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Modelling and controlling outputs of nonlinear systems using feedback

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ABSTRACT

This study aimed to analyze methods for modeling and controlling the output of nonlinear systems using feedback, analytical methods, mathematical modeling, and differential equation theory. Key findings include the mathematical characterization of equations and the analysis of system stability and asymptotic behavior. The study explored various methods for addressing problems in nonlinear systems, emphasizing the importance of identifying effective solutions. The research highlights the significance of developing effective approaches to solving complex problems involving nonlinear systems. Feedback is essential for controlling and correcting dynamic processes in systems with nonlinearities. The study's key finding is the mathematical characterization of equations describing nonlinear systems, providing insight into system structure and behavior under different parameters. Analyzing stability and asymptotic behavior allows for assessing system reliability and predicting long-term stability. This study contributes to the scientific understanding and development of methods for modeling and controlling nonlinear systems using feedback.

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1. INTRODUCTION

This study emphasizes the importance of quality systems management in a rapidly changing environment with dynamic events. The rapid pace of technological development, global variables, and frequent changes in conditions necessitate adaptive and flexible systems. Modeling and feedback are crucial tools for creating adaptive strategies that can respond effectively to these changes. Effective adaptive strategies enable systems to adapt quickly to new environments, minimize risk, and maximize performance. However, managing and modeling nonlinear systems in unpredictable and dynamic contexts can be challenging. The study aims to analyze methods for modeling and controlling output of nonlinear systems using feedback, analytical methods, mathematical modeling, and differential equation theory. Understanding the effectiveness and limitations of numerical methods will provide a framework for selecting and combining these techniques to develop robust and adaptive management strategies for nonlinear systems.

Nonlinear systems exhibit a variety of dynamic properties, such as instability, the possibility of chaotic modes, and nonlinear feedback [1]. Effective management strategies require considering non-linear relationships between system components, uncertainties, external influences, and noise. According to Liu *et al.* [2], nonlinear differential equations appear in many fields and are notoriously difficult to solve.

Researchers have developed the first quantum algorithm for dissipative nonlinear differential equations, which is efficient if the dissipation is strong and the solution doesn't decay rapidly.

Awan *et al.* [3] showed a mathematical model in the form of a partial differential equation (PDE) that is constructed under certain assumptions. The study transforms nonlinear PDEs into dimensionless ordinary differential equations (ODEs) using the MATLAB bvp4c numerical method. The results show that physical parameters impact dimensionless profiles of concentration, temperature, micro polarity, velocity, and induced magnetic field. Alimhan *et al.* [4] considered the problem of global practical tracking results for a class of uncertain high-order nonlinear systems with time delay via state feedback. A uniform state feedback controller with adjustable scaling factor was constructed using the uniform dominance method, focusing on delay-related nonlinearities.

Leylaz *et al.* [5] proposed a technique for the identification of nonlinear dynamical systems with time delay. The sparse optimization algorithm is extended to nonlinear systems with time delay, combining machine learning cross-validation and algebraic operations for signal preprocessing. According to Shu and Zhai [6], closed-loop feedback-based structures are adopted in many control systems because feedback systems can effectively constrain the change of system parameters. Researchers studied dynamic event-driven feedback output control for nonlinear systems under homogeneous growth conditions. They developed a new outgoing feedback control law to ensure bounded system signals and global inclusion of system states in a compact set around the origin.

Zhang *et al.* [7] presented a scheme with two adjustable design parameters based on Lyapunov functional results for the state-entry stability of time-delayed systems. The proposed trigger event control algorithm ensures that finite closed-loop systems are globally asymptotically stable, uniformly bounded, and/or globally attractive for different variants of these parameters. Sufficient conditions for the parameters are derived to rule out Zeno behavior. Two illustrative examples are considered to present the theoretical results. Ma *et al.* [8] considered the topic of fault-tolerant adaptive neural network control for a class of ambiguous switched nonlinear systems with limited feedback and unmodeled dynamics and unmeasurable states. According to the study, in such systems, the uncertain nonlinear components are identified by radial basis function neural networks. The study does not present a comparison of the proposed method with existing control methods for nonlinear systems.

The purpose of the current study was to analyze methods of modeling problems in nonlinear systems using numerical methods. The completion of this task will allow for the investigation the efficiency and stability of numerical methods. The study emphasizes the importance of parameter tuning and the use of MATLAB for solving complex nonlinear differential equations. For the purpose of describing and regulating nonlinear systems with feedback, this research integrates the finite element method, the finite difference method, and the optimal control approach. In contrast to earlier research, which frequently concentrated on a single technique or a particular facet of nonlinear systems, this study offers a comparative evaluation of all three approaches, stressing their unique advantages and disadvantages. In order to improve system management in dynamic and unpredictable contexts, the paper presents a framework for choosing and combining these techniques depending on job needs. This comprehensive way of assessing and utilizing various numerical techniques advances the field of control of nonlinear systems.

2. LITERATURE REVIEW

According to Saeedi *et al.* [9], in recent years, the growing interest in networked control systems (NCS) and cyber-physical systems (CPS) has been caused by the rapid development of digital communications. These new domains are an integration of physical systems, digital controllers, and tools that interact with each other through a common cyber layer to achieve their goals. The emergence of such systems has stimulated the active development of research in the analysis and management of critical infrastructures such as transport systems and energy networks. Modeling nonlinear systems using feedback is an essential aspect in the field of systems analysis and control. Nonlinear systems, unlike linear systems, exhibit complex behavior, which makes them more difficult to analyze and control.

According to studies [10], [11], modern analysis and control methods include the application of various mathematical models, optimization methods, control theory, and artificial intelligence. These approaches allow for complex interactions between system components as well as dynamic changes in the environment to be considered. Moatimid and Amer [12] and Cheng *et al.* [13] consider different approaches to modeling and controlling nonlinear systems, such as differential equation-based methods, stochastic models, neural networks, and hybrid methods. Methods based on differential equations include the use of well-known models such as Lorentz or Van der Pol models, as well as various methods for analyzing the stability and controllability of systems [14], [15]. Stochastic models allow accounting for random effects and uncertainties, which is especially important when modeling real systems with various sources of noise and disturbances. Huijgevoort *et al.* [16] presented a tool for the synthesis of controllers for stochastic continuous state systems, considering the requirements of temporal logic. The tool provides the necessary functions for synthesizing robust controllers and defining formal reliability guarantees. Its specific feature lies in the support of nonlinear dynamics, complex temporal logic specifications, and down-ordering of the model.

Rober *et al.* [17] argued that neural networks are becoming increasingly popular in modeling nonlinear systems because they can approximate complex nonlinear dependencies and learn from the available data. Hybrid methods combine different approaches to obtain more efficient and accurate models and control algorithms. All these approaches play a significant role in the development of network management and cyber-physical systems, ensuring their reliable operation and resilience to various external influences. According to Choi and Yoo [18], nowadays there is a significant spread in the use of interconnected nonlinear systems in engineering and information practice, which requires the development of control methods based on decentralization. It forms an integral part of the current trend in various fields, including robotics, process automation, power grid management, and even financial markets. However, the management of such systems is challenging due to their non-linearity and the interrelationships between the different components. Conventional centralized management methods may be ineffective or even inapplicable in such cases as they involve centralized management of all aspects of the system, which may not be possible or effective due to the high degree of interdependence and dynamic nature of non-linear systems.

3. MATERIALS AND METHODS

The study employed the analytical method, which allowed for in-depth analyses. It involved the application of various mathematical techniques and formulas to analyze the equations of the system under study. Using an analytical approach, a detailed mathematical characterization of the solutions to the equations was carried out, allowing a theoretical study to be carried out. This method has helped to better understand the basic properties of solutions, their asymptotic behavior, structure, and impact on systems and processes in a practical research framework. Various definitions and assumptions have been used in this study, and high-order nonlinear systems with delay [19] (1):

$$\dot{x}_i(t) = x_i + 1(t)^{\wp i} + \phi_i\big(t, x(t), x(t-d), u(t)\big), i = 1,\ldots,n-1,\ \dot{x}_n(t) = u + \phi_n\big(t, x(t), x(t-d), u(t)\big), y(t) = x_1(t) - y_r(t), \tag{1}$$

where: $x(t) = (x_1(t),\ldots,xn(t))T \in Rn, u(t) \in R$ and $y(t) \in R$ are states, input and output of the system; x_1, x_2, x_3 are state variables of the specific nonlinear system example; d are delay term in the system; $y_r(t)$ are reference signal for the output.

This type of system is often encountered in modeling dynamic processes where nonlinearities and delays can have a substantial impact on the dynamics of the system.

Assumption 1. There exist constants and C_1, C_2 and $\tau \geq 0$ such that (2):

$$|\varphi_i(t, x(t), \bar{x}(t-d_i), u(t))| \leq C_1 \left(|x_1(t)|^{\frac{r_i+\tau}{r_1}} + \cdots + |x_i(t)|^{\frac{r_i+\tau}{r_1}} + |x_1(t-d_1)|^{\frac{r_i+\tau}{r_1}} + \cdots + |x_i(t-d_i)|^{\frac{r_i+\tau}{r_1}} \right) + C_2, \tag{2}$$

where: $\bar{x}(t-d_i) = x(t-d_1), x(t-d_2), \ldots, x(t-d_n), r_1 = 1, r_{i+1}p_i = r_i + \tau > 0, i = 1, \ldots, n,\ p_n = 1;$ C_1, C_2 – constants used to bound the nonlinear functions; τ – delay term coefficient.

Assumption 2. The reference signal $y_r(t)$ is continuously differentiable. Moreover, there exists a known constant $D > 0$, such that (3):

$$|y_r(t)| + |\dot{y_r}(t)| \leq D, \forall t \in [0, \infty). \tag{3}$$

where: $\dot{y}_r(t)$ – derivative of the reference signal.

Theorem 1. Under assumptions (1, 2), the global practical output tracking problem of system (1) can be solved using a controller with feedback $u = L^{k_n+1}v$ on a continuous state of the form (2). The proof of the theorem can be seen in Alimahan *et al.* [4].

The theory of differential equations allowed for the analysis the stability of the system. This is important for determining how the system responds to perturbations and changes in input signals. It also helped to investigate the asymptotic behavior of the system. Numerical methods such as finite element and difference methods have been used to solve problems with nonlinear systems. MATLAB programming environment was used to design and implement the numerical methods, perform numerical experiments, and analyze the results. MATLAB provides a wide range of tools and functions for working with numerical methods and data processing, which has greatly simplified the research process.

The finite element method has been used to analyze the structure and dynamics of the system and to optimize its performance considering feedback. The finite element method made it possible to create detailed mathematical models of nonlinear systems, considering complex physical interactions. The finite element method was used to determine the best parameters of the system, considering feedback. The finite difference method was applied to approximate the differential equations and investigate the dynamics of the system in discrete time. The finite difference method was used to discretize the differential equations of the system, which helped to obtain the difference equations. Evaluation of the sensitivity of the system to parameter changes using numerical differentiation. The finite difference method accounted for nonlinearities in the system, such as the nonlinear elastic properties of materials.

Mathematical modeling techniques have been used to describe problems more accurately and completely with a nonlinear system. Specifically, the optimal control method was applied, which provided a mathematical framework to optimize the control of the system. The optimal control method was applied to optimize control signals to achieve certain criteria, such as minimizing costs, maximizing performance, or achieving certain system states. The optimal control method accommodated the nonlinear components in the system, allowing for more accurate modeling and control of systems with nonlinear behavior.

4. RESULTS AND DISCUSSION

4.1. Theoretical framework and analytical approach

The study focuses on understanding nonlinear systems problems using feedback by learning the theory of numerical algorithms. It includes stability and controllability analyses, mathematical methods for solving these problems, and the development of efficient control algorithms. Understanding numerical algorithms is crucial for modeling these problems, as it allows for the development of efficient algorithms considering the specific problem's characteristics and requirements. This includes selecting and optimizing numerical methods, choosing the right mesh and discretization interval, and analyzing the accuracy and stability of numerical solutions. This knowledge enables the application of numerical methods with high confidence and efficiency in solving nonlinear systems with feedback. Analytical analysis is used to identify the main properties of solutions, investigate their asymptotic behavior, structure, and effects on systems and processes. This analytical approach provides a powerful tool for analyzing system behavior and key characteristics.

Differential equation theory plays a key role in solving nonlinear systems using feedback. Differential equations provide a mathematical toolkit to describe the dynamic behavior of a system. In the context of nonlinear systems, they helped to account for nonlinear dependencies between system state variables and time. The theory of differential equations has made it possible to model the complex interactions between the various components of a system. This is important when analyzing non-linear systems where the influence of variables is complex and volatile. Differential equations are widely used in optimal control theory. This allowed finding optimal strategies to manage the system to achieve certain criteria, such as minimizing costs or maximizing performance. Differential equations are easily integrated with feedback theory. Feedback control allowed real-time adjustment of system parameters based on measurements of the current system state.

Mathematical and computer modeling techniques play an essential role in solving nonlinear systems using feedback. Mathematical modeling helped to accurately describe the system, considering all interactions between its components. For nonlinear systems, this is important because they exhibit complex and nonlinear dependencies. Under ideal conditions where an analytical solution is possible, mathematical modeling provides accurate and analytical expressions for system states and control actions. Computer simulation helped to approximate the real behavior of the system, considering its complexity and dynamic changes. It provided the flexibility to introduce changes to the model, allowing it to be adapted to different environments and scenarios. Computer models were used to perform sensitivity analyses, accounting for uncertainties in the system parameters and assessing their impact on the results.

4.2. State feedback tracking control scheme

Global practical tracking problem using state feedback: system (1) was considered, and it was assumed that the reference signal $y_r(t)$ was a time-varying C^1-bounded function on $[0, \infty)$. For any given $\varepsilon > 0$, a state feedback controller is designed with the following structure (4):

$$u(t) = g(x(t), y_r(t)). \tag{4}$$

The practical problem of output tracking using delay-independent state feedback for high-order nonlinear systems with delay (1) under assumptions (1, 2) was considered. For this, the following coordinate transformation is first introduced (5):

$$z_1 = x_1 - y_r,\ z_i = \frac{x_i}{L^{k_i}}, i = 2, \ldots, n, \nu = \frac{u}{L^{k_{n+1}}}. \tag{5}$$

where: z_i is transformed state variables; $L^{k_i}, L^{k_{n+1}}$ are scaling factors; ν is transformed control input.

Then the system (1) was described in new coordinates (6):

$$\begin{aligned} \dot{z_\iota} &= Lz_{i+1}^{p_i} + \psi_i(t, z(t), z(t-d), \nu), i = 1, \ldots, n-1, \\ \dot{z_\iota} &= L_\nu + \psi_n(t, z(t), z(t-d), \nu), y = z_1. \end{aligned} \tag{6}$$

Using assumption (1), the fact that $L \geq 1$ and the boundedness of y_r and $\dot{y_r}$, guaranteed by assumption (2), ensures the existence of constants $\overline{C_\iota}, i = 1,2$ only depending on constants $C_1, C_2, \tau, k_{i,}$ and L, at which (2) becomes (7):

$$|\psi_1(t, z(t), z(t-d), \nu)| \leq \overline{C_1}\left(|z_1(t)|^{\frac{r_1-\tau}{r_1}} + |z_1(t-d)|^{\frac{r_1-\tau}{r_1}}\right) + \overline{C_2}|\psi_1(t, z(t), z(t-d), \nu)| \leq$$
$$\overline{C_1}L^{1-\nu_i}\sum_{j=1}^{i}\left(\left|z_j(t)\right|^{\frac{r_1-\tau}{r_j}} + \left|z_j(t-d)\right|^{\frac{r_1-\tau}{r_j}}\right) + \frac{\overline{C_2}}{L^{k_i}}, i = 2, \ldots, n. \tag{7}$$

For the stability analysis as well as the tracking system design for a nonlinear system with delay, see Alimhan *et al.* [20]. To investigate the modeling of nonlinear systems problems in greater depth, various methods were implemented to understand each method in more detail, namely the finite element method, the finite difference method, and the optimal control method. The global practical problem of tracking the performance of nonlinear systems is one of the key and most challenging issues in the field of nonlinear control. Alimhan *et al.* [21], Tognetti and de Oliveira [22] proposed an approach to the design of a controller with output feedback for a class of high-order nonlinear systems with delay. It was shown that the proposed output controller, which is independent of time delay, can make the tracking error small and reflect the whole trajectory of the closed-loop system as bounded. Wang *et al.* [23], Cui *et al.* [24], and Jiang *et al.* [25] proposed an adaptive control approach, backstepping technique, and finite-time stability theory, and developed an adaptive finite-time tracking controller. The proposed control scheme ensured the performance of elapsed time tracking and the boundedness property of all signals in a closed-loop system.

4.3. Implementation of methods for modelling nonlinear systems

To model the problem with nonlinear systems, we initially set the parameters, initial conditions, and time interval. The authors used MATLAB's built-in finite element method to approximate the differential equations and analyze the system's dynamics. Each finite element represented a part of the system, with discretization enabling the system to be viewed as interconnected elements. The code constrains the derivative values based on assumption (2) and is designed for easy parameter and function customization. For the second method, the finite difference method, the authors modeled the same problem with identical parameters, initial conditions, and time intervals. This method approximates the differential equations in discrete time using local difference approximations of derivatives. The code also constrains derivative values according to assumption (2) and allows for straightforward customization of parameters and functions.

In the implementation of the third method, namely the optimal control method, the same problem in general form was considered. The same initial conditions, time interval, and parameters were set, and the same programming environment was used. The function defined the system equations, describing the dynamics of the system. In this example, the equations are nonlinear, but they can be replaced by the appropriate equations for the particular system. This code provides a general framework for solving the optimal control problem and can be adapted to the specific requirements of a given problem. The following nonlinear system (8) was also considered:

$$\dot{x_1}(t) = x_2^{\frac{5}{3}}(t) + x_1^{\frac{1}{3}}\left(t - \frac{\sin(t)}{5}\right)\sin\left(x_1(t)\right), \dot{x}_2(t) = x_3^{\frac{7}{3}}(t) + 2x_2(t), \dot{x}_3(t) = u(t) +$$
$$2x_3^{\frac{7}{3}}(t), y(t) = x_1(t). \tag{8}$$

When the scaling factor L=100, the obtained tracking error is about 0.41 as presented on Figure 1. When the scaling factor L=400, the obtained tracking error is about 0.17 as presented on Figure 2.

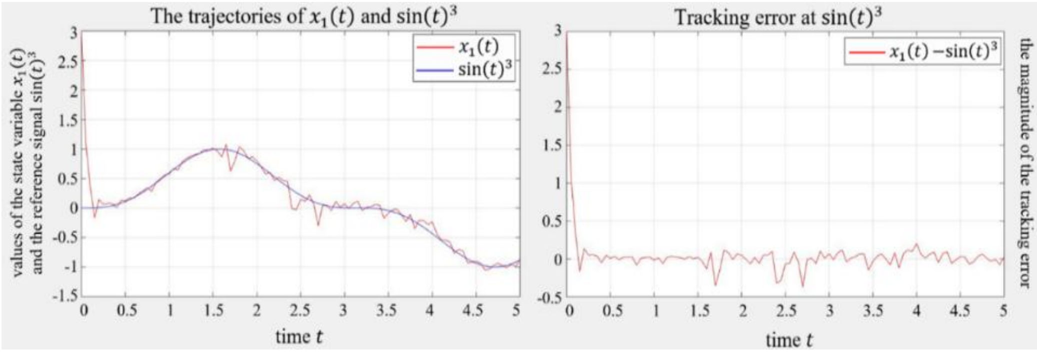

Figure 1. Tracking error at L=100

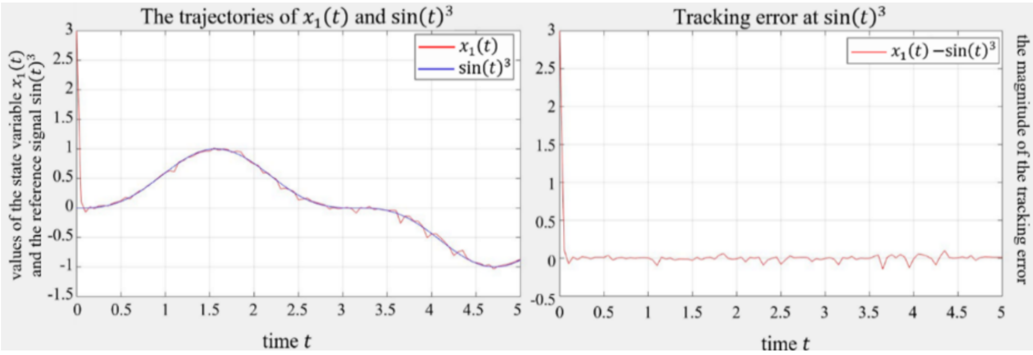

Figure 2. Tracking error at L=400

A mathematical model was used in the study to create a nonlinear system, and a reference signal and state variable were selected for tracking. To guarantee that the output of the system closely follows the reference signal, a state feedback controller was created. Using various scaling factors, simulations were performed throughout a time range of 0 to 5. The system's requirements were taken into account when setting the initial circumstances and settings. The differential equations were solved numerically using techniques like the finite element and finite difference methods. For effective computations, the built-in functions of MATLAB were utilized. Plotting the $x_1(t)$ and $y_r(t)$ trajectories allowed observers to see how the influence affected tracking performance. The tracking error was calculated and shown to give an indication of how accurate the state feedback controller was. It is defined as the difference between $x_1(t)$ and $y_r(t)^3$. As a result, Figures 1 and 2 were compiled to evaluate tracking performance with various scaling factors.

4.4. Analysis of mathematical modelling and control methods in nonlinear systems

The finite element method is a versatile approach for solving nonlinear systems, suitable for large-scale and complex systems, particularly in time-based dynamic processes. However, it can be computationally expensive and requires careful boundary conditions definition. The finite difference method is efficient for uniform grid solutions but decreases efficiency in unstructured meshes, discretization step choices, complex geometries, and interactions with unstructured meshes. The optimal control method is used for optimizing nonlinear systems and control problems, considering control signals' optimality based on specific criteria like cost reduction or system state attainment. Figure 3 provides a structured overview of methods and techniques used in the analysis and control of nonlinear systems.

Figure 3 provides a comprehensive overview of techniques and strategies used in feedback-based analysis and control of nonlinear systems. It emphasizes the importance of a diverse approach to handle the complexity of these systems, enabling the development of effective control plans and algorithms, ultimately improving nonlinear system management and optimization. This study analyzed various methods for modeling complex systems, focusing on their key features and benefits in Table 1. The effectiveness of different modeling methods, focusing on finite element, finite difference, and optimal control methods were identified.

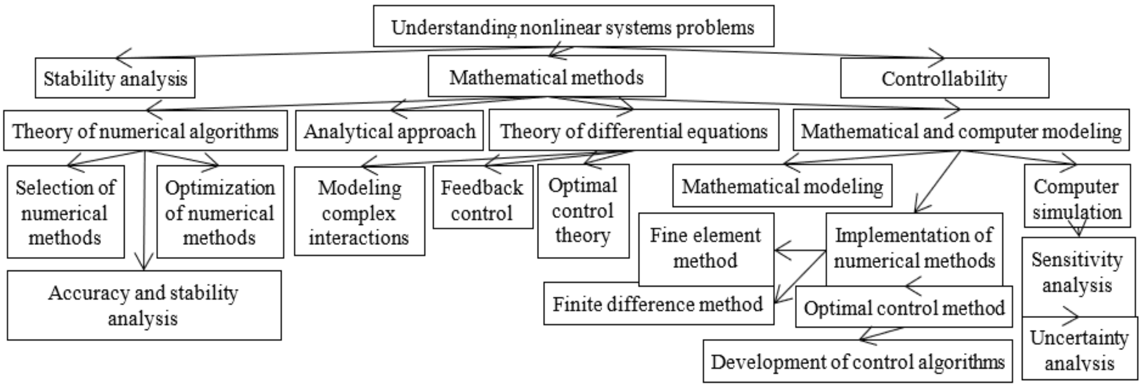

Figure 3. Methods for solving nonlinear systems using feedback

Table 1. Comparative analysis of methods

Criteria	Finite element method	Finite difference method	Optimal control method
Flexibility	High	Moderate	High
Ease of implementation	Moderate	High	Low
Handling of complex geometries	High	Moderate	High
Computational efficiency	Moderate to low	High on uniform grid	Low to moderate
Adaptability to changes	High	Moderate	High
Optimality in control	Moderate	Low	High
Handling of nonlinear interactions	High	Moderate	High
Sensitivity to initial conditions	Moderate	High sensitivity	High sensitivity
Scalability	High	Moderate	Low to moderate
Handling of boundary conditions	Requires careful definition	Moderate difficulty	Requires prior knowledge
Accuracy in modelling	High	Moderate	High
Stability of Solutions	Moderate to high	Moderate	High

The detailed evaluation of the finite element approach, the finite difference method, and the optimal control method allowed the study to fulfill its goal of identifying and analyzing efficient methods for modeling nonlinear systems. The capacity of each technique to manage the innate complexity of nonlinear systems, such as instability, chaotic modes, and nonlinear feedback, was carefully examined. The results corroborated the notion that some approaches would be more appropriate for complex systems by showing how flexible and adaptable the finite element method is to complex geometries and interactions. The study highlights the importance of using feedback in modeling and controlling inputs in nonlinear systems problems. It highlights the use of advanced methods like finite element, finite difference, and optimal control methods for efficient analysis of various systems, including those with nonlinearities and time delays.

4.5. Computational techniques and their efficacy in nonlinear systems

This study utilized the MATLAB programming environment to develop and implement numerical methods, simplifying the process of analysis and experiments. MATLAB's data processing and visualization capabilities helped efficiently analyze results [26]. The finite element method was used to analyze system structure and dynamics, considering feedback. This method allowed for detailed mathematical models of nonlinear systems, allowing for complex physical interactions. The application of this method to optimize system parameters improved system efficiency, considering feedback [27]. The study explores the advantages and disadvantages of different methods for modeling and controlling nonlinear systems in dynamic and uncertain situations. The finite element approach is flexible but computationally intensive, while the finite difference method is effective but has issues with unstructured meshes [28], [29].

The finite difference method is a discrete-time system dynamics study that generates difference equations for numerical simulations. It considers nonlinearities in the system, including materials' nonlinear elastic properties [30], [31]. The optimal control method offers a mathematical framework for optimizing control of nonlinear components, aiming for cost reduction or system performance maximization, thus improving modeling accuracy and control efficiency [32]–[34]. The current research emphasizes the need for a combined approach that leverages the strengths of each method to enhance the robustness and efficiency of systems management in rapidly changing conditions. The computational demands of the finite element method and optimal control techniques, the finite difference method sensitivity to discretization choices, and the general difficulty of properly modeling extremely complex nonlinear systems are some of the study’s limitations.

Study by Perrusquía and Yu [35] on identification and optimal control of nonlinear systems used recurrent neural networks and reinforcement learning for both discrete and continuous time. Researchers discovered that neural networks can approximate dynamical systems using identified elements and different structures. They also explored methods using Lyapunov and Riccati equations to derive neuron update rules. Both studies, which incorporated control theory and numerical methods like recurrent neural networks and optimization techniques, significantly contribute to the field of control and identification of nonlinear systems. Wang *et al*. [36] proposed an adaptive neural network for tracking nonlinear systems with multiple driving constraints. The researchers developed a method to eliminate actuator nonlinearity interference and address difficulty explosion issues. Both studies address nonlinearity in systems using innovative approaches, focusing on adaptation and output management. They also use neural network approximations to improve control efficiency.

Alsalti *et al.* [37] presented an extension of Willems' fundamental lemma to the class of linearized nonlinear discrete feedback systems with multiple inputs and multiple outputs, thereby providing a data-driven representation of their input-output trajectories. Two sources of uncertainty are considered. Both studies focus on feedback in the context of controlling system dynamics. The researchers focus on linearized nonlinear systems with discrete feedback, which is an essential aspect under conditions of limited data availability. Both studies also address the problem of uncertainty. The researchers consider two sources of uncertainty, which emphasizes the significance of considering and managing uncertainty when developing control strategies.

The study of nonlinear systems problems using feedback is a crucial tool in modern engineering and science. Advanced techniques like the finite element method, finite difference method, and optimal control method offer efficient modeling and analysis of various systems, including those with nonlinearities and time delays. The study validates the original theory by showing that each approach has distinct benefits and drawbacks. The study suggests that improved nonlinear system management in dynamic and unpredictable situations can be achieved by combining different strategies according to task-specific needs.

5. CONCLUSION

This study highlights the significance of flexible and adaptive quality systems management in an environment that is changing quickly due to global unpredictability, frequent changes, and technology advancements. It emphasizes how crucial modeling and feedback are to developing adaptive techniques that may successfully adjust to these changing circumstances. The paper offers a thorough overview of modeling techniques for nonlinear systems, with a focus on numerical techniques due to their stability and efficiency. It also emphasizes the value of using MATLAB and parameter adjustment while attempting to solve complicated nonlinear differential equations. The use of the finite difference and finite element methods provides information on the benefits and drawbacks of each technique. The finite element method turns out to be a flexible and powerful tool for modeling systems with complex geometry and interactions. The finite difference method is simple to implement and is well-suited for approximating differential equations in discrete time. The optimal control method provides the ability to optimize the control of a system to achieve given objectives. A comparative study of various approaches showed that the particular needs of the job have a major role in the technique selection. The study came to the conclusion that a combined strategy that made use of each method's advantages may offer a more adaptable and effective solution for complicated nonlinear systems.

The findings highlight the necessity for adaptive solutions in system management, particularly in situations marked by fast change and uncertainty. They have important implications for the area of study as well as for the wider community. In order to provide more precise and adaptable answers to new problems, future research could investigate more sophisticated numerical techniques, create new algorithms to overcome limitations, and include real-time data and machine learning approaches.

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