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Fuzzy integral tracking control of an activated sludge process

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ABSTRACT

This paper addresses the issue of tracking the output of an activated sludge process using fuzzy integral control. First, the dynamics of the nonlinear process are modeled with a dynamic state space fuzzy model integrating the effect of external disturbances, and then an additional integral state of the output tracking error is introduced to obtain an augmented Takagi-Sugeno (TS) fuzzy model. The TS fuzzy model is able to describe the dynamics of complex nonlinear systems with an excellent degree of accuracy. It is formulated by fuzzy if-then rules which can give local linear representation of the overall nonlinear system. Second, the design of the fuzzy integral control is performed, in which the state feedback gains are obtained by solving linear matrix inequalities (LMI). The objective is to ensure trajectory tracking of an activated sludge process (ASP) by controlling two key variables: the substrate concentration and the level of dissolved oxygen. To assess the performance of the proposed control strategy, a comparative analysis is carried out with a gain scheduling PI (GS-PI) controller. Simulation results are provided to illustrate the effectiveness of the proposed approach. Where, the fuzzy integral control reduces the high energy consumption in water treatment plants.

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1. INTRODUCTION

Wastewater treatment plants (WWTP) are complex processes with significant nonlinearities and subject to external disturbances, which are usually unmeasurable [1]. Within these plants, interdependent biological and biochemical phenomena take place. These processes are stiff systems with time constants ranging from minutes to several days, as characterized by wide fluctuations in influent component concentrations. A WWTP comprises chemical, mechanical and biological treatment processes. During the final stage of biological treatment, the common practice involves activated sludge technology, recognized as the most intricate process within a WWTP.

The activated sludge process (ASP) [2] stands as a widely employed system for the biological treatment of wastewater. Numerous research studies have delved into the control of this process, driven by the inherent complexity of ASPs. These systems exhibit substantial nonlinearities and are characterized by numerous uncertainties. Furthermore, many wastewater treatment plants lack comprehensive measurement devices. Despite this, they must operate continuously to comply with stringent regulations. The aim goal is the maintenance of effluent substrate concentration below specified limits (20 mg/l).

Given the difficulty in monitoring certain parameters such as ammonia and nitrates, indirect measurements such as pH and dissolved oxygen are often used to control ASP [3]. The level of dissolved oxygen concentration in aerobic reactors must be adjusted to provide enough oxygen to the microorganisms in the activated sludge [4]. This type of system is characterized by significant variations in parameters and unknown kinetics arise from time-varying characteristics and different interactions induced by living microorganisms. Thus, we are dealing with a highly nonlinear system [5], [6]. The objective of this study is to present a Takagi-Sugeno (TS) representation [7], [8] of the ASP model, incorporating disturbances. To transform the problem into a trajectory tracking problem, we consider an augmented system that includes not only the state but also the integral of the error. This choice is motivated by the potential application of the proposed controller to the actual process. Furthermore, utilizing optimization tools involves integrating stability conditions which are described by linear matrix inequalities (LMIs) [9]–[11] allowing to take disturbances into consideration.

This represents a significant advantage compared to parallel distributed compensation control (PDC) [12] or linear quadratic integral control (LQI) mentioned in the literature [13]–[15], which does not consider disturbances in its LMIs formulations. Moreover, there are several comparative studies with the fuzzy PI controller [16]–[19] and fuzzy PID controller [20]–[24]. In order to stand out and demonstrate the effectiveness of our approach, we focused on the control of dissolved oxygen concentration. We compared the global fuzzy state feedback controller with PI gain scheduling. Although the latter is effective in terms of robustness [25], it exhibits a significant energy consumption compared to the fuzzy global state feedback controller.

The structure of this paper is as follows: section 2 describes the fuzzy modeling of the system. Section 3 presents the design of the fuzzy global state feedback controller using the LMI approach, as well as the design of the Gain Scheduling PI controller. Finally, the dynamic model of the bioreactor is presented in section 4, where simulation studies concerning the wastewater treatment process are also presented.

2. FUZZY MODELING

Mechanistic activated sludge models include numerous stoichiometric and kinetic parameters related to different organism masses. It is important to recognize that the various variations of IAWQ-type models lead to differences in model structure. The ASP model details the substrate concentration and biomass in the biological reactor.

Consider:

$$\begin{aligned} \dot{x} &= h(x,u) \\ y &= g(x) \end{aligned} \tag{1}$$

with $x \in R^n$ and $u \in R^m$ are respectively the state and the control the output $y \in R^p$, h and g are nonlinear functions. We consider TS representation, where the i^{th} rule of a fuzzy rule base is:

$$\begin{aligned} &RULE\ i\text{: IF } z_1 \text{ is } A_1^i \text{ and ... and } z_r \text{ is } z_1^r \\ &\text{THEN} \begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t) + R_i d(t) \\ \qquad y(t) = C_i x(t) \end{cases} \end{aligned} \tag{2}$$

where the premise variables vector $z \in R^r$ is a subset of x, u, θ and y. A_j^i is a fuzzy subset of membership function $\mu_{A_j^i}\colon R \to [0,1]$. The membership function $\mu_{A_j^i}(z_j)$ is defined in the i^{th} rule that applies to the j^{th} premise variable.

$$A_i = \left.\frac{\partial h}{\partial x}\right|_{(x_i,u_i)}, B_i = \left.\frac{\partial h}{\partial u}\right|_{(x_i,u_i)}, C_i = \left.\frac{\partial g}{\partial x}\right|_{(x_i)}, R_i = \left.\frac{\partial h}{\partial d}\right|_{(x_i,u_i)} \tag{3}$$

where $d = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}$, with $d_1 = Do_{in}$ and $d_2 = S_{in}$. The representation of TS system is given as:

$$\begin{aligned} &\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} h_i\big(z(t)\big)\big(A_i x(t) + B_i u(t) + R_i d(t)\big) \\ \qquad y(t) = \sum_{i=1}^{r} h_i\big(z(t)\big)\, C_i x(t) \end{cases} \\ &w_i(z) = \frac{\mu^i(z)}{\sum_{k=1}^{M} \mu^k(z)} \mu^i(z) = \prod_{j=1}^{r} \mu_j^i(z_j) \end{aligned} \tag{4}$$

with $\sum_{i=1}^{M} w_i(z) = 1$ with M number of rules.

3. METHOD

3.1. Fuzzy output tracking control

The fuzzy feedback controller shown in Figure 1 is based on the "state vector", which uses an information content richer than the output feedback. The fuzzy logic controllers are based on fuzzy sets to describe the inputs and output. The crisp value of the output is obtained by defuzzification (center of gravity). However, the TS control is based on IF-THEN rules with functions in the output.

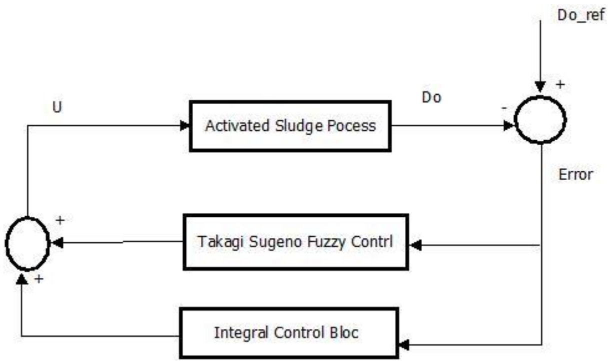

Figure 1. Fuzzy integral control of dissolved oxygen

The local state feedback controller is designed as (5):

$$u(t) = k_{1i}x(t) + k_{2i}x_y(t) \tag{5}$$

where k_{1i}, k_{2i} are the control gains and

$$\dot{x_y}(t) = y_{ref} - y(t) \tag{6}$$

in which $x_{y(t)} \in R^p$ is an integral state variable. Then, the fuzzy feedback controller is given as (7):

$$u(t) = \sum_{i=1}^{r} h_i\big(z(t)\big)\left[k_{1i}X(t) + k_{2i}x_y(t)\right] \tag{7}$$

Then, the fuzzy controller (7) is introduced into the state equation (4) in the closed loop system

$$\dot{x}(t) = \sum_{i=1}^{r} h_i\big(z(t)\big)\left(A_i x(t) + B_i\left[\sum_{j=1}^{\Omega} h_j\big(z(t)\big)\big(k_{1j}x(t) + k_{2j}x(t)\big)\right] + R_i\, d(t)\right) \tag{8}$$

$$\dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{\Omega} h_i\big(z(t)\big)\, h_j\big(z(t)\big)\left[A_i x(t) + B_i k_{1j}x(t) + B_i k_{2j}x_y(t) + R_i d(t)\right] \tag{9}$$

$$\dot{x_y}(t) = \sum_{i=1}^{r} h_i\big(z(t)\big)\big(y_{ref} - C_i x(t)\big) \tag{10}$$

We define an augmented system such as (11).

$$\tilde{x}(t) = \begin{bmatrix} x(t) \\ x_y(t) \end{bmatrix} \tag{11}$$

The augmented system is expressed as (12).

$$\dot{\tilde{x}}(t) = \sum_{i=1}^{r}\sum_{j=1}^{\Omega} h_i\big(z(t)\big)\, h_j\big(z(t)\big)\left[\big(\tilde{A}_i x(t) + \tilde{B}_i k_j\big)\tilde{x}(t) + \tilde{R}_i \tilde{d}(t)\right] \tag{12}$$

where

$$\tilde{A}_i = \begin{bmatrix} A_i & 0 \\ -C & 0 \end{bmatrix},\ B_i = \begin{bmatrix} B_i \\ 0 \end{bmatrix},\ k_j = \begin{bmatrix} k_{1j} & k_{2j} \end{bmatrix},\ \tilde{d} = \begin{bmatrix} d \\ y_{\mathrm{ref}} \end{bmatrix},\ \tilde{R}_i = \begin{bmatrix} R_i & 0 \\ 0 & 1 \end{bmatrix}. \tag{13}$$

3.2. LMI problem

We will demonstrate in this section that, in order to verify the stability conditions of the closed-loop system (12) using Lyapunov conditions, the problem becomes one of solving LMIs.
Lemma 1: There exists tow real matrices X and Y and for any ε, the inequality (14) is verified

$$X^T Y + Y^T X \leq \epsilon X^T X + \frac{1}{\epsilon} Y^T Y \tag{14}$$

We begin by defining the Lyapunov function:

$$V\big(\tilde{x}(t)\big) = \tilde{x}(t)^T P \tilde{x}(t) \tag{15}$$

Then, the derivative of this function along the trajectories of the system is:

$$\dot{V}\big(\tilde{x}(t)\big) = \dot{\tilde{x}}(t)^T P \tilde{x}(t) + \tilde{x}^T P \dot{\tilde{x}}(t) \tag{16}$$

Then

$$\dot{V}\big(\tilde{x}(t)\big) = \sum_{i=1}^{r}\sum_{j=1}^{\Omega} h_i(\, z(t)) h_j(\, z(t))(\tilde{x}^T(t)\big(\tilde{A} + \tilde{B}k_j\big)^T P\tilde{x}(t) + \widetilde{d^T}\widetilde{R_i^T} P\tilde{x}(t) + \tilde{x}^T(t)P\big(\tilde{A} + \tilde{B}k_j\big)\tilde{x}(t) + \tilde{x}^T(t)P + \tilde{R}_i{}^T + \tilde{d}^T) \tag{17}$$

$$\dot{V}\big(\tilde{x}(t)\big) = \sum_{i=1}^{r}\sum_{j=1}^{\Omega} h_i\,\big(z(t)\big)h_j\big(z(t)\big)\big(\tilde{x}^T(t)P\widetilde{A}_i + \tilde{B}k_j\big) + \Big(\big(\widetilde{A}_i + \tilde{B}k_j\big)^T P\Big)\,\tilde{x}(t) + \tilde{x}^T(t)P\widetilde{R}_i\tilde{d} + \tilde{d}^T + \tilde{d}^T\tilde{R}_i^T\tilde{P}\; x(t)) \tag{18}$$

According to Lemma 1, the following inequality is obtained

$$\tilde{x}^T(t)P\widetilde{R}_i\tilde{d} + \tilde{d}^T + \tilde{d}^T\tilde{R}_i^T P\tilde{x}^T \leq \tilde{x}^T(t)P\widetilde{R}_i P\tilde{x}(t) + \tilde{d}^T\tilde{d} \tag{19}$$

The equation is rewritten as (20):

$$\dot{V}\big(\tilde{x}(t)\big) \leq \sum_{i=1}^{r}\sum_{j=1}^{\Omega} h_i\,(\,z(t))\{h_j(z(t))\}[\tilde{x}^T(t)(P\big(\widetilde{A}_i + \tilde{B}k_j\big) + \big(\widetilde{A}_i + \tilde{B}k_j\big)^T P + P\widetilde{R}_i\widetilde{R}_i{}^T P)\tilde{x}(t) + \tilde{d}^T\tilde{d}] \tag{20}$$

Theorem 1: There exist two positive definite matrices $P \in R^{nxn}$ and $Q \in R^{nxn}$ such that the system (17) is asymptotically stable. There exists matrix Q such that:

$$P\big(\widetilde{A}_i + \tilde{B}k_j\big) + \big(\widetilde{A}_i + \tilde{B}k_j\big)^T)P + \tilde{R}_i\tilde{R}_i^T P + Q \leq 0 \tag{21}$$

For $Q = Q^T > 0$, we can verify that

$$\dot{V}\big(\tilde{x}(t)\big) \leq -\tilde{x}^T(t)Q\tilde{x}(t) + \tilde{d}^T\tilde{d}$$

The existence of the minimum eigen value λ_{min} which verify

$$-\lambda_{min}\|\tilde{x}(t)\|^2 \geq \big\|\tilde{d}(t)\big\|^2$$
$$then \qquad \dot{V}\big(\tilde{x}(t)\big) \leq 0$$

In order to obtain the LMIs form, we take into account the following variables: $X = P^{-1}$, $K_i = M_i X^{-1}$, $Q = PYP$, with $X > 0, Y \geq 0$ and $M_i (i = 1, \dots, r)$, we obtain the following stability condition:

$$\begin{bmatrix} A_{ii}X + XA_{ii}^T + B_{ii}M_i + M_i^T B_{ii}^T + Y_{ii} + \tilde{R}_i\tilde{R}_i^T & XQ \\ QX & -X \end{bmatrix} \leq 0 \tag{22}$$

$$\begin{bmatrix} \frac{1}{2}(A_{ii}X + XA_{ii}^T + A_{ij}X + XA_{ij}^T + B_{ii}M_j + M_j^T B_{ii}^T + B_{ij}M_i + M_i^T B_{ij}^T + \widetilde{R}_i\widetilde{R}_i^T + \widetilde{R}_J^T \widetilde{R}_j^T + Y_{ij}) & xQ \\ QX & -x \end{bmatrix} \leq 0 \tag{23}$$

where

$$y = \begin{bmatrix} Y_{11} & \cdots & Y_{1r} \\ \cdots & \cdots & \cdots \\ Y_{r1} & \cdots & Y_{rr} \end{bmatrix} > 0 \tag{24}$$

Hence, if the above conditions are satisfied, the closed loop system is asymptotically stable.

3.3. Gain scheduling PI controller

We propose a gain scheduling PI controller for the local linear subsystems. Figure 2 shows strategy for controlling dissolved oxygen.

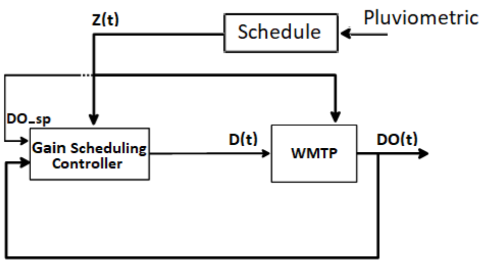

Figure 2. Gain adjustment strategy for controlling dissolved oxygen

The wastewater treatment process can be approximated as (25):

$$H(s) = \frac{K}{1+Ts} \tag{25}$$

It is about using a PI controller in the dissolved oxygen concentration control loop, considering a simplified process model. It identifies symbols K and T as representing the static gain and time constant, respectively. the closed-loop transfer function is given by (26):

$$H(s) = \frac{KK_p(T_i+1)}{T_iTs^2+(KK_pT_i+T_i)s+KK_p} \tag{26}$$

A prefilter has been added to the dissolved oxygen control loop to mitigate the effects of a potentially disruptive zero in the closed-loop system, which could cause significant overshoot in the dynamics.

$$F(s) = \frac{1}{1+T_is} \tag{27}$$

As a result, the transfer function is:

$$H_{BO}(s) = \frac{KK_p}{T_iTs^2+(KK_pT_i+T_i)s+KK_p} \tag{28}$$

where K_p and T_i are the parameters of the PI controller which is found through the pole placement approach. Since the transfer function of the closed-loop system in (27) is:

$$H(s) = \frac{\omega_n^2}{s^2+2\epsilon\omega_n s+\omega_n^2} \tag{29}$$

where

$$\omega_n^2 = \frac{KK_p}{T_iT};\ 2\epsilon\omega_n = \frac{KK_p+1}{T}$$

By defining criteria based on ilutionion time, response time, and percentage overshoot (%OS), and considering that:

$$H(s) = \frac{-ln(\%OS)}{\sqrt{\pi^2+ln^2(\%OS)}} \tag{30}$$

and

$$\omega_n = \frac{4}{t_r\epsilon} \tag{31}$$

The calculation of the parameters of the PI controller is given by $K_p = \frac{8T-T_r}{Kt_r}\ and\ T_i = \frac{t_r\epsilon^2(8T-T_r)}{16T}$.

4. RESULT AND DISCUSSION
4.1. Process discerption
The activated sludge bioprocess studied in this work consists of two tanks as shown in Figure 3. The initial tank is a biological reactor aerated with bacteria and various microorganisms, which serve to eliminate the organic matter. The author tank is a clarifier that separates the sludge from the purified effluent. A portion of the sludge is removed, while the other portion is recycled back into the aerated tank.

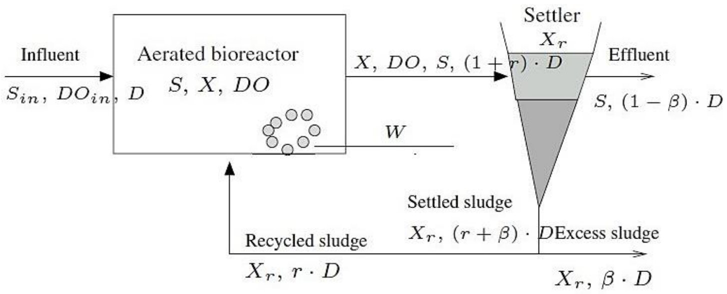

Figure 3. Aerobic wastewater treatment process: ASP

The dynamic response of the bioprocess and the different hypothesis are ilution in [19]; with r is the rate of sludge recycling and β is the rate at which sludge is removed. Then the dynamics of the plant are described as (32):

$$\frac{dX}{dt} = \mu(t)X(t) - D(t)(1+r)X(t) + rD(t)X_r(t) \tag{32}$$

$$\frac{dS}{dt} = -\frac{\mu(t)}{Y}X(t) - D(t)(1+r)S(t) + D(t)S_{in} \tag{33}$$

$$\frac{dDO}{dt} = -K_0\frac{\mu(t)}{Y}X(t) - D(t)(1+r)DO(t) + \alpha W\big(DO_{max} - DO(t)\big) + D(t)DO_{in} \tag{34}$$

$$\frac{dX_r}{dt} = D(t)(1+r)X(t) - D(t)(\beta+r)X_r(t) \tag{35}$$

where β is the rate of sludge removal. X is the biomass, S is the substarte, DO and X_r denote the dissolved oxygen and the recycled biomass, respectively. The non-linear rate of specific growth $\mu(t)$ is given by (36):

$$\mu(t) = \mu_{max}\frac{S(t)}{k_s+S(t)}\frac{DO(t)}{K_{DO}+DO(t)} \tag{36}$$

where $D(t)$ is the ilution rate and $W(t)$ is the aeration rate, substrate in the influent S_{in} and dissolved oxygen concentrations Do_{in}

In our study, we have used a reduced ASP mathematical model (analytic model). Table 1 describes the various parameters:

Table 1. Parameters of the system

Y =0.65	Biomass yield factor
β =0.2	The rate of sludge removal
α =0.018	Oxygen transfer rate
K_{DO} =2,mg/l	Saturation constants
K_0 =0.5	Model constants
μ_{max} =0.15,mg/l	Maximum specific growth rate
k_s =100,mg/l	Saturation constants
DO_{max} =10,mg/l	Maximum dissolved oxygen
r =0.6	Ratio of recycled flow to the influent

In order to describe the nonlinear model by a TS fuzzy model. The scheduling vector is given by (37):

$$z(t) = [D(t) \quad S(t)]^T \tag{37}$$

In order to find the fuzzy model $r = 2$ (number of nonlinearities) and $n = 4$ number of sub-models with the minimum or maximum values for decision variable z_i are $d_1 \leq z_1 \leq D_1, d_2 \leq z_2 \leq D_2$, respectively. Figure 4 shows the time variation of the system state variables:

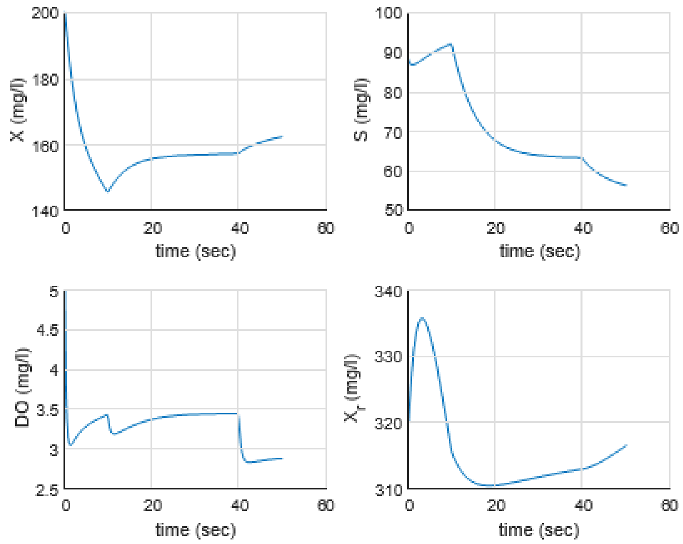

Figure 4. State variables of the ASP

The TS representation is based on $r = 4$ rules. We replace the parameters in the Table 1, and we obtain:

$$A_1 = \begin{bmatrix} -0.0432 & 0.0011 & 0.0003 & 0.0455 \\ -0.1202 & -0.1230 & -0.0005 & 0 \\ -0.0601 & -0.0008 & -0.6205 & 0 \\ 0.1214 & 0 & 0 & -0.0607 \end{bmatrix} \quad A_2 = \begin{bmatrix} 0.0192 & 0.0006 & 0.0002 & 0.0214 \\ -0.1171 & -0.0579 & -0.0003 & 0 \\ -0.0585 & -0.0005 & -0.3007 & 0 \\ 0.0569 & 0 & 0 & -0.0285 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 0.0068 & 0.0009 & 0.0339 & 0.0127 \\ 0.0127 & -0.0352 & -0.0522 & 0 \\ -0.0312 & -0.0007 & -0.1210 & 0 \\ -0.0007 & 0 & 0 & -0.0169 \end{bmatrix} \quad A_4 = \begin{bmatrix} -0.0012 & 0.0009 & 0.1196 & 0.0074 \\ -0.0286 & -0.0212 & -0.1841 & 0 \\ -0.0143 & -0.0007 & -0.1706 & 0 \\ 0.0198 & 0 & 0 & -0.0099 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 & 140.7595 \\ 0 & -192.6664 \\ -0.5592 & -65.7072 \\ 0 & -189.6118 \end{bmatrix} \quad B_2 = \begin{bmatrix} 0 & 140.7359 \\ 0 & -182.1245 \\ -0.5532 & -65.1720 \\ 0 & -188.7495 \end{bmatrix}$$

$$B_3 = \begin{bmatrix} 0 & 126.6169 \\ 0 & -159.6921 \\ 0.1375 & -3.7803 \\ 0 & -171.1206 \end{bmatrix} \quad B_4 = \begin{bmatrix} 0 & 101.1574 \\ 0 & -127.8967 \\ 0.1661 & -1.2369 \\ 0 & -138.5586 \end{bmatrix}$$

$$R_1 = \begin{bmatrix} 0 & 0 \\ 0.0758 & 0 \\ 0 & 0.0758 \\ 0 & 0 \end{bmatrix} \quad R_2 = \begin{bmatrix} 0 & 0 \\ 0.0356 & 0 \\ 0 & 0.0356 \\ 0 & 0 \end{bmatrix}$$

$$R_3 = \begin{bmatrix} 0 & 0 \\ 0.0211 & 0 \\ 0 & 0.0211 \\ 0 & 0 \end{bmatrix} \quad R_4 = \begin{bmatrix} 0 & 0 \\ 0.0124 & 0 \\ 0 & 0.0124 \\ 0 & 0 \end{bmatrix}$$

Table 2 provides the considered control restrictions and the input of the system is given in Figure 5:

Table 2. System parameters

Parameters	Value
rate h_1	$0.01 \leq D \leq 0.38$
Influent substrate *g/l*	$60 \leq S_{in} \leq 140$

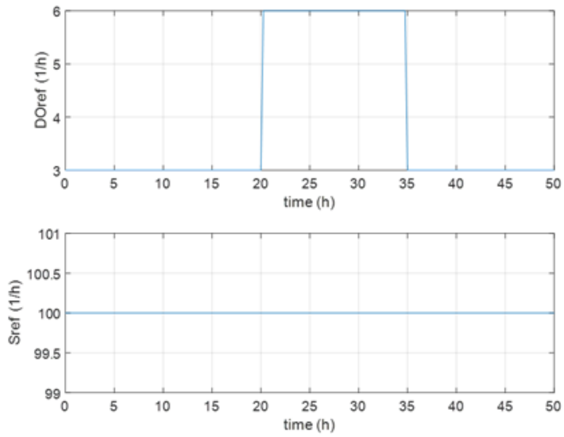

Figure 5. Input of the system

4.2. Fuzzy tracking controller

The advantages from the LMIs resolution using MATLAB's YALMIP solver are as follows, with a feasibility index of $t = 0.435$.

$$K_{1i_1} = \begin{bmatrix} 10.9377 & -1.1886 & 303.4125 & -14.5537 \\ 0.1133 & 0{:}\,3857 & -3.59865 & 0.0282 \end{bmatrix} \quad K_{1i_2} = \begin{bmatrix} 5.0340 & -26.3194 & 304.1947 & -4.9961 \\ -0.1272 & 0.4872 & -3.5652 & 0.0328 \end{bmatrix}$$

$$K_{1i_3} = \begin{bmatrix} -3.9090 & -3.5153 & -80.7389 & 6.0214 \\ -0.0071 & 0.0955 & 0.8762 & 0.0100 \end{bmatrix} \quad K_{1i_4} = \begin{bmatrix} 0.2918 & -3.6229 & -76.1472 & 8.5326 \\ 0.0076 & 0.0632 & 1.0669 & 0.0141 \end{bmatrix}$$

$$K_{2i_1} = \begin{bmatrix} 1.9456 & -109.9350 \\ -0.3336 & 0.9615 \end{bmatrix} \quad K_{2i_2} = \begin{bmatrix} 20.1603 & -93.1369 \\ -0.4694 & 0.7528 \end{bmatrix}$$

$$K_{2i_3} = \begin{bmatrix} 1.4729 & 29.3508 \\ -0.1793 & -0.6075 \end{bmatrix} \qquad K_{2i_4} = \begin{bmatrix} 1.6630 & 28:1754 \\ -0.1463 & -0.6349 \end{bmatrix}$$

Figure 6 displays the simulation findings. The output tracking is completed.

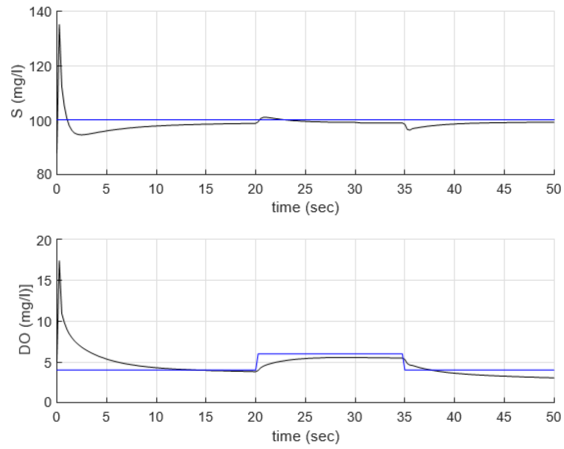

Figure 6. Fuzzy tracking control of the dissolved oxygen

4.3. GS-PI controller

The reduction transfer function:

$$H(s) = \frac{-14,9677}{s + 0.2268} \tag{38}$$

The PI controller's parameters are determined in the following manner:

$$K_p = \frac{8\mathrm{T}-T_r}{Kt_r} = 1.7 \quad \text{and} \quad T_i = \frac{t_r\varepsilon^2(8\mathrm{T}-T_r)}{16T} = 0.079$$

The comparison result between the two controllers is shown in Figures 7 and 8. The design of a fuzzy controller offers significant advantages, particularly the ability to regulate multiple parameters simultaneously through the mathematical approach provided by LMI. Furthermore, the seamless integration of standard numerical optimization tools is facilitated by the use of LMIs.

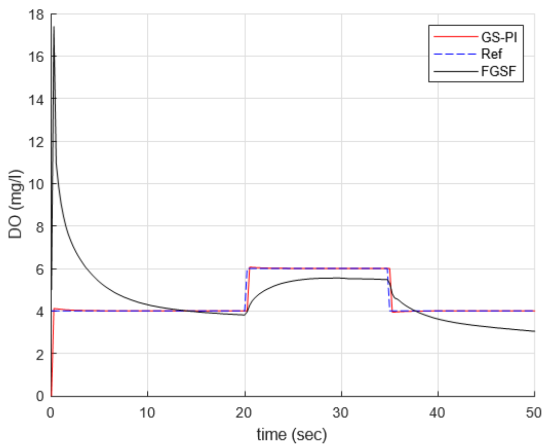

Figure 7. Evolution of the system outputs

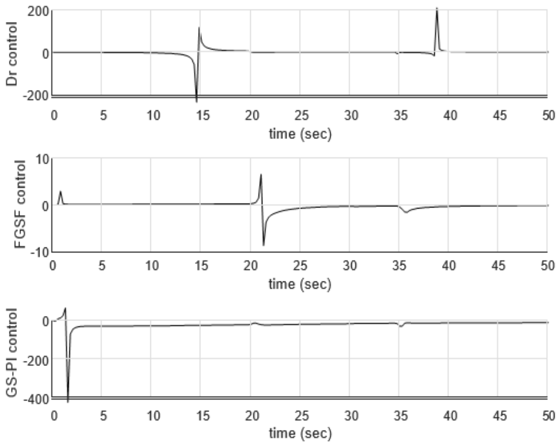

Figure 8. Evolution of control signals

Given the specificity of biological wastewater treatment processes, which are highly complex and nonlinear, heavily dependent on the incoming water quality, it is difficult to assess the controller's performance using classical metrics. We rely on other more important criteria, particularly energy effort. The advantage lies in the reduced energy consumption in large water treatment plants compared to GS-PI controllers shown in Figure 8, the fuzzy tracking controller appears to be the preferred choice.

5. CONCLUSION

This paper proposes a Takagi-Sugeno fuzzy integral control to achieve trajectory tracking in an ASP. The state feedback gains are determined through linear matrix inequalities. In order to demonstrate the robustness of our control, a comparison is drawn between the fuzzy integral control and the GS-PI controller for regulating the dissolved oxygen concentration. The results show the effectiveness of both controllers. However, in terms of energy consumption and implementation of optimization techniques, the fuzzy integral control controller outperforms the GS-PI controller, despite the latter's speed and the presence of a filter in its architecture. This study focuses exclusively on the control of the bioprocess in the absence of defects, paving the way for future work on the development of fault-tolerant control for this process.

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