A novel one-dimensional chaotic map with improved sine map dynamics

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ABSTRACT

These days, keeping information safe from people who should not have access to it is very important. Chaos maps are a critical component of encryption and security systems. The classical one-dimensional maps, such as logistic, sine, and tent, have many weaknesses. For example, these classical maps may exhibit chaotic behavior within the narrow range of the rate variable between 0 and 1and the small interval's rate variable. In recent years, several researchers have tried to overcome these problems. In this paper, we propose a new one-dimensional chaotic map that improves the sine map. We introduce an additional parameter and modify the mathematical structure to enhance the chaotic behavior and expand the interval's rate variable. We evaluate the effectiveness of our map using specific tests, including fixed points and stability analysis, Lyapunov exponent analysis, diagram bifurcation, sensitivity to initial conditions, the cobweb diagram, sample entropy and the 0-1 test.

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1. INTRODUCTION

Human habits have changed significantly since the information age. The most significant achievements of this time were digital data generation, storage, and transmission. Quick accessibility and the many benefits of digital data make it vulnerable to alteration without proper security measures [1], [2]. Digital data encryption, signatures, and hashing are used in several scientific fields to prevent unauthorized access and enhance security [3], [4]. These pseudorandom number generators (PRNGs), crucial in various fields, generate 32-bit random sequences and utilize the IEEE 754-2008 standard for floating-point arithmetic [5]. However, because these methods are inner-linear by nature, correlation and algebraic attacks are more likely to be successful against them. Chaotic maps with complex nonlinear dynamics are now being considered as a viable alternative source for generating pseudo-random numbers [6]. A chaotic map is a mathematical equation that is both iterative and recursive, exhibiting nonlinearity. It is bounded and highly sensitive to the initial condition [7]. Chaotic maps can be designed in a variety of ways, but lately, efforts have been made to create chaotic maps using dimension expansion techniques [8]. Chaotic maps are discrete, which makes them easy to simulate quickly, even though they are recursive and require knowledge of all previous results in order to calculate the current output [9]. They can also be used in digital engineering domains such as secure data transmission [10]–[13].

The classical maps, such as logistic, sine, and tent, have many weaknesses. These weaknesses include limited complexity, unpredictability in certain parameter ranges, and a tendency to exhibit regular or periodic behavior rather than true chaotic dynamics in some scenarios. Additionally, they may suffer from low sensitivity to initial conditions, making them less effective in applications requiring high levels of unpredictability or randomness. The rate values' small range, which is limited to values between 0 and 1, is another important limitation, that restricts their applicability in scenarios that demand a broader range of values. Some applications, especially those involving complex systems or requiring extensive variability in response, necessitate a larger range of control parameters to function effectively. This limitation can reduce the versatility of classical maps in certain practical or theoretical contexts. Furthermore, these maps exclusively have a single parameter. While in certain applications, it is necessary to work with multiple parameters.

Over the last decade, experts have proposed several ways to address these problems. Zhou *et al.* [14] introduced a cascade chaotic system, which cascades chaotic initial maps to produce numerous chaotic maps. New chaotic maps outperform similar seed maps in performance and parameters, resulting in greater unpredictability and security in PRNGs based on cascading chaotic systems. According to authors in [15] using a permutation with several recursive generators may increase PRNG performance by avoiding the short period of the chaotic map. Umar *et al.* [16] suggest a better skew tent map (STM) that uses the sine function and perturbation approach to fix its flaws and make it safer and more efficient. Hua *et al.* [17] established the sine chaotification model to improve the chaos complexity of 1-D chaotic maps. In response to the time-delayed dynamical system, Liu *et al.* [18] propose a new PRNG using a multi-delayed Chebyshev map. Adding state values to the chaotic system increases its complexity and makes it harder to anticipate the pseudo-random sequence produced. Hua and Zhou [19] developed an exponential chaotic model (ECM) to create single-dimensional (1-D) chaotic maps with durable chaos. This universal framework may produce many new chaotic maps using any two 1-D chaotic maps as the basis and exponent. In [20] authors proposed a feedback control approach to increase chaotic signal complexity. These characteristics secure the PRNG. Many of these techniques only slightly improve performance because they have problems like chaotic annulling traps and low Lyapunov exponents (LE), which make them harder to use and more complicated [21].

In this paper, we propose a novel one-dimensional chaotic map that significantly improves upon the traditional sine map. Our approach involves the introduction of an additional control parameter, which provides greater flexibility in tuning the system's dynamics. By modifying the mathematical structure of the sine map, we aim to enhance its chaotic behavior, making it more robust and unpredictable across a wider range of conditions. One of the key innovations of our proposed map is the expansion of the interval's rate variable, which overcomes the limitations of the small rate range (typically confined between 0 and 1) found in classical maps. The enhanced chaotic properties of our map are demonstrated through numerical simulations and comparative analysis with the original sine map. Our results show that the proposed map not only achieves a higher degree of chaos but also offers improved control over the system's dynamics, making it a valuable tool for applications in fields such as cryptography, random number generation, and secure communication.

This paper is organized as follows: section 2 introduces and thoroughly discusses the proposed one-dimensional chaotic map, detailing its mathematical formulation and the enhancements made to the original sine map, including the introduction of an additional control parameter. Section 3 is dedicated to validating the chaotic properties of the proposed map, where we present various analytical techniques and simulations to demonstrate its improved chaotic behavior compared to the traditional sine map and other related works. Finally, section 4 presents the concluding remarks, summarizing the key contributions of our work and highlighting potential future research directions.

2. PROPOSED METHOD AND ITS STABILITY ANALYSIS

This section presents a new one-dimensional chaotic map. The map is mathematically defined as:

$$x_{n+1} = \sin\left(\frac{\alpha}{\beta\left(\frac{\pi}{2} - x_n\right)}\right) \tag{1}$$

where x_0 is the initial state and (β, α) are the control parameters.

$$x_{n+1} = x_n = x^* = sin\left(\frac{\alpha}{\beta\left(\frac{\pi}{2} - x^*\right)}\right) \tag{2}$$

When the output of the map in the next iteration does not change and is the same as the output it is currently producing, this is referred to as a fixed point. Under the assumption that x^* is the fixed point, the (2) is obtained by substituting *xn* and *xn+1* with it. It is not possible to solve this equation analytically using elementary functions. Nevertheless, we can attempt to identify approximate answers by employing numerical techniques such as the interval bisection method, the iterative fixed-points method, and the Newton-Raphson method.

In this study, the interval bisection method will be used [22]. Algorithm 1 illustrates all the steps required to determine fixed points and indicates whether it is unstable. A fixed point is considered unstable if the absolute value of the derivative of the function at the fixed point is more than *1* ($|df(x)| > 1$). In instances of this nature, neighboring points tend to deviate from the fixed point rather than converge towards it, suggesting the presence of potentially chaotic behavior.

We select 100 values of β between [0, 1] and α = 3,500. The number of unstable fixed points calculated for each value of β is displayed in Figure 1; for our map and the MSTent map [16], the total number for all β values is 2,079 and 1,940, respectively. Comparing these totals, our map appears to exhibit more chaotic behavior since it has a higher number of unstable fixed points.

Algorithm 1. Find fixed point
```
Procedure findFixedPoint(f (x), α, β, [-1, 1], N , ϵ)
Input: Function f(x), parameters α and β, initial interval [-1, 1], maximum iterations
       N=1,000, tolerance ϵ=1e-10.
Output: Return the value of fixed points.
Procedure:
         Divide the interval [-1, 1] into 100 small intervals [a, b].
         for each small interval [a, b] do
             if f (a, α, β)=a then
                    return a.
             end if
             if f (b, α, β) = b then
                    return b.
             end if
             Initialize iteration counter i=0.
             while i<N do
                    Increment i.
                    if (f(a, α, β)-a)*(f(b, α, β)-b)>0 then
                        Set atemp=a.
                        Set a=a+b
                        if f (a, α, β)=a then
                              return a.
                        end if
                        if (f(a, α, β)-a)*(f(b, α, β)-b)>0 then
                              Set b = a.
                              Set a = atemp.
                        end if
                    else
                        break.
                    end if
             end while
         end for
end procedure
```

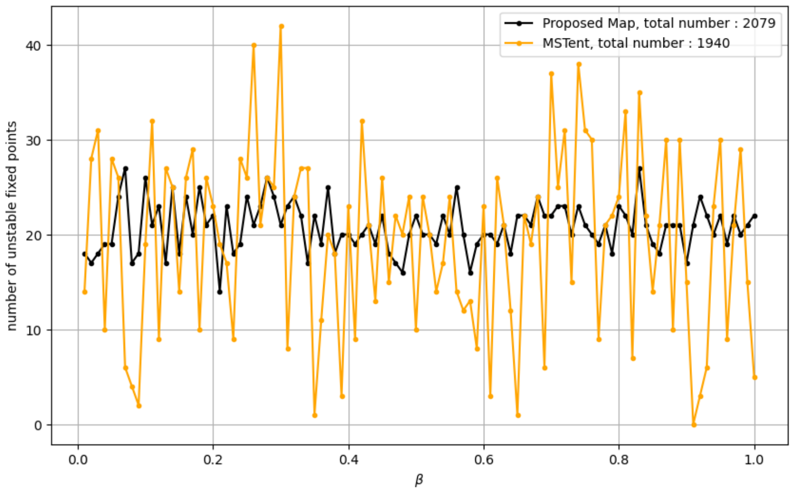

Figure 1. Comparing the number of unstable fixed points between our map and the MSTent map [16]

3. RESULTS AND DISCUSSION

Here, we evaluate the performance and chaotic behavior of the proposed map. To assess the disorderly behavior of our system, we use established assessment techniques like the bifurcation diagram, the Lyapunov exponent, the sensitivity towards the initial conditions, the parameters of the chaotic map, entropy analysis, and the results of the 0-1 test. To showcase the better chaotic qualities of the proposed map, we compare its assessed results with those of other existing chaotic maps.

3.1. Bifurcation diagram

Figure 2 displays the bifurcation diagram of the suggested map, with a fixed value of alpha set at 3,500. The map's output values are confined to the range of [-1, +1]. The range of beta values has been extended from [0, 1] to more than [-50, 50], exhibiting a global chaotic behavior in comparison with the bifurcation diagram of the sine map plotted in Figure 3.

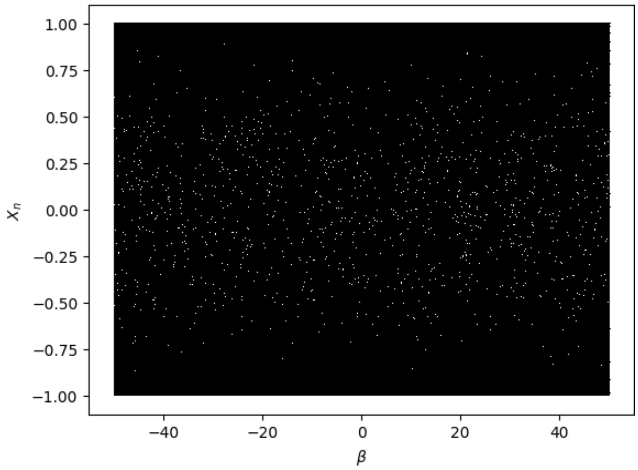

Figure 2. Bifurcation diagram of our map, with α=3,500

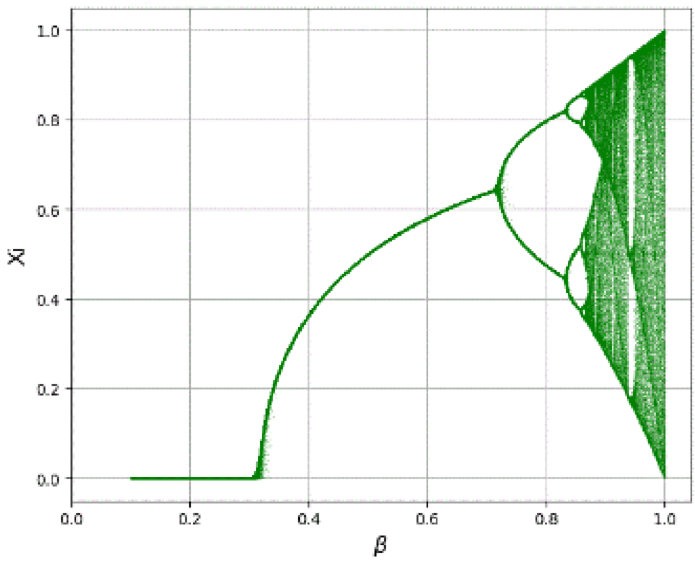

Figure 3. Sine map bifurcation diagram

Additionally, the bifurcation diagram is plotted for eight different values of α in Figure 4. It demonstrates that when this value is greater than 30,000, our map exhibits total chaotic behavior. As a result, for the rest of this study, we fixed α to 35,000 and β between [-50, 50]. For any PRNG or encryption application, a small key space is undesirable.

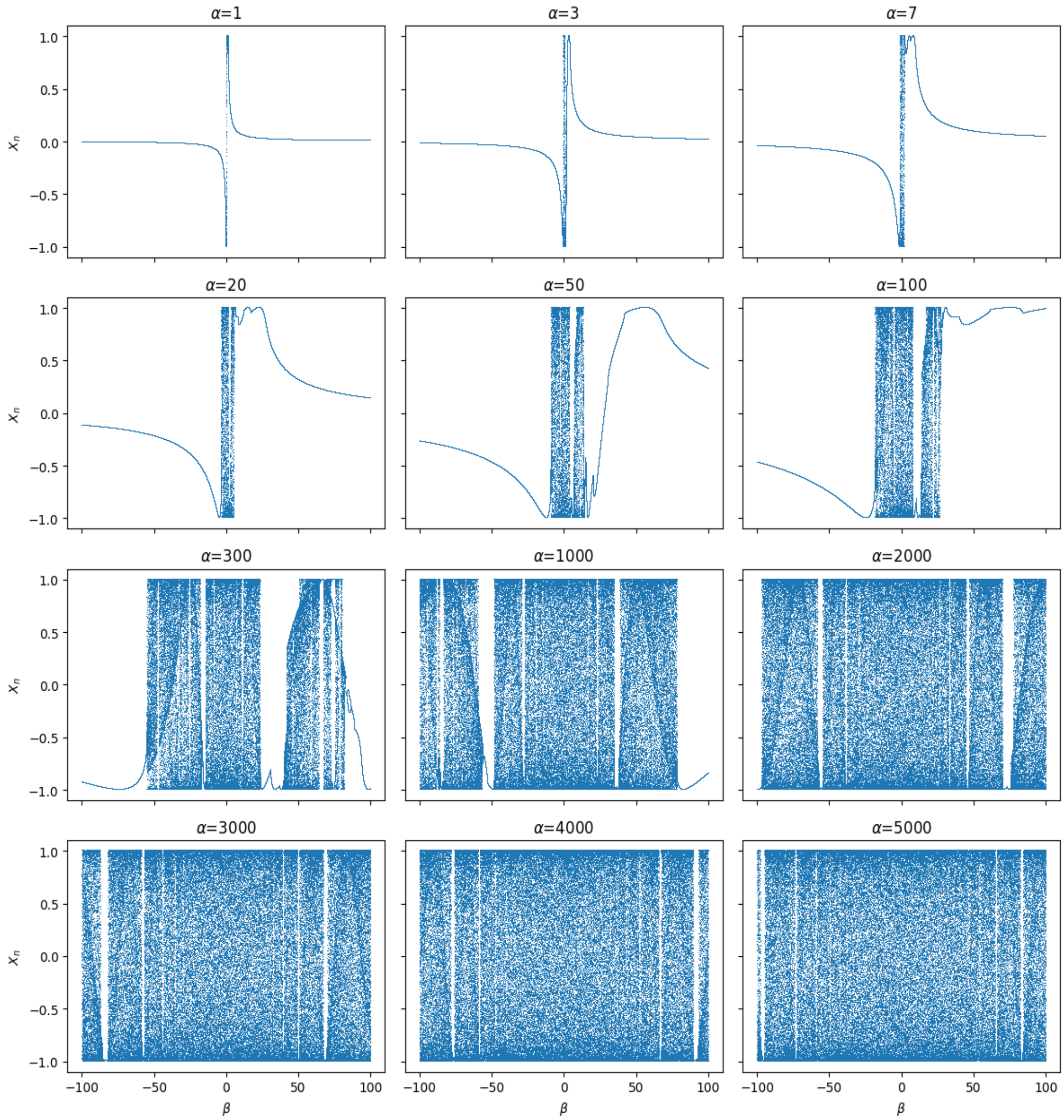

Figure 4. Bifurcation diagram with different values of α for our proposed map

3.2. Lyapunov exponent

The Lyapunov exponent (LE) is an essential tool in computer science for evaluating models and techniques that simulate or describe dynamical systems [23]. Researchers can assess a system's stability and confirm if chaotic dynamics are present by computing the LE. Since it implies that close trajectories diverge exponentially, a positive LE is usually seen as a strong degree of chaos, resulting in an unpredictable and extremely sensitive system. The LE can be mathematically defined using (3), where *F(xi)* represents the state of our system at iteration *i*.

$$LE = \lim_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n-1} ln|F'(x_i)| \tag{3}$$

From the curve in Figure 5, we can observe that our proposed map exhibits a Lyapunov exponent that ranges from a minimum of 5 to a maximum of 12. This indicates a strong level of chaotic behavior, as the Lyapunov exponent is a key measure of the sensitivity of the system to initial conditions. A higher Lyapunov exponent generally signifies more rapid divergence of nearby trajectories, leading to greater unpredictability and complexity in the system's dynamics.

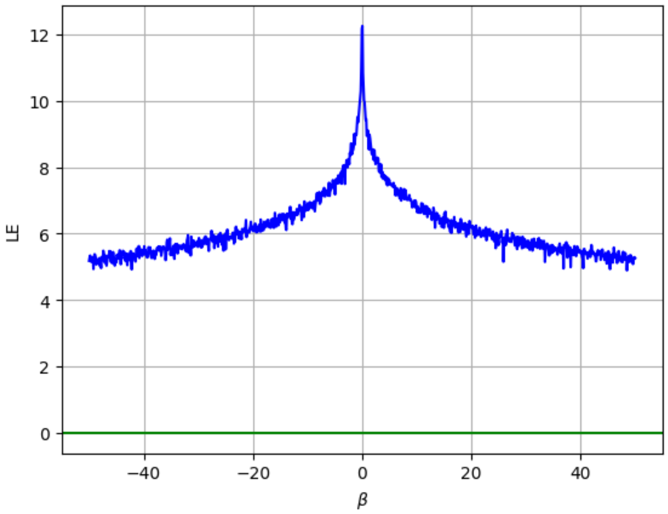

Figure 5. LE evaluation result for α = 3,500

In Table 1, a comparison is made between the LE and chaotic region of the proposed map and other chaotic maps available in literature. The new chaotic map possesses a more expansive chaotic space, a larger Lyapunov exponent, and a heightened sensitivity to initial conditions. We can notice that the Modified Skew Tent Map [16] exhibits a superior maximum LE compared to our map. However, their range of the parameter beta is only within [-1, +1]. For our map it can be selected within in the range of -100 to +100.

Table 1. Comparison of chaotic maps with the chaotic region and maximum Lyapunov exponent (MLE) that correspond to each specific map

References	Chaotic maps	Chaotic region	Maximum LE
--	Sine map	$[3.569, 4]$	0.6724
Ref [24]	Skew tent map	[0,1]	0.7
Ref [25]	Lorenz system	–	2.2991
Ref [21]	Piecewise cubic map	[0,4]	5.4678
Ref [26]	Generalized Sprott-a system	[0,2]	0.9
Ref [16]	Modified skew tent map	$[-1, +1] - \{0\}$	15.98
Proposed work	Our map	$[-100, +100] - \{0\}$	12.26

3.3. Sensitivity to initial conditions

Every chaotic map is required to exhibit sensitivity to initial conditions. Based on this characteristic, any modifications to the initial conditions or parameters should lead to a significant alteration in the subsequent sequence. In order to analyze and assess this attribute, the proposed chaotic map was executed with the identical parameter value $\beta = 0.3$, but with two distinct initial condition values: $x_{01} = 0.02$ and $x_{02} = 0.02 + 10^{14}$. Figure 6 shows the trajectory that the two initial conditions produced; they are noticeably different from one another. Even little alterations in the initial condition values of the chaotic map result in noticeable variations in the generated sequences, providing crucial insights into the chaotic behavior of the proposed chaotic map.

3.4. Cobweb diagram

The cobweb graphic shows how dynamical system functions change over time [27]. The cobweb diagram is an often-employed visual tool for illustrating the dynamics of chaotic maps, particularly in the field of dynamical systems theory. The term "cobweb" is derived from its similarity to the intricate structure of a spider's web. The points on the map can display three different types of behavior: convergence to a single value, oscillation between numerous values, or chaotic activity characterized by unpredictable wandering. The cobweb graphic offers a straightforward means of comprehending the dynamics of chaotic maps and their progression across successive iterations. It is especially beneficial for illustrating concepts such as attractors, periodic orbits, and chaotic behavior. Cobweb plots for $x_0 = 0.2$ and $\beta = 0.34$ are shown in Figure 7. It indicates that our map's series trajectories are not coinciding, demonstrating strong performance.

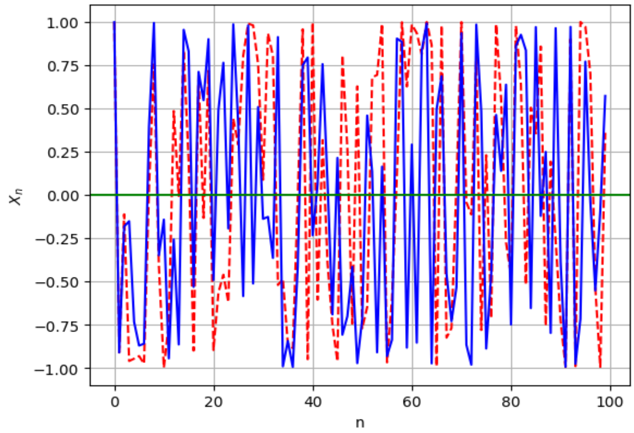

Figure 6. Two time series with a minimal difference in their initial value *x0*

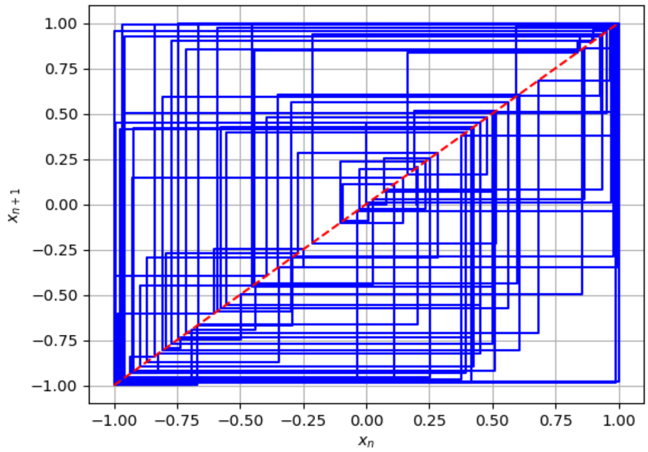

Figure 7. Cobweb diagram of the proposed map

3.5. Sample entropy

Entropy is a fundamental concept in computer science. It was defined by Claude Shannon in 1948 in his paper "mathematical theory of communication". We use it to measure the randomness of our one-dimensional chaotic map generator. A large value of sample entropy means that the maps have better chaos. For a time series X of size $N(x_1, x_2, x_3, \ldots x_N)$, we can generate vectors with the size of m and m+1.

$$U_m(i) = (x_i, x_{i+1}, x_{i+2}, \ldots \ldots x_{i+m-1}) \tag{4}$$

$$V_m(j) = (x_j, x_{j+1}, x_{j+2}, \ldots \ldots x_{j+m-1}) \tag{5}$$

$$\mathrm{U}_{m+1}(\mathrm{i}) = (x_\mathrm{i}, x_{\mathrm{i}+1}, x_{\mathrm{i}+2}, \ldots \ldots x_{i+m}) \tag{6}$$

$$\mathrm{V}_{m+1}(\mathrm{j}) = (x_\mathrm{j}, x_{\mathrm{j}+1}, x_{\mathrm{j}+2}, \ldots \ldots x_{j+m}) \tag{7}$$

$d_m(U(i), V(j))$ is the Cheybyshev distance between U_m and V_m, $d_{m+1}(U(i), V(j))$ is the Cheybyshev distance between U_{m+1} and V_{m+1}.

the sample entropy equation is defined as:

$$SE(m, r, N) = -\log \frac{A}{B} \tag{8}$$

where A is the number of vectors $U(i)$ that satisfy the condition $d_{m+1} < r$, and B is the number of vectors $U(i)$ that satisfy the condition $d_m < r$, r is the acceptable tolerance. We set $m = 2$, and $r = 0.2\ std(X)$ in order to follow the recommendations provided in [28]. Based on the obtained results, it appears that our map generator has demonstrated an improvement in entropy for the three input maps. The entropy value approaches 2, which is notably higher compared to the maximum value observed in the logistic, sine and tent maps. This means that our map generator exhibits a higher degree of chaos, as shown in Figure 8.

3.6. Test 0-1

The 2-dimensional system is driven by the time series *X(n)* for *n* = 1, 2, ..., *N* in the 0-1 test.

$$p_c(n) = \sum_{j=1}^{n} X(j) \times cos(c \times j) \ , q_c(n) = \sum_{j=1}^{n} X(j) \times sin(c \times j) \tag{9}$$

where c ∈ (0.2π), The mean square displacement of this 2-dimensional system is given as (10):

$$M(n) = \lim_{N\to\infty} \frac{1}{N} \sum_{j=1}^{n} ([p(j+n) - p(j)]^2 + [q(j+1) - q(j)]^2) \tag{10}$$

we calculate *K* as (11):

$$K = \lim_{n\to\infty} \frac{log(M(n))}{log(n)} \tag{11}$$

According to Gottwald and Melbourne [29], *K* may be 0 in regular systems but must be 1 in chaos systems.

Figure 9 demonstrates that our system exhibits greater chaos compared to sine, tent, and logistic maps across all β values. While our map maintains a value of 1 throughout the entire range of β, the other maps reach a value of 1 only within a limited range.

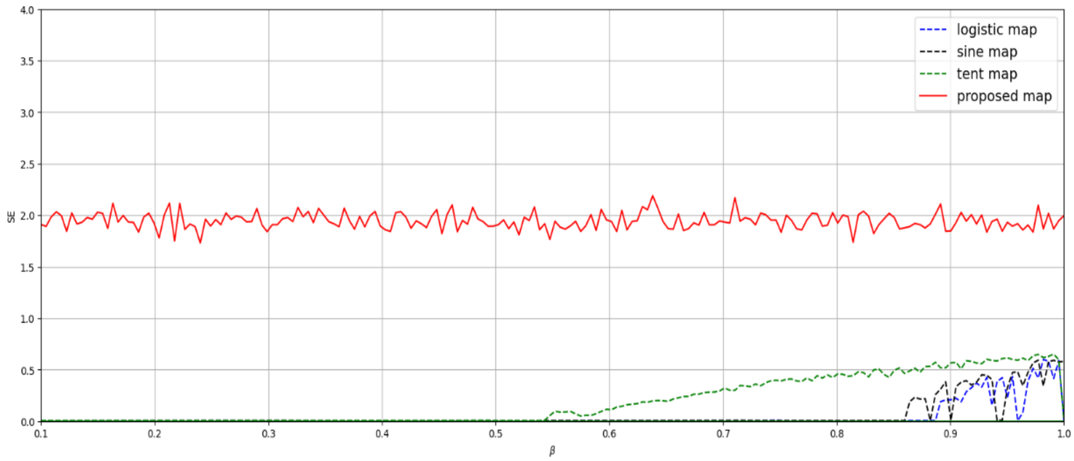

Figure 8. Sample entropy evaluation results

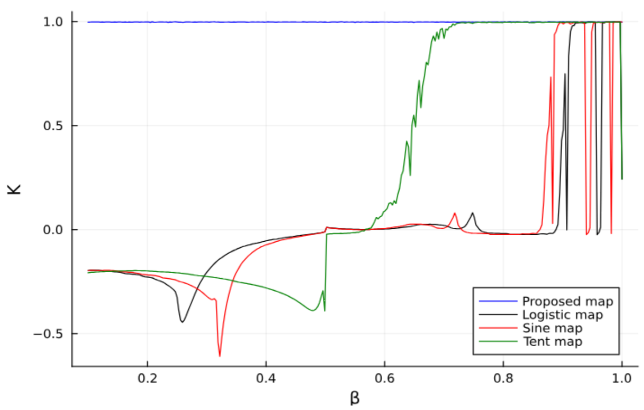

Figure 9. 0-1 evaluation results

4. CONCLUSION

In conclusion, the proposed one-dimensional chaotic map represents a significant advancement over classical maps like the sine map by addressing their inherent limitations. By introducing an additional parameter and modifying the mathematical structure, the new map enhances chaotic behavior and broadens the interval's rate variable, thereby increasing its utility in encryption and security applications. The comprehensive evaluation, including tests such as fixed points and stability analysis, Lyapunov exponent analysis, and sensitivity to initial conditions, demonstrates the map's improved performance and robustness. We plan to leverage these characteristics to develop a new cryptosystem that takes full advantage of the map's capabilities to encrypt images and speech. Additionally, it can enhance the effectiveness of image steganography.

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