Method of undetermined coefficients for circuits and filters using Legendre functions

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ABSTRACT

This article presents a new way to implement matching networks and filters using the method of undetermined coefficients. A method is proposed for approximating the transmission coefficient of the synthesized filter, taking into account the required amplitude-frequency characteristics. To synthesize the filter, an approximating function (AF) was used using orthogonal Legendre polynomials, which is a mathematical description using a system of equations. Filter properties whose implementation is based on modified Legendre approximating functions usually depend on the interval on which they are defined and have the property that they are orthogonal on this interval. An example of seventh order filter synthesis using modified Legendre approximating functions is given. The filter circuit is implemented, the elements of the filter circuit are calculated based on the selected approximating modified function. The criteria used were minimization of the unevenness of the group delay time (GDT) and minimization of the complex approximation error for given values of the AF parameters. As a result, the number of filter elements, the group delay value and the complex approximation error are significantly reduced.

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1. INTRODUCTION

The study of broadband matching-filtering circuits [1]–[3] is of great practical importance due to the growing number of wireless communications devices from individual ones (navigation devices and mobile gadgets) to space and terrestrial satellite communication systems [4]–[6]. Also, the enormous progress in the technology of satellite and mobile telecommunication systems is largely associated with new developments of electrical filters and matching circuits [7]–[9], which serve as important components in modern communication systems and as one of the key components of interface modules. Requirements for matching-filtering circuits are constantly growing. For example, the 5G communication system standard implies expansion of the operating frequency band [10]–[12], and this characteristic largely depends on the parameters of the filtering circuits used. Filter-matching circuits are becoming increasingly important as they are used in more and more different applications, and with the evolution of 6G technology, the operating mode and standard of the communication system will become increasingly diversified [13], [14]. Filters that meet various requirements are of great importance when developing devices with high functionality (communication devices, and unmanned aerial vehicles). Therefore, the search for new ways to improve the functional characteristics of filters is an urgent problem in modern electronics and communications. The article aims to present a new method for direct filter synthesis by solving a system of nonlinear equations using modified Legendre approximating functions [15], [16].

Researchers continuously continue to search for new effective approaches and methods to achieve the desired characteristics of matching-filtering circuits. Many different broadband matching and filtering circuits have been proposed for wireless systems. By combining different design concepts and material choices, wideband or ultra-wideband bandwidth performance can be achieved. The main efforts of the developers of these elements in the last two decades have been aimed at increasing their selectivity, simplifying the design and structure, and reducing insertion losses and spectral distortions of signals with minimal weight and dimensions [17]. One of the directions of these efforts was the emergence of new mathematical functions that approximate the frequency characteristics of filters and matching circuits (modified functions). In most cases, approximation is performed using polynomials. An essential feature of these functions is the presence of transmission zeros in the frequency region adjacent to the passband (built-in transmission zeros) [18]–[20]. Along with all the advantages, when using such functions, some problems arise at the stage of their implementation, namely at the synthesis stage. Finding a quadrupole of a canonical form using classical methods [21], [22], using modified functions, becomes difficult due to the significant complication of mathematical calculations.

2. METHOD

Particular attention is paid to analytical approaches related to precise synthesis methods since they have a rigorous solution, as well as the ability to obtain matching devices that form the initially specified frequency characteristics of power transmission [23]. The approximating function can be any corresponding function, such as a Legendre polynomial that satisfies the conditions of practical realizability and modified associated Legendre polynomials of the m-th series [24]. Legendre polynomials reduce the solution of various problems in engineering and scientific applications to the solution of a system of algebraic equations, thereby greatly simplifying the problem. The properties of polynomials usually depend on the interval over which they are defined. Legendre polynomials, as special cases of Jacobi polynomials, are orthogonal on the interval $[-1, 1]$ concerning the weight function $(1 - x2)\ \alpha - 1/2$. They can be used to approximate functions defined on this interval [25].

The proposed filter synthesis options must be adequate for the selected implementation technology. An important circumstance for the synthesis of matching circuits is the choice of an analytical implementation method, which allows solving broadband matching problems for arbitrary complex loads using both classical and modified approximating functions. The approximation problem is the first step in the synthesis of frequency-selective circuits, which can be synthesized in a variety of ways. This circumstance makes it possible to use, to select from a set of options, the best one according to some criterion, which will allow for optimal synthesis, as a result of which one or a set of parameters (characteristics) of the circuit is optimized (minimized or maximized).

To solve the problem, it is proposed to use direct synthesis by solving a system of nonlinear equations. The initial data for composing the system of equations is the input resistance obtained from the approximating function (1) and the resistance of a quadrupole of a canonical form, which has the order of the approximating function taking into account the number of transmission zeros (2).

$$Z_{\text{BX}}(s) = \frac{a_0 + a_1 s + a_2 s^2 + a_3 s^3 + \ldots + a_{\text{n}} s^n}{b_0 + b_1 s + b_2 s^2 + b_3 s^3 + \ldots + b_{\text{n}} s^n}, \tag{1}$$

where n is the order of the approximating function. To find the resistance of a quadripole network of a canonical form, it is necessary to specify the structure of a circuit in which the number of arms coincides with the order of the approximating function. The diagram must have a ladder structure and be symmetrical. The number of transmission zeros determines the number of resonant arms in the synthesized circuit. Examples of constructing fifth-order circuits are presented in Figure 1.

After determining the structure of the circuit, you can ascertain the resistance of the four-terminal network. This process involves analyzing the coefficients of the variable s, as they represent the necessary parameters for the circuit. These coefficients are directly influenced by the values of the elements within the synthesized circuit. By examining these coefficients, you can gain a clear understanding of how the resistance is distributed and how it impacts the overall functionality of the four-terminal network.

$$Z_{\text{вх,ц}}(s) = \frac{a_{0ц}+a_{1ц}s+a_{2ц}s^2+a_{3ц}s^3+\ldots+a_{nц}s^n}{b_{0ц}+b_{1ц}s+b_{2ц}s^2+b_{3ц}s^3+\ldots+b_{nц}s^n}, \quad (2)$$

To compose nonlinear (3), it is necessary to equate the coefficients of polynomials (1) and (2):

$$\begin{gathered} a_{0ц} = a_0, a_{1ц} = a_1, a_{2ц} = a_2, a_{3ц} = a_3, \ldots, a_{nц} = a_n \\ b_{0ц} = b_0, b_{1ц} = b_1, b_{2ц} = b_2, b_{3ц} = b_3, \ldots, b_{nц} = b_n, \end{gathered} \quad (3)$$

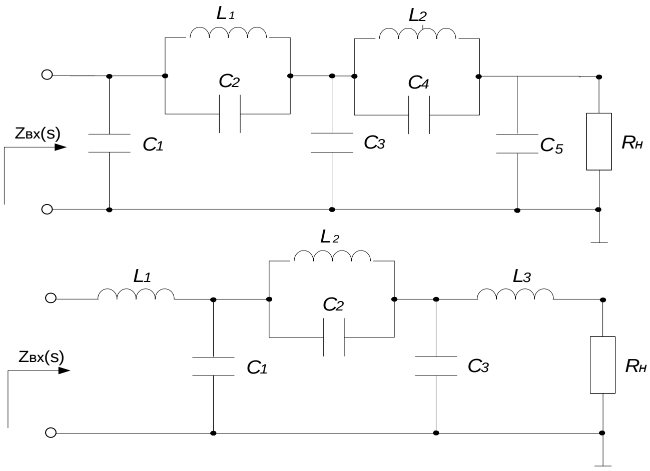

Figure 1. Examples of constructing a fifth-order circuit

Thus, by solving a system of nonlinear equations, we find the values of the parameters of the synthesized circuit. As a rule, when choosing an approximation of the amplitude-frequency response of a filter, preference is given to power functions, as in Butterworth filters, and Chebyshev polynomials of the first kind. Also, one of the well-known ways to calculate a low-pass filter is to use Legendre functions. Let's look at an example of calculating a low-pass filter using Legendre functions. The transfer function of the synthesized filter, when using approximating Legendre polynomials, gives the frequency response high linearity. This means that the unevenness of the frequency response will be limited compared to the use of Chebyshev polynomials of the first kind with the same filter orders. In addition, the use of Legendre functions reduces the group delay time compared to the use of power Butterworth functions. This also provides a gain in filter order, making the method more efficient (4).

$$K_m(-s^2) = \frac{k\prod_{q=1}^{n}(s_q^2-s^2)^2}{\prod_{q=1}^{n}(s_q^2-s^2)^2+\varepsilon^2\prod_{q=1}^{n}\left(s_q-1\right)^2 P_{\lg}^2(s)}, \quad (4)$$

where P_{lg} is the Legendre correction polynomial; s is coordinates of the input transmission zeros. The matching network circuit is first selected according to the requirements to match the load. To implement the filter, we set the requirements for the frequency response of the implemented filter: cutoff frequency 50 MHz; the filter order should not exceed 7; the attenuation at frequency $1.2\omega_c$ must be at least 30 dB, and at frequency $1.4\omega_c$ at least 55 dB; unevenness in the filtering band should not exceed 0.5 dB. To solve this problem, it is necessary to search for the coordinates of the input zeros of the transfer of the modified approximating function with 7th order Legendre correcting polynomials, at which the established requirements will be most accurately met. Research has shown that the same slope of the amplitude-frequency response of the filter is provided by the Legendre polynomial at order $n_{lej} = 7$. For this purpose, we use expression (5) for the desired frequency band:

$$I = \int_0^1 \left|1 - K_p(s)\right| ds + \int_1^{1.4} \left|0 - K_p(s)\right| ds \to \min \quad (5)$$

The surface describing the value of the complex approximation error is shown in Figure 2. Frequency $1.4\omega_c$ is optimal for the location of zeros on the s-plane according to a given criterion, maximum linearity in the filter passband. This optimal frequency is determined according to a specific criterion, which prioritizes maximum linearity in the filter's passband. By achieving this linearity, the filter performs more effectively within the desired frequency range, reducing distortion and improving overall signal quality.

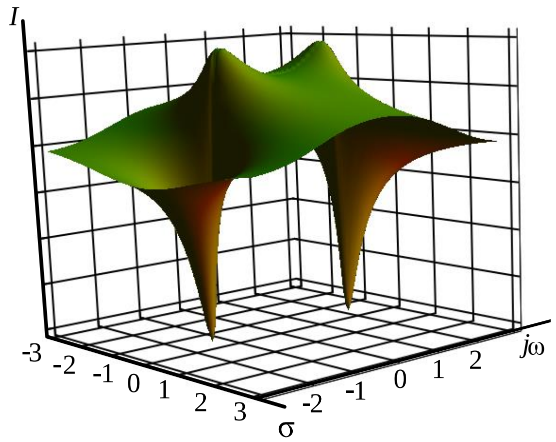

Figure 2. Surface describing the magnitude of the complex approximation error for modified functions with correcting Legendre polynomials

The power transfer function has the form (6):

$$K_p(s) = \frac{3.84-3.92s^2+s^4}{A+B} \quad (6)$$

here $A = 3.84 + 0.942s^2 - 86.52s^4 + 586.36s^6; B = -1852s^8 + 2978.8s^{10} - 2359.94s^{12} + 730{,}46s^{14}$. The relationship between the reflection coefficient and the power transfer function has the form (7).

$$\rho(s)\rho(-s) = 1 - K(-s^2) \quad (7)$$

Isolating the poles and zeros of function (7) in the left half-plane, we obtain an expression for $\rho(s)$. After the reflection coefficient factorization procedure has been carried out, the input resistance function can be determined (8).

$$Z_{\text{вх}}(s) = \frac{1+4.259s+14{,}373s^2+17{,}316s^3+33{,}2s^4+14{,}281s^5+19{,}846s^6}{1+6{,}51s+14{,}373s^2+37{,}566s^3+33{,}2s^4+58{,}831s^5+19{,}846s^6+27{,}587s^7} \quad (8)$$

The next stage of synthesis is the determination of the chain structure. After determining the structure of the circuit in this case, the structure is shown in Figure 3, you can determine the resistance value of the four-terminal network. To solve the problem of implementing a circuit, it is proposed to use direct synthesis by defining a system of nonlinear equations. The initial data for composing a system of nonlinear equations are the input resistance (8) and the resistance of a quadrupole of a canonical form, which has the order of an approximating function taking into account the number of transmission zeros (9).

$$Z_{chains}(s) = \frac{A_0+A_1s+A_2s^2+A_3s^3+A_4s^4+A_5s^5+A_6s^6}{B_0+B_1s+B_2s^2+B_3s^3+B_4s^4+B_5s^5+B_6s^6+B_7s^7} \quad (9)$$

where
$A_0 = 1;$
$A_1 = C_1R_{\text{н}} + C_3R_{\text{н}} + C_2R_{\text{н}};$
$A_2 = C_1L_2 + C_1L_4 + C_1L_5 + C_3L_5 + C_2L_4 + C_2L_5 + C_2L_3;$
$A_3 = C_1C_3L_2R_{\text{н}} + C_1C_3L_4R_{\text{н}} + C_1C_2L_2R_{\text{н}} + C_3C_2L_4R_{\text{н}} + C_1C_2L_3R_{\text{н}} + C_3C_2L_3R_{\text{н}};$
$A_4 = C_1C_3L_2L_5 + C_1C_3L_4L_5 + C_1C_2L_3L_4 + C_1C_2L_3L_5 + C_1C_2L_2L_3 + C_1C_2L_4L_3 +$
$+C_1C_2L_5L_3 + C_3C_2L_5L_3;$

$A_5 = C_1C_3C_2L_3L_4R_н + C_1C_3C_2L_2L_3R_н + C_1C_3C_2L_4L_3R_н;$
$A_6 = C_1C_3C_2L_1L_4L_5 + C_1C_3C_2L_2L_5L_3 + C_1C_3C_2L_4L_5L_3;$
$B_0 = R_н;$
$B_1 = L_1 + L_2 + L_4 + L_5;$
$B_2 = C_1L_1R_н + C_3L_1R_н + C_3L_2R_н + C_3L_4R_н + C_2L_1R_н + C_2L_2R_н + C_2L_3R_н;$
$B_3 = C_1L_1L_2 + C_1L_1L_4 + C_1L_1L_5 + C_3L_1L_5 + C_3L_2L_5 + C_3L_4L_5 + C_2L_1L_4 + +C_2L_1L_5 + C_2L_3L_4 + C_2L_3L_5 + C_2L_1L_3 + C_2L_2L_3 + C_2L_4L_3 + C_2L_5L_3;$
$B_4 = C_1C_3L_1L_2R_н + C_1C_3L_1L_4R_н + C_1C_2L_1L_2R_н + C_1C_2L_1L_4R_н + C_3C_2L_2L_4R_н +C_3C_2L_1L_3R_н + C_3C_2L_1L_3R_н + C_3C_2L_2L_3R_н + C_1C_2L_4L_3;$
$B_5 = C_1C_3L_1L_2L_5 + C_1C_3L_1L_4L_5 + C_1C_2L_1L_2L_4 + C_1C_2L_1L_2L_5 + C_3C_2L_1L_4L_5 +C_3C_2L_1L_4L_5 + C_3C_2L_2L_4L_5 + C_1C_2L_1L_4L_3 + C_1C_2L_1L_5L_3 + C_3C_2L_1L_5L_3 + +C_3C_2L_2L_5L_3 + C_3C_2L_4L_5L_3;$
$B_6 = C_1C_3C_2L_1L_2L_4R_н + C_1C_3C_2L_1L_2L_3R_н + C_1C_3C_2L_1L_4L_3R_н;$
$B_7 = C_1C_3C_2L_1L_2L_4L_5 + C_1C_3C_2L_1L_2L_5L_3 + C_1C_3C_2L_1L_4L_5L_3;$

The coefficients of the variable s are the required ones since they are determined by the values of the elements of the synthesized circuit. To compose nonlinear equations, it is necessary to equate the values of the coefficients of the polynomial (8) and the four-terminal resistance polynomial (9). The system of nonlinear equations will have the form (10):

$$\begin{cases} A_0 = 1;\ A_1 = 4{,}259;\ A_2 = 14{,}373;\ A_3 = 17{,}316;\ A_4 = 33.2;\ A_5 = 14{,}281;\ A_6 = 19{,}846 \\ B_0 = 1; B_1 = 6.51; B_2 = 14{,}373; B_3 = 37{,}566; B_4 = 33.2; B_5 = 58{,}831; B_6 = 19{,}846; B_7 = 27{,}587 \end{cases} \quad (10)$$

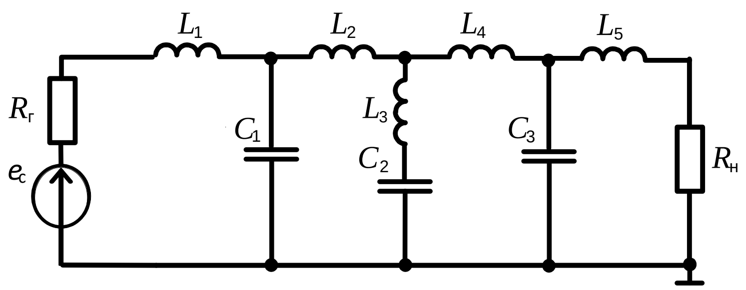

Figure 3. Canonical circuit shape for input impedance

3. RESULTS AND DISCUSSION

By solving the system of nonlinear (10) using known methods, it is possible to determine the values of the elements of the synthesized circuit. These methods allow you to obtain the exact and optimal values necessary for the implementation of a given scheme. After solving the equations, you can find the normalized values of the circuit elements, which simplifies their subsequent configuration and use. The normalized values of the circuit elements are given in (11), which provides a standard approach to their implementation and facilitates comparison with other circuits.

$$C_1 = 1{,}563; C_2 = 1{,}133; C_3 = 1{,}563; L_1 = 1{,}39;\ L_2 = 1{,}865; L_3 = 0{,}45; L_4 = 1{,}865; L_5 = 1{,}39 \\ R_н = 1. \quad (11)$$

The elements are denormalized to a given frequency using the following relations (12). This process involves converting the normalized values of circuit elements into their actual values corresponding to the operating frequency. The ratios used allow you to fine-tune the circuit elements for optimal operation at a given frequency. Thanks to these transformations, the correct operation of the circuit in real conditions is ensured, taking into account the specifics of the frequency range.

$$C = \frac{C_{norm}}{2\pi f_0 R} \text{ and } L = \frac{L_{norm}}{2\pi f_0} R \quad (12)$$

where f_0 is the filter cutoff frequency, R the active load resistance. Having carried out the indicated replacement and calculation for a cutoff frequency of 50 MHz as shown in Figure 4, we obtain the filter circuit diagram shown in Figure 4(a) and the frequency response as shown Figure 4(b).

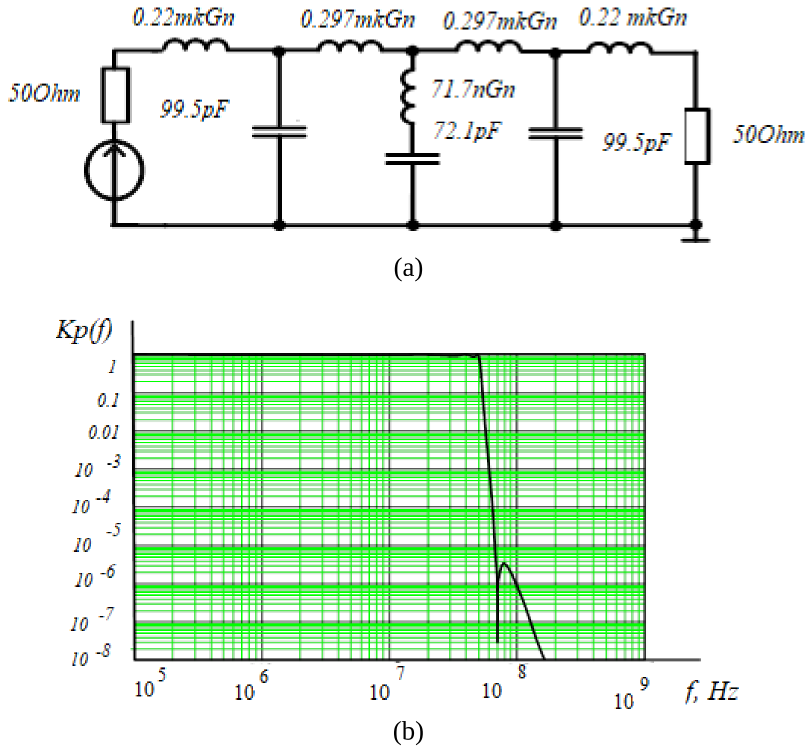

Figure 4. Calculation for cutoff frequency 50 MHz: (a) schematic diagram and (b) its frequency response

Analyzing the frequency response of the circuit, we can conclude that the filter satisfies the specified requirements, namely: cutoff frequency 50 MHz; filter order 7th; attenuation at frequency $1.2\omega_c$ is at least 30 dB; at frequency $1.4\omega_c$ 55 dB; the unevenness in the filtering band does not exceed 0.5 dB. It is very difficult to construct a general method for solving the problem of optimal filter synthesis. Traditional filter approximations may not be sufficient to ensure that the implemented filters will work well with real components or when various factors (temperature and parameter changes) vary. It follows from this that the highest quality choice of approximating function (AF) for the synthesis of a frequency-selective circuit can be achieved by increasing the AF parameters.

The selected Legendre AF best reproduces the required characteristics of the circuit about other AFs based on the criterion by which the matching-filtering circuit is optimized. So, the synthesis of a filter circuit has been implemented using the method of indefinite coefficients, and the properties of the synthesized filters are initially determined by the choice of the transfer function of their prototype filter, which approximates the transfer function of an ideal non-distorting filter with a given accuracy. Ideally, filters with a wide passband and sharp suppression of interference in the transmission band are desirable, i.e., creating a broadband effect with increased selectivity. The symmetry of the chain structure is used to reduce the number of unknowns. A higher-order filter can be created using simpler cascaded filter blocks: the specific decision is made based on filter parameters or implementation requirements (for example, ease of implementation and values of elements from a standard set of values).

4. CONCLUSION

This article discusses the possibility of using methods of undetermined coefficients to solve the problem of finding the parameters of filters with modified approximating functions, which is an extremely difficult task, and impossible for complex functions. The approximating function can be any appropriate function such as a Legendre polynomial, and some parameters can be optimized according to preferred criteria. The chosen method for approximating the transmission coefficient of the synthesized filter is the modified Legendre approximating function, which best reproduces the required characteristics of the circuit about other approximating functions based on the criterion by which the frequency-selective circuit is optimized: minimizing the order of the frequency-selective circuit; minimizing the magnitude of group delay time unevenness; minimizing the approximation error for given values of the parameters of the approximating function. In our case, a seventh order filter is synthesized.

The structure of the synthesized filter circuit and its components has been selected and determined, as well as the approximating function has been selected. The effectiveness of the polynomial approach strongly depends on the type of structure, and the symmetry of the structure helps reduce the number of unknowns. A new method for direct filter synthesis is presented by solving a system of nonlinear equations using modified Legendre approximating functions. This is a breakthrough since the results of this research are very useful for use (indirect design and practical implementation) in many technical fields, since they allow increasing the linearity of the frequency response (reduce ripple) and reduce the group delay time in the transparency band of matching circuits and filters in comparison with classical transfer functions. To check the validity, our obtained results were compared with those previously published and obtained using an analytical approach.

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