

## Investigation of auto-oscillational regimes of the system by dynamic nonlinearities

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### ABSTRACT

The paper proposes a method for the analysis and synthesis of self-oscillations in the form of a finite, predetermined number of terms of the Fourier series in systems reduced to single-loop, with one element having a nonlinear static characteristic of an arbitrary shape and a dynamic part, which is the sum of the products of coordinates and their derivatives. In this case, the nonlinearity is divided into two parts: static and dynamic nonlinearity. The solution to the problem under consideration consists of two parts. First, the parameters of self-oscillations are determined, and then the parameters of the nonlinear dynamic part of the system are synthesized. When implementing this procedure, the calculation time depends on the number of harmonics considered in the first approximation, so it is recommended to choose the minimum number of them in calculations. An algorithm for determining the self-oscillating mode of a control system with elements that have dynamic nonlinearity is proposed. The developed method for calculating self-oscillations is suitable for solving various synthesis problems. The generated system of equations can be used to synthesize the parameters of both linear and nonlinear parts. The advantage is its versatility.

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## 1. INTRODUCTION

The use of dynamic nonlinearities in control systems of technical objects expands the possibilities of synthesis of these systems. We classify dynamic elements as nonlinear elements that contain links that make it possible to obtain, in general terms, the sums of products of coordinates and their derivatives of arbitrary degrees and orders. In addition, the system may include elements with static nonlinear characteristics. In some of these systems, the operating modes are stable periodic oscillations-of self-oscillations. Such properties are typical for control systems for electromechanical objects, electrical power systems, and electric drives with frequency regulation.

Currently, there is practically no information on possible methods for calculating such systems. Attempts to use, for these purposes, methods for studying the calculation of systems with static nonlinear characteristics turned out to be ineffective, due to either low universality or unsuitability for synthesis [1]. Digital modeling, which has become quite widespread in recent years, can be expensive or time-consuming if

it is not preceded by a study of the system using approximate or exact methods [2], [3]. Therefore, it seems appropriate to use a computer in the analytical approach to solve a system of equations, some of which are nonlinear algebraic.

In works [4]–[10] a numerical-analytical method for calculating self-oscillations in systems with elements having static nonlinear characteristics was presented. For the development of the results obtained, it is of interest to be able to extend them to systems with dynamic nonlinearities while retaining such advantages as universality and algorithm city. Let the system under study of the class under consideration satisfy the following restrictions: [11]–[14]: i) They can be divided into two parts - linear (LP) and nonlinear (NP), which in turn can consist of static (NSP) and dynamic (NDP) parts; ii) The function describing the nonlinear static part satisfies the Dirichlet conditions; and iii) The output variable of the system is a periodic arbitrary waveform.

The block diagram of the system from the class under consideration is shown in Figure 1. The transfer function of the linear part is not degenerate and has the form:

$$W(p) = M(p)/N(p) = \sum_{i=0}^m b_i p^i / \sum_{i=0}^n a_i p^i, \tag{1}$$

where  $p = d/dt$ ;  $a_i, b_i$  – polynomial coefficients,  $m < n$ .

The nonlinear static characteristic is described using a polynomial spline of degree  $r$  defect  $k$  on the grid  $\Delta$ :

$$y = F(x) + S_{r,k}(x) = \sum_{i=0}^r a_{si} x^i, \quad \forall x \in \Delta \tag{2}$$

where  $a_{si}$  - spline coefficients  $x^i$ - input variables.

The link of the NDP system performs the transformation of the input variable ( $v_c$ ) weekend ( $y_d$ ) the following kind:

$$(y_d) = \sum_{i,j=0}^{l,j} c_{ij} y_s^i y_s^{(j)}. \tag{3}$$

here  $y_s$  - static characteristic output,  $y_d$  - output dynamic response,  $c_{ij}$  - coefficient transformations.

In what follows, for definiteness, we will consider the NDP link, in which only the coefficients  $c_{00}$  and  $c_{11}$  in expression (3) are different from zero, i.e.,

$$(y_d) = c_{00} y_s + c_{11} y_s p y_s \tag{4}$$

Desired self-oscillations  $z(t)$  write in the form (5),

$$z(t) = Az_0 + \sum_{k=1}^L A_{zk} \sin(k\omega t + \alpha_{zk}) \tag{5}$$

where  $L$ - specified number of harmonics to be considered;  $A_{zk}$  - self-oscillation amplitudes;  $\alpha_{z1}=0$  as a result of the choice of origin. The procedure for analyzing self-oscillations (5) consists of two stages [15]–[18]: ii) calculation of the parameters of all possible periodic modes in a given system and ii) selection of stable modes (self-oscillations) from all modes.

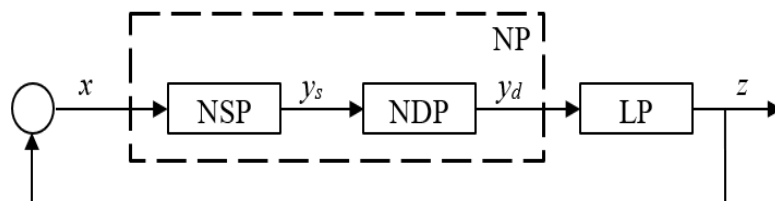


Figure 1. Block diagram of the system

The following is the order in which the material is presented: section 2 explains the method of solving the problem and reveals the essence of the proposed algorithm. Section 3 contains the simulation results that were used to verify the proposed self-oscillatory modes of the system by dynamic nonlinearities. Section 4 concludes with a conclusion and recommendations for further use and development of the proposed approach.

## 2. METHOD

For any system from the class under consideration, the balance equations for amplitudes and phases are valid, which follows from the strong connectivity of the system graph. They look like this:

$$y_{dk}|W(jk\omega_1)| = A_{zk}, k \in [0, L]; \beta_{dl} + \arg W(jl\omega_1) = \alpha_{zl}, l \in [1, L] \quad (6)$$

where  $y_d(t) = y_{d0} + \sum_{k=1}^{\infty} y_{dk} \sin(k\omega_1 t + \beta_{dk})$ ; and  $y_s(t) = y_{s0} + \sum_{k=1}^{\infty} y_{sk} \sin(k\omega_1 t + \beta_{sk})$

Taking into account expression (4), we can obtain a relationship between the coordinate parameters  $y_d(t)$  and  $y_s(t)$ :

$$\begin{aligned} y_{d0} &= c_{00}y_{s0}; \\ y_1 &= \frac{1}{2}c_{11}y_{c1}\omega_1 \sqrt{y_0^2 + y_{s2}^2 + 2y_0y_{s2}\cos(2\beta_{s1} - \beta_{s2} - \beta_{s0}) + \dots}, \\ \beta_{d1} &= \beta'_{d1} + \pi; \operatorname{tg}\beta'_{d1} = \frac{y_0\sin(\beta_{s1} - \beta_{s0}) + y_{s2}\sin(\beta_{s2} - \beta_{s1}) + \dots}{y_0\cos(\beta_{s1} - \beta_{s0}) + y_{s2}\cos(\beta_{s2} - \beta_{s1}) + \dots}, \end{aligned}$$

where

$$y_0 = 2\sqrt{y_{s0}^2 + c_{00}^2/(c_{11}^2\omega_1^2)}; \operatorname{tg}\beta_{s0} = -c_{11}y_{s0}/c_{00} \quad (7)$$

The system of (6), taking into account relations (7), is formally sufficient to determine the parameters  $z(t)$ , however, it is not possible to obtain efficient solution procedures due to its complexity. It is advisable, keeping the system (6), to supplement it with one more equation, which will allow solving the newly formed, extended system of equations.

We use the analytical-numerical method for calculating essentially nonlinear systems [1] to obtain the required additional equation [19]–[21]. Let us write the equations of dynamics of the system under study from the class under consideration in the following form:

$$x + z = 0; \quad y_s = F(x); \quad y_d = c_{00}y_s + c_{11}y_s p y_s, \quad z = W(p)y_d. \quad (8)$$

where  $F(x)$  - functional.

Further coordinates  $y_s(t)$  and  $z(t)$  expanded into the Taylor series in the neighborhood of some arbitrarily chosen expansion point  $t_p, t_p \in [0, 2\pi/\omega_1)$ , in which the y-axis is placed:

$$z(t_p + t) = \sum_{i=0}^{\infty} R_{zi}t^i/i!, \quad y_s(t_p + t) = \sum_{i=0}^{\infty} R_{yi}t^i/i!. \quad (9)$$

where  $R_{zi}$  and  $R_{yi}$  - Taylor series coefficients.

Between odds  $R_{zi}$  and  $R_{yi}$  near (9) there is a one-to-one relationship determined from the relations between the coordinates  $x$ ,  $y_s$ , and  $z$  for instance,

$$R_{y0} = \sum_{i=1}^r R_{si}(-R_{z0})^i, \quad R_{y1} = R_{z1} \sum_{i=1}^r ia_{si}(-R_{z0})^{i-1},$$

Let us transform the third and fourth equations of system (8) into one equation for the possibility of referring to the solution procedure. To do this, we will make the following transformations:

$$\begin{aligned} -M(p)y_d &= -M(p)(c_{00}y_s + c_{11}y_s p y_s) \\ &= -M(p) \left[ c_{00}y_s + c_{11}y_s \left( R_{y1} + \sum_{i=2}^{\infty} \frac{R_{yi}t^i}{(i-1)!} \right) \right] = -M_1(p)y_s - \sum_{i=0}^{\infty} \frac{H_i t^i}{i!} \end{aligned} \quad (10)$$

where  $M_1(p) = (c_{00} + c_{11}R_{y1})M(p)$ ;  $\sum_{i=0}^{\infty} \frac{H_i t^i}{i!} = c_{11}M(p) \left[ \sum_{i=0}^{\infty} \frac{R_{yi}t^i}{i!} \right] \sum_{i=2}^{\infty} \frac{R_{yi}t^{i-1}}{(i-1)!}$ ;  $H_0 = c_{11}b_1 \frac{R_{y0}R_{y2}}{0!1!} + 2c_{11}b_2 \left( \frac{R_{y1}R_{y2}}{1!1!} + \frac{R_{y0}R_{y3}}{0!2!} \right) + \dots$ ,

Taking into account transformations (10), a system of (8) can be written as  $x + z = 0$

$$-a_{s1}x + y_c = \sum_{i=0}^{\infty} \frac{T_i t^i}{i!}, \quad -M(p)y_c + N(p)z = \sum_{i=0}^{\infty} \frac{H_i t^i}{i!}. \quad (11)$$

Next, the system of (11) is transformed according to Laplace, and the solution is found  $z(s)$  ( $s$ -complex variable) according to Cramer's rule, we obtain

$$z(s) = \frac{SQ(s)+M_1(s)\sum_{i=0}^{\infty} T_i s^{-i} + \sum_{i=0}^{\infty} H_i s^{-i}}{N(s)+\alpha_{s1}M_1(s)} \frac{1}{s} = \sum_{i=0}^{\infty} R_{zi} s^{-1} \frac{1}{s}, \tag{12}$$

where  $Q(s)$  - polynomial of preinitial.

From (12) we determine the coefficients  $R_{zi}$ ,  $i \in [0, n]$ . For this, the complementary equation of the analytical-numerical method can be obtained from the expression for  $R_{zn}$ , because the coefficients  $R_{zi}$ ,  $i \in [0, n - 1]$  identically equal  $z_0^{(i)}$ . We have (13):

$$\sum_{i=0}^m (c_{00} + c_{11}R_{y1})b_i T_i + H_0 - \sum_{i=0}^n [\alpha_i + \alpha_{s1}(c_{00} + c_{11}R_{y1})b_i]z_0^{(i)} = 0 \tag{13}$$

In (13) relates the parameters of the system to the parameters of periodic oscillations. After transformations, we obtain an algebraic equation, for example, the amplitude  $A_{z1}$  first harmonic:

$$\sum_{i=0}^{2r} \alpha_i A_{z1}^i = 0, \tag{14}$$

where  $\alpha_i = \alpha_i(p_k, \omega_1, \alpha_{ze}, \alpha_j, b_j, c_{00}, c_{11}, \alpha_{s\lambda}, t_p)$ ;  $p_k = \frac{A_{zk}}{A_{z1}}$ .

The complete system of equations for unknown parameters of periodic modes takes the following form [22]–[24]:

$$\begin{aligned} \sum_{i=0}^{2r} \alpha_i A_{z1}^i &= 0; \quad y_{dk} |W(jk\omega_1)| = p_k A_{z1}, \quad l \in [0, L]; \\ \beta_{dl} + \arg W(jl\omega_1) &= \alpha_{z1}, \quad k \in [1, L]. \end{aligned} \tag{15}$$

We assume that the desired periodic regime exists in the form of two components: some first approximation  $z_{p,p}(t)$ , and a segment of the Fourier series  $z_F(t)$ , obtained from the complete solution  $z(t)$ , minus the first approximation. An iterative procedure for step-by-step calculation of two components with subsequent refinement  $z_{p,p}(t)$ , due to accounting  $z_F(t)$ , ends when the predetermined accuracy of the solution is reached.

Definition of harmonic composition  $z_{p,p}$  and  $z_F$ , based on a preliminary study of the system. As a first approximation  $z_{p,p}(t)$ , includes the fundamental harmonics of the oscillations  $z(t)$ , (for many systems this is one and often the first harmonic), which is  $z_F(t)$ , can contain harmonics both smaller and larger than the harmonics  $z_{p,p}(t)$ , frequencies. Let us write down the system of equations for determining the parameters of the first approximation  $z_{p,p}(t)$ , having the form of a harmonic signal with a constant component:

$$z_{p,p}(t)A_{z0} + A_{z1} \sin \omega_1 t \tag{16}$$

Let us introduce the following notation:

$$\begin{aligned} W(jl\omega_1) &= [M_{Rl}(\omega_1) + jM_{Il}(\omega_1)]/[N_{Rl}(\omega_1) + jN_{Il}(\omega_1)] \\ M_{Rl}(\omega_1) &= \sum_{i=0}^m (-1)^{0.5i} b_i (l\omega_1)^i, \quad N_{Rl}(\omega_1) = \sum_{i=0}^n (-1)^{0.5i} \alpha_i (l\omega_1)^i, \quad i = 0, 2 \dots, \\ M_{Il}(\omega_1) &= \sum_{j=0}^m (-1)^{0.5(j-1)} b_j (l\omega_1)^j, \quad N_{Il}(\omega_1) = \sum_{j=0}^n (-1)^{0.5(j-1)} \alpha_j (l\omega_1)^j, \quad j = 1, 3 \dots \end{aligned}$$

Then from the system of (15), considering relations (7), after transformations, we obtain

$$\begin{aligned} \sum_{i=0}^{2r} d_i A_{z1}^i &= 0, \quad c_{00} b_0 y_{s0} = \alpha_0 A_{z0}; \\ y_{s1} \sqrt{c_{00}^2 + c_{11}^2 y_{s0}^2 \omega_1^2} \sqrt{M_{R1}^2 + M_{I1}^2} &\sqrt{1 + p_{y2}^2 + 2p_{y2} \cos(2\beta_{s1} - \beta_{s2} - \beta_{s0}) + \dots} = A_{z1} \sqrt{N_{R1}^2 + N_{I1}^2}; \\ \frac{\sin(\beta_{s1} - \beta_{s0}) + p_{y2} \sin(\beta_{s2} - \beta_{s1}) + \dots}{\cos(\beta_{s1} - \beta_{s0}) + p_{y2} \cos(\beta_{s2} - \beta_{s1}) + \dots} &= \frac{M_{R1}N_{I1} - M_{I1}N_{R1}}{M_{R1}N_{I1} + M_{I1}N_{R1}}, \end{aligned} \tag{17}$$

where  $p_{y2} = y_{s2}/y_{s0}$ .

Often the components  $y_{sk}$  ( $k > 1$ ) of the coordinate  $y_s(t)$  are significantly smaller than the components  $y_{c0}$  and  $y_{c1}$ . In this case, the second and third equations of system (17) are simplified and have the following form:

$$y_{s1} \sqrt{c_{00}^2 + c_{11}^2 y_{s0}^2 \omega_1^2} \sqrt{M_{R1}^2 + M_{I1}^2} = Az_1 \sqrt{N_{R1}^2 + N_{I1}^2}; \quad \frac{c_{00} \text{tg} \beta_{s1} + c_{11} y_{s0} \omega_1}{c_{00} - c_{11} y_{s0} \omega_1 \text{tg} \beta_{s1}} = \frac{M_{R1} N_{I1} - M_{I1} N_{R1}}{M_{R1} N_{I1} + M_{I1} N_{R1}}. \quad (18)$$

In the case of an unambiguous static nonlinear characteristic and the first approximation in the form (16), the phase is equal to (19):

$$\frac{c_{11} y_{s0} \omega_1}{c_{00}} = \frac{M_{R1} N_{I1} - M_{I1} N_{R1}}{M_{R1} N_{I1} + M_{I1} N_{R1}}. \quad (19)$$

In the case of calculating systems for describing the static nonlinear characteristics of which the cubic splines of the defect are used 1 ( $r=3, k=1$ ), and the degrees of polynomials  $W(p) - (1); m = 1, n = 4$ , coefficients at the lowest  $d_0$  and senior  $d_6$  degrees  $A_{z1}$  are determined by the following expressions:

$$d_0 = c_{00} b_0 (\alpha_{s0} - \alpha_{s1} A_{z0} + \alpha_{s2} A_{z0}^2 - \alpha_{s3} A_{z0}^3) - \alpha_0 A_{z0}; \\ d_6 = 3c_{11} \alpha_{s3}^2 \omega_1 \sin^4 \omega_1 t_p [b_0 \sin \omega_1 t_p \cos \omega_1 t_p + b_1 \omega_1 (5 \cos^2 \omega_1 t_p - \sin^2 \omega_1 t_p)].$$

When determining the parameters and form of the periodic mode by calculating a certain first approximation and its subsequent refinement taking into account harmonics with smaller amplitudes, an approach is proposed that allows one to abandon harmonic linearization and accept the spline approximation [25]. We transform the system of (8) into one nonlinear differential equation.

$$N(p)z = M(p)[c_{00}S(z) + c_{11}F_1(z)pz], \quad (20)$$

where  $S(z) = S_{r,k}(-z) = y_s = \sum_{i=0}^r (-1)^i \alpha_{si} z^i$ ;  $F_1(z)pz = S_{r,k}(-z)pS_{r,k}(-z) = [\sum_{i=0}^r (-1)^i \alpha_{si} z^{i-1}]$ .

When determining the parameters of the first approximation  $z_{p,p}(t)$  from (20) one can obtain a system of equations for any of  $k$  harmonics,  $k \in [k_1, k_n]$ , included in  $z_{p,p}(t)$ :

$$N(p)z_{pk} = M(p)y_{dk},$$

where  $z_{p,p}(t) = \sum_{k=k_1}^{kn} z_{nk}(t)$ ;  $y_d(t) = \sum_{k=k_1}^{kn} y_{dk}(t) = c_{00}s(z_{p,p}) + c_{11}F_1(z_{p,p})pz_{p,p}$ .

To calculate the component  $z_F(t)$  and clarifications  $z_{p,p}(t)$  the following system of equations can be obtained:

$$N(p)z_{pk} = M(p)(y_{dk} + F_{pk}), \quad k \in [k_1, k_n]; \quad N(p)z_{Fl} = M(p)(y_{dl} + F_{Fl}), \quad l \in [l_1, L], \quad (21)$$

where  $z_{Fl}(t) = \sum_{l=l_1}^L z_{Fl}(t)$ ,  $F(t) = F_{p,p}(t) + F_F(t) = \sum_{k=k_1}^{kn} F_{nk}(t) + \sum_{l=l_1}^L F_{Fl}(t) = \delta y_d$ .

At each iteration, the first  $(k_n - k_1 + 1)$  equations of system (21) are used to determine and refine the parameters  $z_{p,p}(t)$  the following  $(L - l_1 + 1)$  equations, to determine and refine the parameters  $z_F(t)$ . When implementing this procedure, it turned out that the calculation time to the greatest extent depends on the number of harmonics taken into account in the first approximation, so it is recommended to choose the number  $k_n$  minimally and increase it only in the following cases: 1) in the absence of  $z_{p,p}$  in a priori chosen form, 2) with poor convergence of the proposed procedure.

In order to single out stable regimes (auto-crawling), it is sufficient to investigate the stability of the oscillations found in the small. Then (20) can be written in vector form in the following form:

$$D_1 pz = Az + c_{00}BS(z), \quad (22)$$

where

$$z = \text{colon}(z_1, z_2, \dots, z_n); \quad pz = \text{colon}(z_1, z_2, \dots, z_n); \quad A = \text{diag}(A_1, A_2, \dots, A_n); \\ B = \text{colon}(B_1, B_2, \dots, B_n); \quad z_i = W_i(p)y_d = \frac{B_i}{p - A_i} y_d; \quad z = \sum_{i=1}^n z_i.$$

From (22) one can obtain the equation of the first approximation in variations  $\delta z$ , which represent deviations from the unperturbed motion - the regime studied for stability  $z(t)$  :

$$p\delta z = D_3\delta z, \quad (23)$$

where

$$\delta z = colon(\delta z_1, \delta z_2, \dots, \delta z_n); \delta z = \sum_{i=1}^n \delta z_i; D_3 = D_2^{-1}D_1;$$

$$D_2 = A + c_{00}BS_\delta; S_\delta = \frac{dS(z)}{dz}.$$

Equation (23) corresponds to some characteristic equation:

$$\varphi(p_0) = \det[D_3 - p_0E] = 0. \quad (24)$$

Having estimated the roots of (24), we can conclude that the periodic regime is stable. To find roots  $p_{oi}, i \in [1, n]$ , it is necessary to determine  $n$  systems of partial solutions of the system of (23) on the period  $T_z = 2\pi/\omega_1$ , satisfying the initial conditions forming the identity matrix. The developed method for calculating self-oscillations is suitable for solving various synthesis problems. The generated system of equations (15) can be used to synthesize the parameters of both the linear and nonlinear parts. The advantage is its versatility.

From the whole variety of synthesis problems, we will consider the most characteristic problem for this class of systems, the problem of synthesis of coefficients  $c_{00}$  and  $c_{11}$  dynamic nonlinear part. For coefficient synthesis  $c_{00}$  and  $c_{11}$  according to the given parameters of self-oscillations, (6) of the balance of amplitudes and phases are used. For definiteness, let us consider the case of the presence of self-oscillations in the system in the form (16) provided that the components  $y_{sk}, k > 1$ , less  $y_{s0}$ , and  $y_{s1}$ . Then the system of equations for determining the coefficients  $c_{00}$  and  $c_{11}$  and unknown frequency  $\omega_1$  according to the given parameters  $A_{z0}$  and  $A_{z1}$  is combined as (25):

$$c_{00} = p_0 a_0 / b_0; (c_{00}^2 + c_{11}^2 y_{s0}^2 \omega_1^2) \frac{M_{R1}^2 + M_{I1}^2}{N_{R1}^2 + N_{I1}^2} = p_1^2;$$

$$c_{11} = \frac{c_{00}}{y_{s0} \omega_1} \frac{M_{R1} N_{I1} - M_{I1} N_{R1} - (M_{R1} N_{I1} + M_{I1} N_{R1}) \operatorname{tg} \beta_{s1}}{M_{R1} N_{R1} + M_{I1} N_{I1} - (M_{R1} N_{I1} - M_{I1} N_{R1}) \operatorname{tg} \beta_{s1}}; \quad (25)$$

where  $p = A_{zk}/y_{sk}, k \in [0, 1]$ .

Substituting the third equation into the second of the system (25), we obtain an algebraic equation for the frequency  $\omega_1$ ;

$$c_{00} (M_{R1}^2 + M_{I1}^2) = p_1 \left| (M_{R1} N_{R1} + M_{I1} N_{I1}) \cos \beta_{s1} + (M_{R1} N_{I1} - M_{I1} N_{R1}) \sin \beta_{s1} \right|. \quad (26)$$

### 3. RESULTS AND DISCUSSION

Let us find the parameters of self-oscillations in the system shown in Figure 1, the nonlinear dynamic part of which is described by (4). The characteristic of the NSP link is approximated by a spline  $S_{3,1}(x)$  on an uneven grid  $\Delta: -18 \leq x \leq 18$ . The parameters of the links of the NDP and LP are as follows:  $c_{00} = 1; c_{11} = 1; m = 0; b_0 = 15; n = 4; \alpha_0 = 1; \alpha_1 = 4.9; \alpha_2 = 8.3; \alpha_4 = 1.2$ .

As a first approximation  $z_{p,p}(t)$  chooses a harmonic signal with a constant component (16). To find its parameters  $A_{z0}, A_{z1}$  and  $\omega_1$  we use the first and second equations of system (17), the first equation of system (18), and (19). As a result, we get (27):

$$z_{p,p}(t) = 3.08 + 10.9 \sin 1.06t \quad (27)$$

Next, we calculate the component  $z_F(t)$ . The number of harmonics taken into account by it is limited, for example, by the condition  $A_{zk} \geq 0.1 A_{z1}, k > 1$  and specify the parameters  $z_{p,p}(t)$ . Having performed two iteration steps, we finally obtain the following values of the oscillation parameters  $z(t)$ .

$$A_{z0} = 3.5; A_{z1} = 9.2; A_{z2} = 2.6; A_{z3} = 0.9; \alpha_{z1} = 1.27; \alpha_{z2} = 4.04; \alpha_{z3} = -0.71.$$

$$\left| 1 - A_{zkl} / A_{zkl(l-1)} \right| \leq 0.1, \quad k \in [0, 1].$$

where  $A_{zkl}$  and  $A_{zk(l-1)}$  – harmonic amplitude values  $z(t)$  respectively on  $l$ -th and  $(l-1)$ -th iteration.

Using the first Lyapunov method, it was found that the oscillations found are stable, i.e., self-oscillations. Let's find the coefficients  $c_{00}$  and  $c_{11}$  of the NDP link, described by (4), providing obtaining in the system of Figure 1 self-oscillations in the form (16) with given parameters  $A_{z0}=3$ , and  $A_{z1}=11$ . The frequency is determined during the synthesis.

From decomposition  $y_s(t)$  in a Fourier series according to known  $A_{z0}$ ,  $A_{z1}$  and by the NSP link we find:  $y_{s0}=0.61$ ;  $y_{s1}=5.36$ ;  $\beta_{s1}=\pi$ . From the first equation of system (25) we calculate  $c_{00}=0.328$ . Of the eight roots of (26), only three are positive:  $\omega_{1-1}=0.59$ ;  $\omega_{1-2}=2.65$ ;  $\omega_{1-3}=2.56$ . From the third equation of system (25) we determine the corresponding values of the coefficient  $c_{11}$ :  $c_{11-1}=-0.9$ ;  $c_{11-2}=-10.47$ ;  $c_{11-3}=9.69$ . As shown by checking the stability of given oscillations in a system with synthesized coefficients  $c_{00}$  and  $c_{11}$ , frequency only  $\omega_{1-1}=0.59$  corresponds to self-oscillations:

$$z(t) = 3 + 15 \sin 0.5t, \quad (28)$$

In this case, the parameters of the NDP link are as follows:  $c_{00}=0.328$ ;  $c_{11}=-0.9$ . To check the correctness of the synthesis, we calculated self-oscillations in a system with synthesized  $c_{00}$  and  $c_{11}$ , and oscillations (28) were chosen as the first approximation. As a result of one step of iterations, self-oscillations are found that actually take place in the system.

$$z(t) = 3 + 14.97 \sin(0.59t + 0.16) + 3.05 \sin(2 * 0.59t - 0.8) + 1.16 \sin(3 * 0.59t + 3.9).$$

#### 4. CONCLUSION

An algorithm for determining the self-oscillating mode of a control system with elements having dynamic nonlinearity is proposed. When determining the parameters and form of the periodic regime by calculating a certain first approximation and then refining it, taking into account harmonics with smaller amplitudes, we use an approach that allows us to abandon the harmonic linearization and accept the spline approximation. The developed method for calculating self-oscillations is suitable for solving various synthesis problems. A comparative analysis of the proposed approach with such well-known methods as the spectral analysis method showed its effectiveness associated with the linearization of nonlinear characteristics of the control system. The generated system of equations can be used to synthesize the parameters of both the linear and nonlinear parts. The advantages include its versatility.




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