The potential of virtual representations to help students in learning mathematics

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ABSTRACT

The study aims to conclude the benefits of virtual representation in helping students learn mathematics. To answer this goal, this research utilizes the meta-analysis method of two-group difference design. The data collection process used inclusion criteria. The data collection process was carried out with the PRISMA. The results of the effect analysis of the p-value $< 5\%$ (95% confidence interval) and the total effect of 1.1761 so that virtual representation has a significant influence. The analysis showed that i) implementation in each continent showed identical positive effects, ii) the number of students subjected to the treatment did not make a difference, iii) themes in mathematics were equally well affected with the help of virtual representation, iv) the effect of virtual representation in junior high school, high school, and university was identical, v) the development of competencies in attitude, knowledge, and skills was equally good, vi) among the many applications, GeoGebra was the application that had the greatest impact in helping students understand mathematics subject matter, and vii) the use of smartphones had a greater effect than other devices such as computers and calculators. To produce the maximum effect in understanding students, it is recommended to use mobile devices and GeoGebra software.

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1. INTRODUCTION

Mathematics, as a complex discipline by nature, involves essential skills such as linking, abstraction, and generalization [1]. In recent years, research in mathematics education has highlighted the complexity of teaching challenges and strategies [2]. Mathematics teaching generally undergoes a transition from concrete to abstract [3], [4]. This indicates an understanding that concrete mathematical thinking skills are taught in the early years of education at the primary level, while mathematical thinking skills that require further inference and justification are provided in more advanced educational processes. Several studies have been conducted on the complexity of mathematics which is not only viewed as a series of concepts, but also as a series of interrelated cognitive skills [2]. Researchers assert that effective mathematics teaching requires a deep understanding of how students build relationships between concepts, perform abstractions, and make generalizations.

Students often experience difficulties in learning mathematics because they face difficulties in understanding concepts, performing abstractions, and generalizing [5], [6]. This is because some students do not understand mathematical topics thoroughly so they face difficulties establishing connections between mathematical concepts. Students' difficulties in understanding the basics of mathematical knowledge can hinder the understanding of more complex mathematical operation concepts [7]. Abstraction ability is an obstacle for students, because they have difficulty linking mathematical concepts to real-world situations or formulating problems abstractly [8]. In addition, difficulties in generalization also often occur, because students have difficulty applying the concepts they learn to different contexts or understanding general principles that can be applied to various problems [9]. Therefore, the importance of a deep understanding of how students build relationships between concepts, perform abstraction, and generalize is key in overcoming mathematics learning difficulties.

In learning mathematics, from the concrete to the abstract level, one of the strategies that teachers can use to improve understanding of mathematical concepts is to use visual representations or real objects [10]. At the concrete stage, for example on the topic of numbers, teachers teach basic math operations with number blocks, which helps students understand the concepts better [11]–[13]. In the representational stage, students are introduced to real objects such as number blocks. In the last stage, the abstract stage, students learn math through mathematical symbols in a more general way after gaining concrete and representational foundations. If students face difficulties, it is important to return to the concrete level [14]. Mathematical manipulation and visual representation are essential for solving students' math learning difficulties.

Technological approaches to creating visual presentations have transformed the learning environment by providing a variety of ways to present information in dynamic and engaging ways [15]. A variety of digital tools and software enable teachers and students to create engaging and informative images [16], [17]. From graphic animations to interactive simulations, technology helps students understand complex concepts by creating visual presentations that enrich the learning experience [16], [18]. These benefits create a more dynamic and adaptive learning environment that supports multiple ways to meet students' diverse needs in understanding the material.

Technological developments have led to the creation of various applications and tools for virtual representation in mathematics learning. Some technological approaches in mathematics learning include the use of various tools such as GeoGebra, 3D models and Google Sites applications. GeoGebra can create visualizations to explain mathematical concepts and 3D models provide a visual and interactive experience, making learning more fun and immersive [19]. Google Sites application is a useful tool for practicing visual representation of mathematical models [20]. This technology also facilitates various visualizations, including drawing scenes, creating graphs, and using general visualization tools. By using this technology, math learning becomes more interactive and fun, supporting students' concept understanding through better visual representation.

Mathematics learning integrated with electronic devices, such as tablets or laptops, has been the subject of significant research over the past 10 years [21]. The development of technology-integrated mathematics learning tools, such as e-worksheets and interactive e-books, has successfully increased the effectiveness of mathematics learning [22], [23]. This shows that the integration of technology in mathematics education has shown a positive impact on students' abilities. Studies show that the key success factors of advanced technology in mathematics education involve the design of computer-based tools and the way they are implemented [24]. The use of mobile devices has also been shown to have a positive effect on math learning, where the integration of technology and computers can help students better understand math concepts [25]. In addition, the application of a realistic mathematics education approach with technological support has succeeded in improving students' ability to relate mathematical concepts to everyday life [26]. Thus, the portrait of learning success using electronic devices includes increasing learning effectiveness, developing student abilities, and integrating technology to enrich the mathematics learning experience. There are several opinions against digital learning or using electronic devices [27], [28].

First, the use of electronic screens can lead to eyestrain and decreased focus, which can hinder learners' comprehension of the material [28]. Other studies have also shown that reading comprehension on paper books is better than on electronic books [29]. This suggests that electronic devices can have a negative impact on reading comprehension. In addition, too much time spent in front of a screen can also lead to fatigue and a decline in students' eye health. Secondly, learning to draw with electronic devices is often unable to develop drawing skills properly. The use of tablets or computers for drawing does not provide the same experience as using paper and pencil, which can be detrimental to the development of students' artistic skills and visual sensibilities [27]. Thus, while technology provides access to digital resources and flexibility in learning, the opposition lies in its potential negative impact on health and specific skills, such as drawing ability and reading skills.

In mathematics learning, there are two different views on the integration of technology in visual representation. The positive view emphasizes that the use of technology such as GeoGebra software or 3D models can improve understanding of mathematical concepts and make learning more interactive. However, the negative view states that the use of electronic devices can cause eyestrain and decreased focus, and potentially hinder the development of skills such as drawing. Therefore, comprehensive meta-analysis research is needed to conclude the impact of technology integration in visual representation in mathematics learning. This study will combine the results of various existing studies to provide a more complete picture of the effectiveness and impact of technology integration in mathematics learning contexts. Thus, meta-analysis research will be an invaluable tool to summarize various perspectives and provide a holistic view of the influence of technology in visual representations in mathematics learning.

2. METHOD

This research related to the potential of virtual representation in helping students learn mathematics is meta-analysis research. Research using the meta-analysis method is a technique for summarizing various research results with the same type and characteristics [30], [31]. The meta-analysis method helps us to understand the conclusions of various studies based on statistical data displayed in articles or research reports [32]. The conclusion is a general condition of the potential of virtual representation in helping students learn mathematics. The basis of inference is the total effect size found based on various effect sizes on each data [33].

2.1. Data sources

This research takes data from various articles that have been published in Scopus database. The selection of the database is based on the assumption that Scopus has good management in managing the quality of journals in its community. Thus, researchers can at least assume that the data collected are in articles that have been rigorously reviewed so that only articles with good quality can be published.

2.2. Research procedure

The data collection process carried out on the Scopus database portal was carried out by utilizing various relevant keywords. Some of the relevant keywords used by researchers are "geogebra," "cabri," "MATLAB," "maple," and "learning." To get more detailed and extensive search results, the search process utilizes Boolean logic based on the operators "AND" and "OR" as a link between various keywords. Articles found by the search engine were then selected based on the criteria set by the researcher, namely inclusion and exclusion criteria. The following are the inclusion and exclusion criteria in this research related to virtual representation, shown in Table 1.

Table 1. Inclusion and exclusion criteria

Aspect	Inclusion criteria	Exclusion criteria	Aspect	Inclusion criteria	Exclusion criteria
Year	2018-2023	Else	Data component	Sample size, Mean, and Standard deviation	Not found
Language	English	Else			
Articles type	Research article	Else			
Data type	Quantitative	Qualitative	Data analysis	Quantitative	Qualitative
Research design	Group contrast	Else	Theme	The influence of virtual representation on learning mathematics	Else
Research model	Control and experiment	Else			
Data base	Scopus	Else			

In the process of searching for articles on the Scopus database, researchers use the PRISMA data collection model. The PRISMA data collection model helps researchers to administer the data search process in a systematic and well-documented manner [34]. The ability of the PRISMA model makes it easier for researchers to be accountable for data collection process. The PRISMA model is summarized in Figure 1.

The results of data searches in accordance with the inclusion criteria with the PRISMA method obtained 24 articles. The 24 articles obtained 54 data that can be used as material for analyzing the effect of virtual representation in mathematics learning. Examples of articles that have more than one data are researches [35] which displays data on procedural and conceptual abilities. In addition, another example can be seen in the research results [36] which displays the impact of virtual representation on students' mathematical knowledge, skills, and attitudes.

2.3. Prerequisite tests and publication bias

The first prerequisite test is about data heterogeneity. Based on a review of the characteristics of the data analyzed, it was found that the data was heterogeneous. This conclusion was drawn based on the profile of research data conducted in various countries and various ages. Thus, the meta-analysis research will be conducted with a random effect model. However, claims based on data characteristics must be strengthened with statistical evidence [37]. The statistics used to prove the assumption of heterogeneity are the Q parameter, τ^2, and I^2. The three methods are used in the hope that they can correct each other so that the assumption can be proven steadily. The decision-making criteria for proving the assumption is that the data is said to be substantially heterogeneous if the value of I^2 is between 75% and 100% [38], p-value $Q < 0.05$, and τ^2 is more than zero [39].

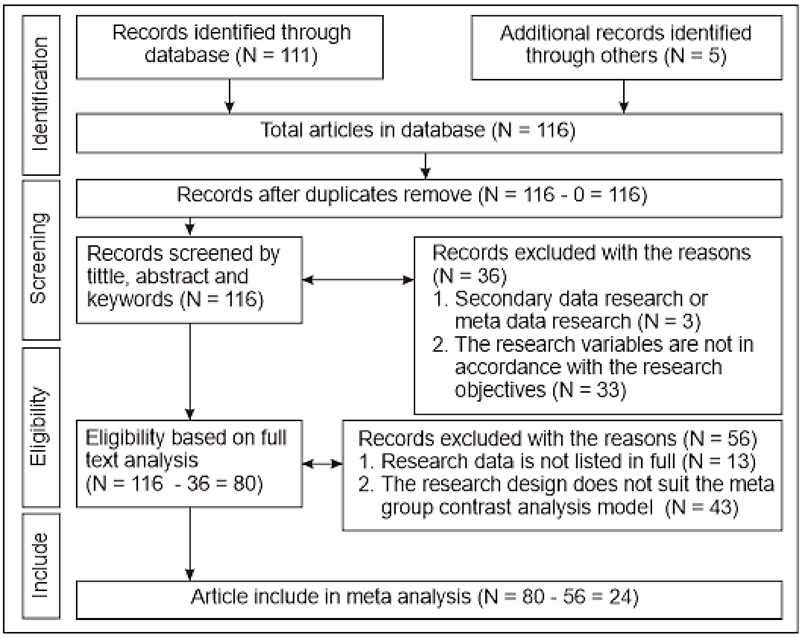

Figure 1. PRISMA data selection

Furthermore, the assumption to be proven is the freedom of data from publication bias. In meta-analysis, this proof is a must as a guarantee that the data and analysis results can be trusted. The proof of freedom from publication will use two methods, namely identification of funnel plot-based point distribution and inference of the results of the file-safe N calculation based on passing grade. Data is free from publication bias when the distribution of points on the funnel plot is identified as symmetrical and the file-safe N value is greater than the criterion of $5K + 10$ where K is the number of data [40].

2.4. Determining effect size

After the prerequisite assumptions have been proven, the analysis process can continue to fin d the total effect size. This research is a meta-analysis of standardized experimental and control group comparison models [41]. The standardization process aims to equalize data that has highly variable aggregates. The results of standardized data analysis will eventually find an effect size (d). To minimize the bias of the effect size calculation results, the data is transformed [42]. The analysis and calculation process are assisted by the meta and metaphor packages in R software. The results of the effect size findings can be classified as i) no effect for ES below 0.19, ii) small effect for ES between 0.20 to 0.49, iii) medium effect for ES between 0.5 to 0.79, iv) large effect for ES between 0.80 to 1.29, and v) very large effect for ES above 1.30.

2.5. Moderator variable analysis

Moderator variable analysis is used to obtain supporting information in interpreting the findings of the total size effect related to the influence of virtual representation in helping students learn mathematics. To deepen the study, this research involves 8 moderator variables, namely continent, theme, level, number of students in a class, targeted competencies, applications used, devices used, and the era of virtual representation implementation. Analysis of moderator variables utilizes an analysis of variance model. The results of the analysis of variance can be used as a basis for concluding whether or not there are differences in categories for each moderator variable. The decision criterion in the analysis of variance is the p-value. There is a difference between categories if the p-value <0.05 (5% error). If the analysis results show that there is a significant difference, the analysis process can be continued by looking at the effect size of each category to conclude which category has the strongest or weakest effect. The description between variables is shown in Table 2.

Table 2. Moderator variables

Moderator variables	Categories	Freq.	%
Continent	Asia	17	31.48%
	America	34	62.96%
	Euro	3	5.56%
Theme	Geometry	29	53.70%
	Calculus	12	22.22%
	Trigonometry	6	11.11%
	Algebra	7	12.96%
Level	JHS	20	37.04%
	SHS	15	27.78%
	University	19	35.19%
Sample size	Small ($n <= 30$)	25	46.30%
	Medium ($31 < n < 60$)	20	37.04%
	Large ($n > 60$)	9	16.67%
Competence	Attitude	13	24.07%
	Knowlegde	28	51.85%
	Skill	13	24.07%
App	GeoGebra	37	68.52%
	Cadri	4	7.41%
	Sketchpad	10	18.52%
	GeoHepta	1	1.85%
	Graphing Calculator	2	3.70%
Device	Computer	45	83.33%
	Smartphone	7	12.96%
	Calculator	2	3.70%

3. RESULTS AND DISCUSSION

There are 54 data from 24 articles analyzed in this study. The data analyzed are the results of research conducted on mathematics learning in various themes, conducted in various countries spread across various continents, implemented at various school levels, and implemented in various eras. Based on this profile, it is assumed that the data is heterogeneous so that the analysis process uses a random effect model. However, assumptions based on the data profile must be supported by statistical analysis.

3.1. Proving the assumption of heterogeneity

The assumption of statistical heterogeneity can be proven by the parameters Q, $Ʈ^2$, and I^2. The analysis results show that the value of $Ʈ^2$ with an error tolerance of 5% is 1.1466 [0.8392; 2.0359]. Based on the results of the analysis, $Ʈ^2 > 0$, which means that the data is heterogeneous. Thus, based on the analysis model $Ʈ^2$ shows that the assumption of heterogeneous data is proven. The results of the analysis with the statistical model I^2 with an error tolerance of 5% show a value of 90.1% [87.9%; 91.9%]. When compared with the classification of data heterogeneity, the data in this study are included in the substantial heterogeneous category because they are in the range of 75% to 100%. Then based on the Q parameter analysis, it is found that the p-value is very small, which is close to zero. Based on the results of this analysis, it is proven that the data is heteron. Based on the three proofs of assumptions, the data is proven to be statistically heterogeneous so that the random effect model selection in the meta-analysis is appropriate.

3.2. Proof of freedom from biased publication

Data freedom from bias publication is another thing that must be proven. This proof is used to guarantee that the data and analysis results can be trusted. Proof of freedom from publication bias uses the interpretation of the funnel plot and compares the Fail-Safe N value with the minimum standard. Figure 2 is an image of the funnel plot.

Based on the funnel plot, the distribution of points cannot show perfect symmetry. Thus, based on the figure, the interpretation results can be disputed. Based on this, it is necessary to conduct a fail-safe N-based proof to demonstrate freedom from publication bias. The fail-safe N calculation results in 16,714 (5% confidence interval). The value limit to be said to be free from publication bias is $N>5K+1$ with the information N is the fail-safe N value and K is the amount of data. This research involves 54 data so that $5K+1=271$ so that the value is far below 16714. Based on the fail-safe N method, it can be concluded that the data is free from publication bias. This means that the data and results of meta-analysis related to the impact of virtual representation in helping students learn mathematics in this study can be trusted.

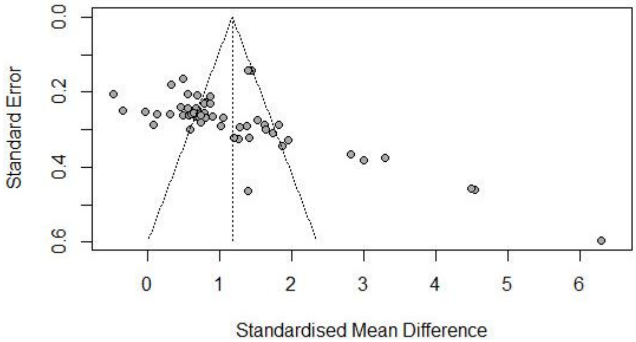

Figure 2. Funnel plot

3.3. Total effect size

After the assumption of heterogeneity was proven as the basis for the correct selection of the random effect model and freedom from publication bias was also proven, the analysis entered the stage of calculating the effect size of each study and the total effect size. The meta-analysis showed that the total effect size with the random effect model was 1.1761 [0.8802; 1.4719] with a p-value<0.001 (95% confidence interval). These results can be translated that there is a significant effect of virtual representation in helping students learn mathematics. The effect size of 1.1761 can be categorized as having a large effect [39]. The results of the analysis of each study and the total can be seen in the forest plot shown in Figure 3.

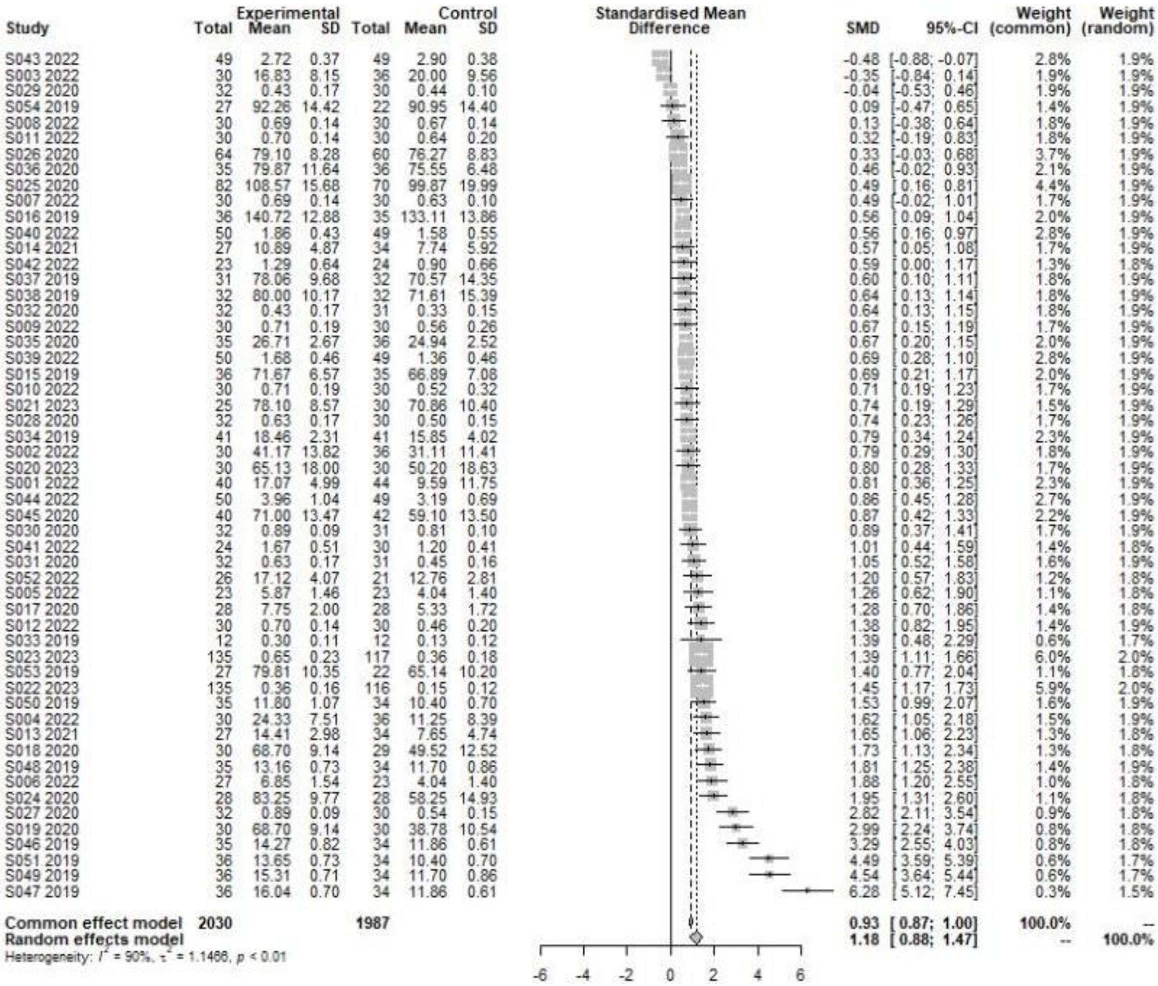

Figure 3. Forest plot

Virtual representation has great potential to facilitate the learning process of mathematics for students. Various technologies such as GeoGebra software, 3D models, and Google Site applications can be used to explore mathematical concepts visually and interactively. For example, GeoGebra allows the creation of images or visualizations that can help students understand mathematical problems more clearly, facilitate problem solving, and make mathematical concepts more accessible [43]. The use of 3D models in building space learning also allows students to explore concepts in real time, while the Google Site application can integrate visual representations of mathematical models with technology, creating a more interesting and enjoyable learning experience [20]. With this approach, students can develop a deeper understanding of mathematical concepts through visual representations that provide a real and concrete picture of the learning material.

3.4. Moderator variables analysis

Based on the total effect size analysis, it shows that there is a positive influence of virtual representation in making it easier for students to learn mathematics. The findings were deepened by analyzing moderator variables to get more detailed findings. The variables analyzed were the continent where the research was conducted (Africa, Asia, and Europe), the number of students in one class represented by the number of research samples (small, medium, and large), the theme of the mathematics material studied (geometry, calculus, trigonometry, and algebra), the level of students who received the treatment (junior high school, high school, and university), competencies targeted in learning (attitude, knowledge, and skills), applications used to display representations (GeoGebra, Cadri, Sketchpad, GeoHepta, and Graphng Calculator), and devices used in the learning process (computers, smartphones, and calculators). Figure 4 is a forest plot of the moderator variable analysis results.

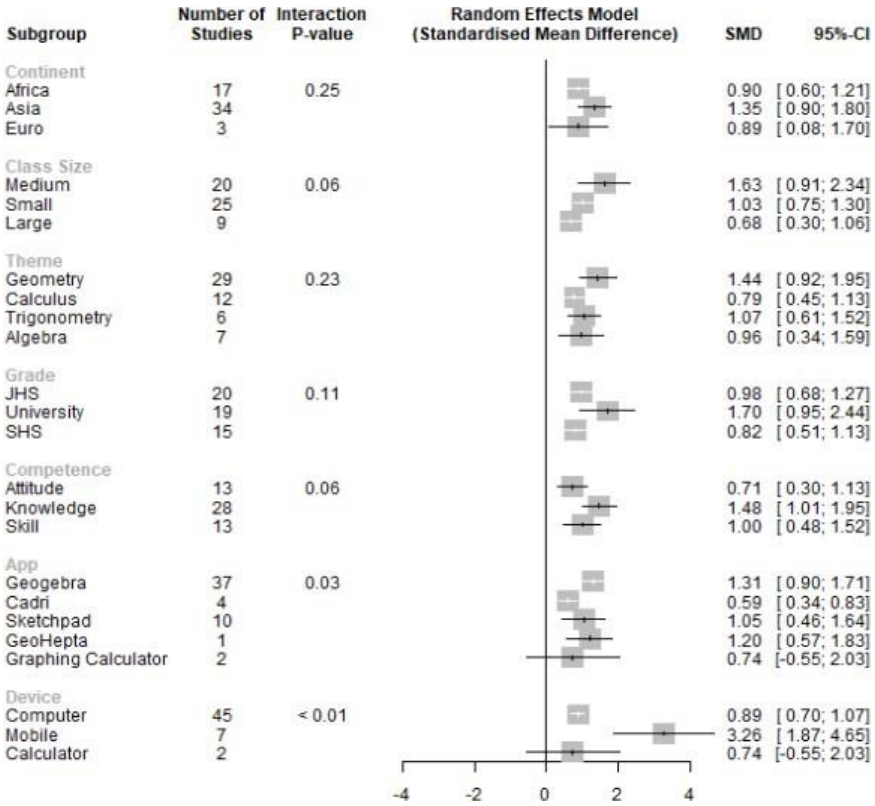

Figure 4. Interaction and forest plot of moderator variables

Based on the data, it appears that the utilization of virtual representation in mathematics learning is mostly done in Asia. More than 50% of the data comes from Asia. This is due to the rapid growth of technology and the adoption of innovation in Asian countries. Countries such as Japan, South Korea and China have become centers of technological innovation, including in the development of mathematics applications that utilize virtual representations [44]. This suggests that technological advances in Asia have encouraged the use of virtual representations in mathematics learning. In addition, educational policies in some Asian countries also support the integration of technology in the mathematics curriculum. For example, the use of virtual reality and digital math tools can be integrated in an effort to improve mathematical concept understanding skills [44]–[46]. However, when viewed from the effect, the positive effect of virtual representation on various continents does not show any significant difference. The conclusion is based on the p-value=0.25>0.05 (95% confidence interval). Thus, virtual representation can be recommended to be implemented in all locations. This is based on the characteristics of mathematics learning which has universal principles and can be applied equally in various global contexts. In addition, the characteristics of students in different continents can also be similar in response to certain learning methods, especially with adaptation and personalization in the use of virtual representations. While cultural and contextual aspects remain important, these results provide a rational basis for recommending the use of virtual representations in all locations in an effort to improve mathematics learning globally.

The second moderator variable is sample size. This variable shows the number of students in one treatment. The result of the moderator variable analysis shows that the p-value=0.06>0.05 (95% confidence interval) so there is no significant difference in the effect of using virtual representation in small, medium, and large sample sizes. This ratio can be attributed to the fact that the use of virtual representation in mathematics learning has a tendency to increase learning independence [47], [48]. With virtual representations, students can engage in learning activities using their own devices, regardless of the number of students in a learning session [49]. In this context, the insignificant difference in effect across different sample sizes may be due to the adaptive nature of using virtual representations. Students, whether in small, medium or large groups, can effectively engage in mathematics learning activities with the same level of independence, thanks to the advantages of virtual representation technology that supports individualized learning.

Mathematics has many learning themes. Based on the data collected, there are 4 themes that are studied by utilizing virtual representation, namely geometry, calculus, trigonometry, and algebra. Although they have different characters between themes in mathematics, they are interrelated with each other. Among the four themes, the geometry theme is the most common theme in mathematics [50]. The use of virtual representation in learning mathematics is dominated by geometry because it is in accordance with its nature. In this context, virtual representation is mainly focused on the use of simulations to visualize geometric objects and concepts in 2D and 3D [51]. Geometry provides advantages in the form of concrete visualization of space, so geometric objects such as triangles and circles can be clearly shown in a virtual environment [12]. The concept of geometry also opens up opportunities for intense interaction, allowing students to directly explore geometric objects and observe changes in parameters. In addition, the simulation of geometric events such as shifts, rotations, and scale changes become relevant in virtual representation, making it an effective tool for understanding these concepts. In the simulation of 2D and 3D objects, geometry provides the basis for a rich and immersive learning experience. Thus, the use of geometry themes in virtual representation allows for more real, interactive learning of mathematics and deepens the understanding of geometric concepts.

The results of the analysis show that the p-value=0.06>0.05 (95% confidence interval) so that there is no significant difference in the effect of using virtual representation on the themes of geometry, calculus, trigonometry, and algebra. This is because mathematics has a nature as an abstract science, where the use of virtual representation aims to facilitate the understanding of these abstract concepts by students who are learning mathematics [52], [53]. In this context, the insignificant difference between mathematical themes in the influence of virtual representations can be explained by the fact that the abstract nature of mathematics is uniformly reflected through visual representations, resulting in uniform analysis results.

In its implementation, virtual representation helps facilitate the learning of mathematics at various levels. The analysis showed p-value=0.11>0.05 (95% confidence interval). The meaning of the analysis results is that virtual representation provides equally good assistance to students in learning mathematics at junior high school, high school and university levels. At the junior high school level, mathematical materials are generally more concrete, visual representations can help students understand concepts better [54]. In high school, the level of abstraction of the material increases, but virtual representation remains effective because it is able to visually reduce the complexity of the concept [55]. At the university level, where mathematical material reaches the highest level of abstraction, virtual representations remain relevant by providing visual images that assist students in understanding complex concepts. Thus, the uniformity of assistance provided by virtual representations at different educational levels can be explained through their flexibility in presenting mathematical concepts, creating a learning environment that supports student understanding at each level of abstraction.

In a learning process, the competency domains (attitude, knowledge and skills) should develop in a balanced manner. Thus, teachers are considered successful if they choose learning methods or strategies that can develop all three. This includes the implementation of virtual representation which should be able to target the development of the three domains of competence. The results of the analysis show that the p-value =0.06>0.05 (95% confidence interval), meaning that there is no significant difference in the effect of virtual representation implementation on the development of attitudes, knowledge, and skills. This also supports that learning mathematics using virtual representations has significant benefits for student motivation [56]. One of the benefits is through concept visualization, which can make learning more interesting and increase student engagement. In addition, virtual representations can also transform abstract mathematical concepts into more concrete ones, thus helping students in understanding complex ideas [13]. Moreover, through simulation and interactivity, virtual representations can also develop students' practical skills, thus creating a holistic learning experience [53]. Given these positive impacts, virtual representations have an important role in improving students' motivation, concept understanding, and skills in learning mathematics.

Among the software used in the implementation of virtual representation, GeoGebra is the most popular software. 68.5% of the data is research that utilizes GeoGebra as a tool to create visual illustrations. Uniquely, GeoGebra is also the software that has the best influence compared to other software. Statistically, this conclusion is based on the p-value<0.001<0.05 (95% confidence interval), which means that there is a significant difference in the effect of applications on the ease of learning mathematics. This conclusion is the foundation for identifying the effect in each category which shows that GeoGebra is the software with the greatest effect compared to other software. GeoGebra has advantages in several aspects that distinguish it from other math software. One of these advantages is the ability to integrate concepts, such as geometry, algebra, and calculus, which gives students the flexibility to understand and apply mathematical concepts holistically [57]. With a high level of interactivity and dynamic features, GeoGebra allows students to interact directly with graphic visualizations and mathematical objects, thus deepening concept understanding [58]. The discovery learning approach supported by GeoGebra also provides exploration space for students in exploring mathematical concepts independently, which in turn stimulates creative thinking and intuitive understanding. The flexibility of using this software from elementary school to college level shows its adaptability in various learning contexts [43], [58]. Moreover, the presence of an active community and abundant learning resources provide additional support, making GeoGebra a superior choice in improving students' concept understanding and visualization skills in mathematics learning.

The next variable analyzed is the device used in operating the software. This variable is important to analyze because it can be related to the relevance of using virtual representation to make it easier for students to learn mathematics. There are three devices identified as being used, namely laptops, smartphones, and calculators. Among the three, laptop/computer is the device that has the greatest impact among others. This conclusion is based on the p-value=0.006>0.005 (95% confidence interval). This means that there is a significant difference in the impact of devices on students' ease of learning mathematics. The analysis continued by looking at the effect value. Based on the effect value and error range, the most effective learning with virtual representation is using laptop. The use of virtual representation on laptops in learning has significant advantages compared to the use of other devices such as tablets or smartphones. One of the main advantages of laptops is their larger screen, which allows for more detailed and clear visualization [59]. This can enhance students' visual experience in understanding mathematical concepts. In addition, the keyboard and touchpad on laptops also provide more complete and precise input control, which makes it easier to interact with math software [59].

4. CONCLUSION

The results of this study show that the data collected are heterogeneous (indicating that the selection of the random effect model is appropriate) and free from publication bias (indicating that the results of the analysis can be trusted). The results of the analysis show that virtual representation has a positive influence on students' ease of learning mathematics. The ease of learning is manifested from the student learning achievement index at the end of learning. Overall, it was found that the p-value <5% (95% confidence interval) and the total effect of 1.1761 so that virtual representation has a significant influence and falls into the category of having a large effect in influencing mathematics learning outcomes. The global conclusion was translated more specifically through moderator variables. The analysis showed that i) implementation in each continent showed identical positive effects; ii) the number of students subjected to the treatment did not make a difference; iii) themes in mathematics were equally well affected with the help of virtual representation; iv) the effect of virtual representation in junior high school, high school, and university was identical; v) the development of competencies in attitude, knowledge, and skills was equally good; vi) among the many applications, GeoGebra was the application that had the greatest impact in helping students understand mathematics subject matter; and vii) the use of smartphones had a greater effect than other devices such as computers and calculators. Thus, virtual representation learning is highly recommended in mathematics learning. To produce the maximum effect in understanding students, it is recommended to use mobile devices and GeoGebra software.

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