elements g_1 and g_2 in order to minimize variance *var*(θ, φ in the works familiar to the authors, which concern the optimization of the antenna array topology to increase accuracy, the gain and the radiation patterns of the array elements are not accounted for at all.

Consider an example, two circular array configurations are studied: a 3-element and a 2-element array of directional antennas. The task is the variance of the estimates of the three-element array $var(\theta, \varphi)_3$ must be equal to the variance of the two-element one $var(\theta, \varphi)_2$. In this case the sum of the gains of the elements of the second array must be consistent to the sum of the gains of the three-element circular antenna array. The formula of the Cramer-Rao lower bound for the two-element antenna array must be remembered (7) and expression for the three-element array:

$$\mathrm{var}(\theta,\varphi)_3 \approx \frac{\sigma^2}{2K}\left\{\begin{pmatrix} (g_1^2 g_2^2 (a_1'-a_2')^2 + g_3^2 g_1^2 (a_1'-a_3')^2 + g_2^2 g_3^2 (a_2'-a_3')^2 + (g_2' g_1 - g_1' g_2)^2 \ldots \\ +(g_3' g_1 - g_1' g_3)^2 + (g_2' g_3 - g_3' g_2)^2) \end{pmatrix}^{-1}\right\} \tag{8}$$

According to the condition the following equation must be met:

$$var(\theta,\varphi)_2 = var(\theta,\varphi)_3 \tag{9}$$

In addition, it is assumed that two- and three-element circular arrays are located at the same radius from the center, as well as the noise power σ^2 and the number of samples *K* of the correlation matrices are the same for $var(\theta,\phi)_2$ and $var(\theta,\phi)_3$. Thus,

$$\frac{\sigma^2}{2K}\left\{\frac{1}{(g_1^2 g_2^2 (a_1'-a_2')^2 + (g_2' g_1 - g_1' g_2)^2)}\right\} = \frac{\sigma^2}{2K}\left\{\frac{1}{a+b}\right\} \tag{10}$$

$$a = (g_1^2 g_2^2 (a_1' - a_2')^2 + g_3^2 g_1^2 (a_1' - a_3')^2 + g_2^2 g_3^2 (a_2' - a_3')^2 \tag{11}$$

$$b = (g_2' g_1 - g_1' g_2)^2 + (g_3' g_1 - g_1' g_3)^2 + (g_2' g_3 - g_3' g_2)^2) \tag{12}$$

here and further g_{i2} and g_{i3} are the radiative characteristics of the elements of the considered two- and three-element arrays respectively.

The equal terms, *i.e.*, the noise power and the number of samples of the correlation matrix can be eliminated, therefore it can be written that:

$$\frac{1}{g_1^2 g_2^2 (a_1'-a_2')^2 + (g_2' g_1 - g_1' g_2)^2} = \frac{1}{\begin{matrix} g_1^2 g_2^2 (a_1'-a_2')^2 + g_3^2 g_1^2 (a_1'-a_3')^2 + g_2^2 g_3^2 (a_2'-a_3')^2 \ldots \\ +(g_2' g_1 - g_1' g_2)^2 + (g_3' g_1 - g_1' g_3)^2 + (g_2' g_3 - g_3' g_2)^2 \end{matrix}} \tag{13}$$

Consider only the denominators of the expression (13):

$$\begin{aligned} g_{12}^2 g_{22}^2 + (g_{22}' g_{12} - g_{12}' g_{22})^2 = g_{13}^2 g_{23}^2 + g_{33}^2 g_{13}^2 + g_{23}^2 g_{33}^2 + (g_{23}' g_{13} - g_{13}' g_{23})^2 + \\ (g_{33}' g_{13} - g_{13}' g_{33})^2 + (g_{23}' g_{13} - g_{23}' g_{33})^2 \end{aligned} \tag{14}$$

Since it is clear that the product of the squares of the radiation patterns is much greater than the other terms, the simplification can be continued and:

$$g_{12}^2 g_{22}^2 \approx g_{13}^2 g_{23}^2 + g_{33}^2 g_{13}^2 + g_{23}^2 g_{33}^2 \tag{15}$$

Now it is required to determine the gain of the pattern of the synthesized antenna array. It is hypothesized that the maximum value of the pattern of the rectangular antenna element is equal to 6, and the minimum value to be 0.1. Then it can be written that:

$$g_{12}^2 0.1_{22}^2 \approx 6_{13}^2 0.1_{23}^2 + 0.1_{33}^2 6_{13}^2 + 0.1_{23}^2 0.1_{33}^2 \tag{16}$$

$$g_{12}^2 \approx \frac{6_{13}^2 0.1_{23}^2 + 0.1_{33}^2 6_{13}^2 + 0.1_{23}^2 0.1_{33}^2}{0.1_{22}^2} \tag{17}$$

After simple calculations it can be claimed that $g_{12} = 8.5$. The value makes it possible to obtain the accuracy using a smaller number of antenna elements. Here, the missing elements are compensated by means of a higher gain, as it can be seen from the CRLB formula (8, 10).

It may be stated; the proposed approach allows designing antenna arrays having radiation patterns which minimize the variance in spatial coordinate estimates. On the contrary, it can be utilized to evaluate the accuracy of existing arrays by means of the closed form equations. The variance based on (7)-(8) for the spatial coordinate in azimuth equal to 90° can be calculated using (18) and (19):

$$var(\theta = 90°)_{2-flat\_top} \approx \frac{1}{g_{12}^2 g_{22}^2} \approx \frac{1}{2.3_{12}^2 1.4_{22}^2} \approx 0.0964° \tag{18}$$

$$var(\theta = 90°)_3 \approx \frac{1}{g_{13}^2 g_{23}^2 + g_{33}^2 g_{13}^2 + g_{23}^2 g_{33}^2} \approx \frac{1}{4.6_{13}^2 0.2_{23}^2 + 0.2_{33}^2 4.6_{13}^2 + 0.2_{23}^2 0.2_{33}^2} \approx 0.5902° \tag{19}$$

As it can be seen from expressions (18)-(19), according to the CRLB and the proposed methodology, the variance of the estimates using the dual-element array will promote smaller values in comparison with the tri-element digital antenna array. With all this, there will only be mutual relationships between different geometries of antenna arrays, among which comparisons can be made and then the best one can be selected from the point of view of obtaining high accuracy of the DOA-estimation.

Now, based on the obtained antenna gain value, a radiation pattern must be synthesized satisfying the requirements. In addition, the proposed hypothesis will be confirmed by fulfilling comparative modeling. The results are given below in the section 3.

3. PERFORMANCE ANALYSIS

3.1. Simulation study of the proposed methodology

In the section the introduced strategy of designing an the dual-element array for DOA-estimation with super-resolution is will be approved via simulation study. A circular antenna array out of three directional elements is used for the comparison. In order to obtain the same accuracy as the three-element array. Each antenna of the dual-element array will have gain equal to $g_{12} = 8.5$ as it was proven in the previous part. The following antenna array structure is proposed for DOA-estimation, as represented in Figure 2. As can be seen from Figure 2, each antenna of the array must have radiation patterns which have to be synthesized and possess the gain $g_{12} = 8.5$ in order to minimize (7). The proposed patterns decreasing DOA-estimation errors are showed in Figure 3.

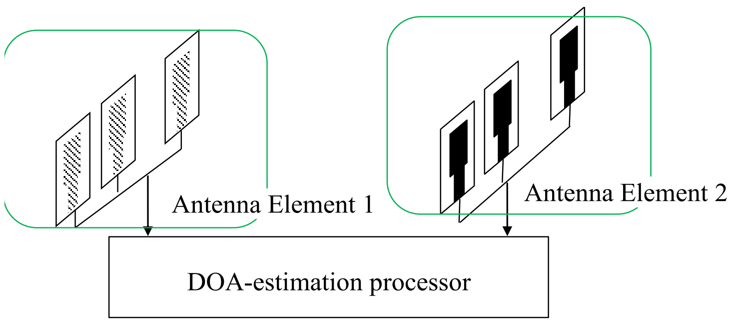


Figure 2. Proposed digital antenna array design for DOA-estimation with two antenna elements for 360° scanning on azimuth

Figure 3(a) reveals the peak values are picked up in such a way that they coincide with the minimal of the pattern of the neighboring element. At the same time, the main part of the gain is focused in a wide azimuth range. A template radiation pattern is shown in Figure 3(a) that will be used for antenna synthesis. Further these radiation patterns are utilized to synthesize a linear antenna array, which is consists of ten directional elements with a uniform half-wave interelement distance. The linear antenna element shown in Figure 2 consists of directional antenna elements. Mathematically, the spatial pattern of each element is defined as (20) [23]:

$$g_n(\varphi, \theta) = \frac{D}{2^{2m}}(1 + sin(\varphi))^m(1 + cos(\theta - \frac{2\pi n}{N}))^m, \tag{20}$$

where D is the directivity, m controls D.

The radiation pattern of the linear array consisting of (N = 10) directional elements without taking into account mutual coupling can be obtained as (21):

$$E(\theta) = \sum_{n=1}^{N} g_n(\theta) \cdot a_n e^{j(k(n-1)d\cos(\theta+\beta_n))}, \quad (21)$$

The linear array has inter-element spacing d, where each $n^{\text{-th}}$ element is weighted by amplitude a_n and phase β_n. Their individual radiation patterns $g_n(\theta)$ is characterized by (8).

A genetic optimization algorithm is applied to synthesize the patterns of the entire linear array by means of MATLAB optimization toolbox [24]. The relative error which is difference between the resulting field level and the intended pattern value across the M common sampling points is calculated using the formula (22):

$$e_m = E_{actual}(\theta_m)\text{-}E_{desired}(\theta_m), i = 1,2,\dots,M \quad (22)$$

where $E(\theta_m)$ denotes the electric field level of the linear array at a point with azimuth coordinate θ_m. The value of least mean squares is used as the objective function according to the following expression [25]:

$$F(\alpha_1,\dots,\alpha_N,\beta_1,\dots,\beta_N) = \left(\frac{1}{M}\sum_{m=1}^{M}|e_m|^2\right)^{(1/2)} \quad (23)$$

The values of phase β_n and amplitude excitation a_n of the linear array feeding network are optimized. Following optimization, the determined parameters (phase shifts β_n and amplitude attenuations a_n) appear in Table 1.

Table 1. The values of phases and amplitudes for linear antenna array

Antenna element index (n)	1	2	3	4	5	6	7	8	9	10
Phase (β_n), deg.	-279.02	298.6	140.4	359.99	-255.67	-342.97	137.22	-49.16	-256.9	253.5
Amplitude (α_n)	0.999	1	1	0.999	0.999	0.999	0.999	0.999	0.912	0.852

As evident from Figure 3(b) the synthesized linear array's radiation pattern closely matches the reference pattern. An obvious peak in the -5° to 5° azimuth is manifesting, as well as gain from -90° to 90° to achieve intended azimuthal coverage. In addition, it was possible to obtain the radiation pattern of the linear array that are higher than those of the single antenna, such as, for example, a rectangular microstrip. The achieved gain is the basis for reducing the error variance of DOA-estimation as it can be realized from (7). Thus, the steering vector $a(\theta)$ for the two-element array is given by:

$$\vec{a}(\theta) = \begin{bmatrix} \left(\sum_{n=1}^{N} g_n\left(\theta-\frac{\pi}{2}\right)\cdot a_n e^{j\left(k(n-1)d\cos\left(\theta+\beta_n-\frac{\pi}{2}\right)\right)}\right) e^{j\frac{2\pi}{\lambda}r\cos\left(\theta-\frac{\pi}{2}\right)} \\ \left(\sum_{n=1}^{N} g_n\left(\theta+\frac{\pi}{2}\right)\cdot a_n e^{j\left(k(n-1)d\cos\left(\theta+\beta_n+\frac{\pi}{2}\right)\right)}\right) e^{j\frac{2\pi}{\lambda}r\cos\left(\theta+\frac{\pi}{2}\right)} \end{bmatrix} \quad (24)$$

where r is the distance between antenna elements, β_n and a_n are given in Table 1.

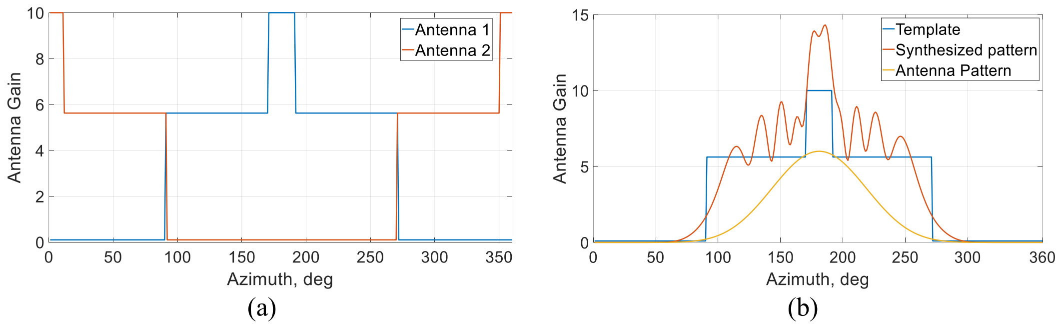


Figure 3. Radiation patterns of designed antenna elements (a) intended radiation patterns (red and blue are indicated for corresponding antenna elements) and (b) radiation patterns: blue – template, red – linear antenna array, yellow – common antenna

Further a simulation is fulfilled in order to verify the hypothesis. The super-resolution algorithm MUSIC (4) is utilized to estimate the spatial coordinates of signal source. The correlation matrix (3) uses 100 samples with a 0 dB signal-to-noise ratio. The signal's azimuth coordinate varies from 0° to 180°, and we compute the RMSE at each angular position:

$$RMSE = \frac{1}{L-1}\sqrt{\sum_{l=1}^{L}\left(\theta - \hat{\theta}_l\right)^2} \quad (25)$$

Thus, the simulation process goes through several stages. At the first stage, the variance is set using (7) and (8). Then it is necessary to calculate the gain of the antenna pattern in the same way as (8) to (17) in order to obtain the acceptable level of variance from the previous step. The obtained values of g(θ, φ) are substituted in (1) and (24). At the last step, the predefined signal coordinates are calculated according to epy expressions (2) to (4). The process continues to obtain (25).

Figure 4 represents circular arrays consisting of two (Figure 4(a)) and three (Figure 4(b)) directional elements respectively. Further they are compared with the array demonstrated in Figure 2. The single-element radiation pattern of the circular arrays from Figure 4 is obtained from expression 8 for *D* = 6 *db*.

The radiation patterns of circular arrays of two and three elements are shown in Figure 5(a) to 5(c) depicts the obtained flat-top pattern. Figure 5 reveals the circular antenna arrays from Figure 4(a) and 4(b) have radiation patterns in which local maxima coincide with local minima. Consequently, their product is minimal. At the same time, as evident from Figure 5(c), the maximum is distributed along the azimuth in order to receive the signal uniformly over the entire range. In addition, the maximum here has a greater value in order to compensate for the minimum gain of the neighboring element.

In Figure 6 several curves of RMSE of the MUSIC method are shown for the following types of digital antenna arrays: the two-element array from Figure 2, as well as the circular arrays depicted in Figure 4. Figure 6 demonstrates that the proposed digital array design method reduces RMSE across all azimuth scanning angles while maintaining accuracy comparable to (or better than) the tri-element circular array. The RMSE of the estimates of the proposed dual-element antenna array having the synthesized patterns, which is shown in Figure 5(c), has a value of about 1°, which corresponds to the circular antenna array of the two elements (Figure 4(b)) having radiation patterns as in Figure 5(b). Thus, the simulation confirmed the correctness of the proposed method of designing antenna arrays for DOA-estimation. At the same time, the error values obtained analytically from section 2 coincide with the graph in Figure 6.

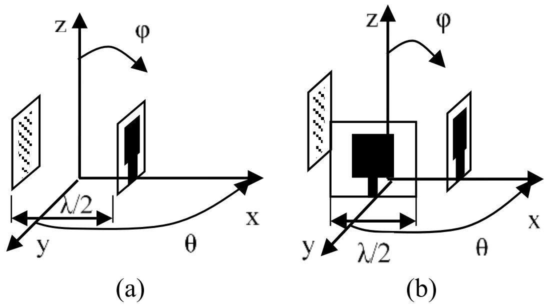


Figure 4. Circular antenna arrays consisting of (a) two and (b) three elements

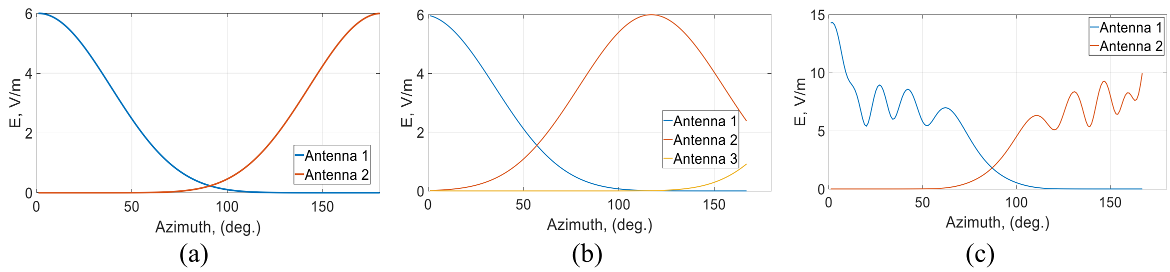


Figure 5. The simulated radiation characteristics of the digital antenna array (a) depicted in Figure 4(a), (b) illustrated in Figure 4(b), and (c) presented in Figure 3

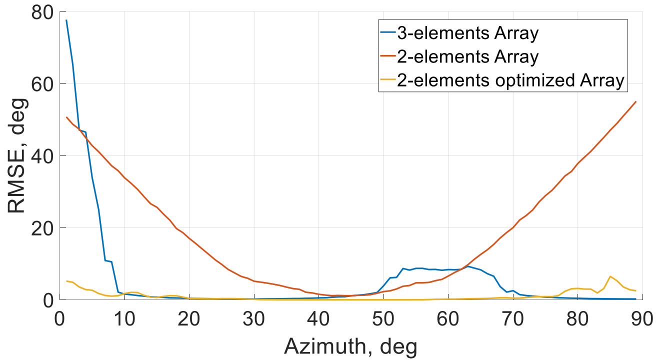
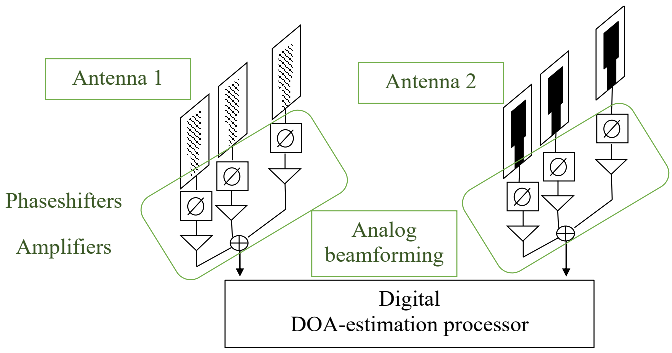


Figure 6. Root mean square errors of the considered digital antenna arrays

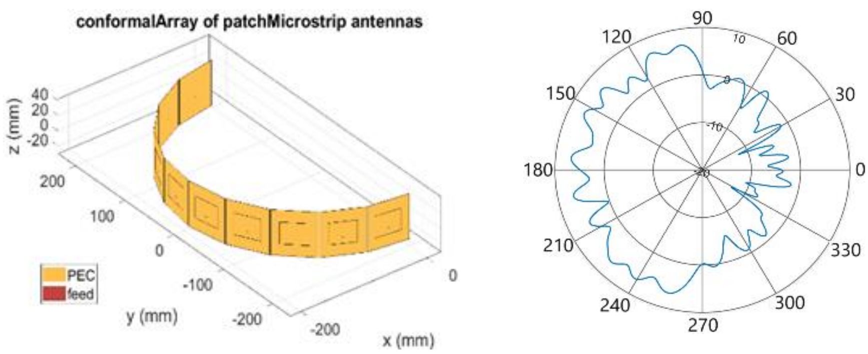
3.2. Simulation of DOA-estimator prototype

Now in this section the approach explained in the section 2 is implemented by means of the Method of moments. The method allows obtaining antennas that are closest to real samples. The example of developing a microstrip antenna array with a synthesized radiation pattern which maximally reduces the variance of the DOA-estimation is described. Consider a two-element antenna array and its elements pattern shown in Figure 7.

The circuit in Figure 7(a) has two antennas, *i.e.*, "Antenna 1" and "Antenna 2". An idealized perfect conductor (PEC) material is used as an emitter. Each antenna is represented in Figure 7(b) and must have a radiation pattern illustrated in Figure 5(c). This directional pattern can be achieved using several antenna elements, phase shifters and amplifiers that perform the phase shift β_n and amplitude excitation a_n. The synthesized radiation pattern of the antenna in Figure 7(b) is depicted in Figure 7(c). The optimization is fulfilled according to (22) and (23). The characteristics of the antenna arrays described in this section are obtained using the method of moments by 'MATLAB antenna toolbox' [26]. The calculated amplitude excitations and phase shifts for obtaining the radiation pattern are given in Table 2.



(a)



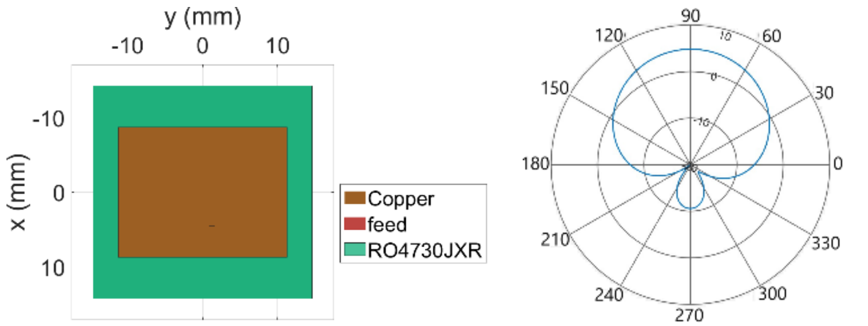
(b) (c)

Figure 7. Two element DOA-estimator (a) sketch, (b) its antenna element, and (c) pattern of an element

Table 2. The values of phases and amplitudes for antenna array depicted in Figure 7(b)

Antenna element index (n)	1	2	3	4	5	6	7	8	9	10
Phase (β_n), deg.	123.9	1.4	-252.1	-1.5	270.8	-4.9	-88	305.4	92	345.4
Amplitude (α_n)	0.4	1.4	1.8	0.6	3	3	2.8	0.8	2.1	1.6

In addition, the dual- and tri-element circular antenna arrays are considered in Figure 5, which consist of rectangular patch antennas, which are shown in Figure 8(a) and its pattern in Figure 8(b). As it can be seen in Figure 8(d), the patch element has the gain equal to 4.7 dBi while the gain of the antenna from Figure 7(b) is approx. 7 dBi that can be viewed in Figure 7(c). At the same time, it is concentrated in a wide azimuth range. Thus, the hypothesis is that the designed antenna array will allow estimation direction-of-arrival coordinates with higher accuracy. The following figure shows the graphs of the pseudospectrum of the MUSIC method for the antenna arrays under consideration: the two-element array (blue curve), the three-element circular (red curve) and the two-element circular digital antenna array (yellow curve). The coordinate of the signal in the azimuthal plane is equal to 75°, SNR=5 dB. Thus, the steering vector (1) and the spatial correlation matrix (3) are calculated using the modified radiation patterns *g(θ)* after the method-of-moments.



(a) (b)

Figure 8. Electrodynamic model of the elements of the circular antenna array (a) rectangular patch antenna based on RO4730JXR dielectric and (b) its directional pattern

As it can be seen from Figure 9, the two-element antenna array shown in Figure 7(a) and possessing the elements radiation patterns depicted in Figure 7(c), allows obtaining spatial pseudospectra as sharp as the three-element circular antenna array. This is explained by the fact that the elements "Antenna 1" and "Antenna 2" have the gains which are greater than the elements of the circular antenna array, *i.e.*, 7 dBi (Figure 7(c)) and 4.7 dBi (Figure 8(b)) respectively. Thus, as depicted in Figure 9, higher gain results in higher accuracy in DOA-estimation. Based on the procedure described in sections 2 and 3 the antenna array prototype is implemented and the results are consistent. The implemented scheme can be named as beamspace [27], [28].

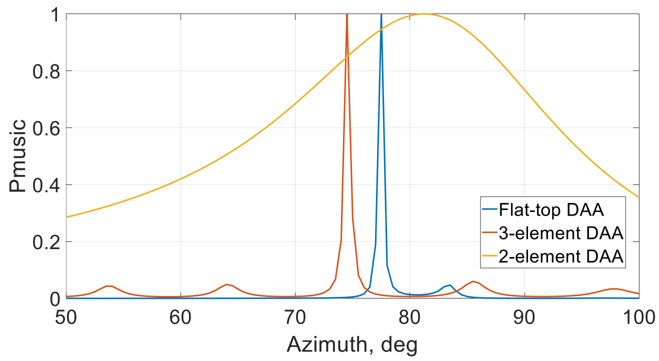


Figure 9. The pseudospectrum of MUSIC

4. CONCLUSION

The paper details a method for designing the digital antenna arrays for increasing DOA-estimation accuracy with super-resolution. Unlike the referenced methods, the proposed design approach accounts for the gain and directional patterns of individual antenna elements. Commonly, they focus on creating new DOA-estimation methods, adding more antennas, or optimizing array layouts, while neglecting antenna-specific properties by treating them as omnidirectional. It has been shown based on simulations and analytical expressions that the higher gain compensates for the lack of antennas. For example, the gain equaling to 8.5 makes it possible to obtain the same DOA-estimation accuracy as an antenna array composed of more elements having the gain equaling to 6.

Additionally, this technique has been utilized for designing a hybrid array architecture for direction-of-arrival estimation with super-resolution. Modeling based on the Method-of-Moments incontestably demonstrated that the hybrid array with fewer antenna elements does not result in a decline in the DOA-estimation accuracy. As a result, the reduced-element antenna array allows achieving the same level of accuracy in DOA estimation. In other words, the proposed approach can be implemented in practice as beam space hybrid construction. Thus, it can be argued that the considered technique can be used as a theoretical substantiation for the design of sensor arrays for spectral spatial processing. Thus, in the paper the methodology of the design flow of arrays for DOA-estimation with super-resolution has been represented, beginning from the analytical analysis on the closed-form equations up to the prototyping close to experimental solutions. Additionally, it handles the location of the antennas, the radiation patterns of the elements, the value of DOA-estimation errors simultaneously. All that allows moving from an abstract model to a final prototype vividly.

REFERENCES

[1] B. Yang, Z. Yu, J. Lan, R. Zhang, J. Zhou, and W. Hong, “Digital beamforming-based massive MIMO transceiver for 5G millimeter-wave communications,” *IEEE Transactions on Microwave Theory and Techniques*, vol. 66, no. 7, pp. 3403–3418, Jul. 2018, doi: 10.1109/tmtt.2018.2829702.

[2] N. Ruan, H. Wang, F. Wen, and J. Shi, “DOA estimation in B5G/6G: Trends and challenges,” *Sensors*, vol. 22, no. 14, p. 5125, Jul. 2022, doi: 10.3390/s22145125.

[3] W. Yi, S. Wei, and Y. Xu, “An improve mmW automotive radar DOA estimation approach: Joint matrix completion and spectrum extrapolation,” in *2024 IEEE 17th International Conference on Signal Processing (ICSP)*, Oct. 2024, pp. 16–19, doi: 10.1109/icsp62129.2024.10846245.

[4] Y. Fang, X. Wei, and J. Ma, “High-precision DOA estimation based on synthetic aperture and sparse reconstruction,” *Sensors*, vol. 23, no. 21, p. 8690, Oct. 2023, doi: 10.3390/s23218690.

[5] Y. Wang, G. Gui, H. Gacanin, T. Ohtsuki, O. A. Dobre, and H. V. Poor, “An efficient specific emitter identification method based on complex-valued neural networks and network compression,” *IEEE Journal on Selected Areas in Communications*, vol. 39, no. 8, pp. 2305–2317, Aug. 2021, doi: 10.1109/jsac.2021.3087243.

[6] V. Janoudi *et al.*, “Antenna array design for coherent MIMO radar networks,” in *2023 IEEE Radar Conference (RadarConf23)*, May 2023, pp. 1–6, doi: 10.1109/radarconf2351548.2023.10149789.

[7] D. Bonacci, F. Vincent, and B. Gigleux, “Robust DoA estimation in case of multipath environment for a sense and avoid airborne radar,” *IET Radar, Sonar & Navigation*, vol. 11, no. 5, pp. 797–801, May 2017, doi: 10.1049/iet-rsn.2016.0446.

[8] L. C. Godara, “Application of antenna arrays to mobile communications. II. Beam-forming and direction-of-arrival considerations,” *Proceedings of the IEEE*, vol. 85, no. 8, pp. 1195–1245, 1997, doi: 10.1109/5.622504.

[9] J.-F. Gu, S. C. Chan, W.-P. Zhu, and M. N. S. Swamy, “DOA estimation and tracking for signals with known waveform via symmetric sparse subarrays,” in *2012 IEEE 55th International Midwest Symposium on Circuits and Systems (MWSCAS)*, Aug. 2012, pp. 952–955, doi: 10.1109/MWSCAS.2012.6292179.

[10] C. Stöckle, J. Munir, A. Mezghani, and J. A. Nossek, “DoA estimation performance and computational complexity of subspace- and compressed sensing-based methods,” *Proceedings of 19th International ITG Workshop on Smart Antennas, WSA 2015*, 2015.

[11] N. Z. Katz, M. Khatib, Y. Ben-Horin, J. D. Rosenblatt, and T. Routtenberg, “Geometry design for DOA estimation in seismic 2D-arrays: Simulation study,” *IEEE Access*, vol. 12, pp. 35827–35843, 2024, doi: 10.1109/access.2024.3371977.

[12] O. Lange and B. Yang, “Antenna geometry optimization for 2D direction-of-arrival estimation for radar imaging,” in *2011 International ITG Workshop on Smart Antennas*, Feb. 2011, pp. 1–8, doi: 10.1109/wsa.2011.5741909.

[13] A. Ghani, F. Keyvani, and S. H. Sedighy, “Antenna array placement on limited bound for isotropic and optimal direction-of-arrival estimation,” *IET Signal Processing*, vol. 12, no. 3, pp. 277–283, May 2018, doi: 10.1049/iet-spr.2016.0472.

[14] H. Gazzah and J.-P. Delmas, “Optimization of the antenna array geometry based on a Bayesian DOA estimation criterion,” in *2011 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, May 2011, pp. 2544–2547, doi: 10.1109/icassp.2011.5947003.

[15] U. Baysal and R. L. Moses, “On the geometry of isotropic arrays,” *IEEE Transactions on Signal Processing*, vol. 51, no. 6, pp. 1469–1478, Jun. 2003, doi: 10.1109/tsp.2003.811227.

[16] R. Zhang, J. Zhou, J. Lan, B. Yang, and Z. Yu, “A high-precision hybrid analog and digital beamforming transceiver system for 5G millimeter-wave communication,” *IEEE Access*, vol. 7, pp. 83012–83023, 2019, doi: 10.1109/access.2019.2923836.

[17] X. Huang, Y. J. Guo, and J. D. Bunton, “A hybrid adaptive antenna array,” *IEEE Transactions on Wireless Communications*, vol. 9, no. 5, pp. 1770–1779, May 2010, doi: 10.1109/twc.2010.05.091020.

[18] M. A. L. Sarker, M. F. Kader, and D. S. Han, “Hybrid beam selection scheme in millimeter-wave beamspace MIMO-NOMA with lens antenna array,” *AEU - International Journal of Electronics and Communications*, vol. 120, p. 153204, Jun. 2020, doi: 10.1016/j.aeue.2020.153204.

[19] A. Y. J. Chan, “MUSIC and maximum likelihood techniques on two-dimensional DOA estimation with uniform circular array,” *IEE Proceedings - Radar, Sonar and Navigation*, vol. 142, no. 3, p. 105, 1995, doi: 10.1049/ip-rsn:19951756.

[20] R. Schmidt, "Multiple emitter location and signal parameter estimation," *IEEE Transactions on Antennas and Propagation*, vol. 34, no. 3, pp. 276–280, Mar. 1986, doi: 10.1109/tap.1986.1143830.
[21] Y. Nechaev, I. Peshkov, and N. Fortunova, "Evaluation and minimization of Cramer-Rao bound for conformal antenna arrays with directional emitters for DOA-estimation," *Progress In Electromagnetics Research C*, vol. 90, pp. 139–154, 2019, doi: 10.2528/pierc18111802.
[22] I. Peshkov, "An approach for designing a two-element antenna array for estimating the spatial coordinates of radio signals with increased accuracy," in *2023 Systems of Signal Synchronization, Generating and Processing in Telecommunications (SYNCHROINFO*, Jun. 2023, pp. 1–4, doi: 10.1109/synchroinfo57872.2023.10178622.
[23] B. R. Jackson, S. Rajan, B. J. Liao, and S. Wang, "Direction of arrival estimation using directive antennas in uniform circular arrays," *IEEE Transactions on Antennas and Propagation*, vol. 63, no. 2, pp. 736–747, Feb. 2015, doi: 10.1109/tap.2014.2384044.
[24] "MATLAB Ga - Genetic Algorithm," *mathworks.com*. https://www.mathworks.com/help/gads/ga.html (accessed Jul. 18, 2024).
[25] H.-J. Zhou, Y.-H. Huang, B.-H. Sun, and Q.-Z. Liu, "Design and realization of a flat-top shaped-beam antenna array," *Progress In Electromagnetics Research Letters*, vol. 5. Wiley, pp. 159–166, Apr. 2008, doi: 10.2528/PIERL08111911.
[26] "Antenna Toolbox," *mathworks.com*. https://www.mathworks.com/help/antenna.html (accessed Jul. 18, 2024).
[27] C. Lv, Y. Liu, T. He, Y. Li, J. Liang, and X. Zhao, "A Low-complexity Beamspace-based Method for DOA Estimation in Massive MIMO Systems," in *2020 IEEE 20th International Conference on Communication Technology (ICCT)*, Oct. 2020, pp. 334–338, doi: 10.1109/icct50939.2020.9295911.
[28] J. Pan and F. Jiang, "Low complexity beamspace super resolution for DOA estimation of linear array," *Sensors*, vol. 20, no. 8, p. 2222, Apr. 2020, doi: 10.3390/s20082222.

BIOGRAPHIES OF AUTHORS

Ilia Peshkov was born in Yelets, Russia. He received his M.Sc. in radio electronics and its service at Bunin Yelets State University in 2008. He received the Ph.D. degree in radio physics from Voronezh State University, Voronezh, Russia, in 2012. Currently, he is an associate professor at the Department of Agricultural Engineering, Mechatronic and Radioelectronic Systems, Bunin Yelets State University, Yelets, Russia. His research interests include simulation and digital signal processing. In addition, he is a reviewer for leading optical journals such as IET Microwaves, Antennas & Propagation and IEEE Access. He has authored or coauthored more than 100 refereed journal and conference papers. He can be contacted at email: ilvpeshkov@gmail.com.

Natalia Fortunova was born in Yelets, Lipetskaya Oblast, Russia. She received her M.Sc. and Ph.D. from Chernihiv Institute of Technology, Ukraine. She is currently Head of the Department of Agricultural Engineering, Mechatronic and Radioelectronic Systems at Bunin Yelets State University. Her research interests include analog electronics and simulation. She can be contacted at email: fortuna@mail.ru.

Irina Zaitseva was born in Lipetsk, Lipetskaya Oblast, Russia. She received her M.Sc. from Lipetsk Polytechnic Institute, Russia, and Ph.D. from Bunin Yelets State University. Currently, she is an associate professor at the Department of Agricultural Engineering, Mechatronic and Radioelectronic Systems, Bunin Yelets State University, Yelets, Russia. Her research interests include analog electronics and simulation. She can be contacted at email: irina-zai@mail.ru.