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Optimal load management of autonomous power systems in conditions of water shortage

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ABSTRACT

The issues of optimizing the operation of micro hydropower plants in conditions of water scarcity, performed by additional connection to the grid of an energy storage system and wind power turbine, as well as optimal load management, are considered. It is assumed that the load of the system is a concentrated autonomous power facility that consumes only active power. The paper presents a rigorous mathematical formulation of the problem, the solution of which corresponds to the minimum cost of an energy storage system and a wind turbine, which allows for uninterrupted supply of electricity to power facilities in conditions of water shortage necessary for the operation of micro hydropower plants (under unfavorable hydrological conditions). The problem is formulated as a nonlinear multi-objective optimization problem to apply metaheuristic stochastic algorithms. At the same time, a significant part of the problem is taken out and framed as a subproblem of linear programming which will make it possible to solve it by a deterministic simplex method that guarantees to find the exact global optimum. This approach will significantly increase the efficiency of solving the entire problem by combining metaheuristic algorithms and taking into account expert knowledge about the problem being solved.

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1. INTRODUCTION

Recently, close attention has been paid to small-scale energy using renewable energy sources. The interest in the use of renewable energy sources (solar, wind, river water.) is explained by the lack of fuel purchases and the possibility of power energy supply to hard-to-reach areas. The latter is especially important for countries with mountainous terrain and sparsely populated areas, where laying a power grid is economically impractical. At the same time, these countries and regions are characterized by a large hydropower potential, which has led to the accelerated development of small and micro hydropower plants (HPPs) in them. The active construction and operation of these stations, which, as a rule, do not have reservoirs, is also associated with the desire to avoid [1], [2].

Micro HPPs (power up to 100 kW) can be installed almost anywhere. The hydraulic unit includes a water intake device, a power unit and an automatic control device. Micro HPPs are used as sources of power energy for farms, country villages, farms and small industries in hard-to-reach areas, where it is unprofitable to lay networks [3], [4]. The main advantage of small hydropower is safety from an environmental point of view. During the construction of facilities in this industry and their further operation, there are no harmful effects on the quality and properties of water. Modern stations have simple designs and are fully automated, i.e., they do not require the presence of a person during operation. The electric power of micro hydroelectric power plants meets the requirements of regulatory documents on voltage and frequency, and the stations are able to operate autonomously. The total service life of the station is more than 40 years (at least 5 years before the overhaul) [5], [6].

As is known, there is no common universal solution for the construction of microgrids in isolated areas. The layout of systems depends on financial and economic conditions, logistical opportunities and potential for the use of relevant renewable energy source (RES) on the ground, and the availability of relevant technological competences of engineering companies building microgrids. At the same time, there is an obvious tendency of transition from simpler solutions to more complex and technically advanced ones. For instance, the initial stage of evolution of "old" local systems is the addition of additional generating capacities based on RES to the diesel generator. The most common and popular types of such systems are wind-diesel and solar-diesel complexes. Since the generation of wind and solar power plants is stochastic, it is quite logical to move to the next stage, the efficiency of the system is increased by energy storage. For example, the lack of generation at night and the need for load balancing during the day do not allow photovoltaic generation to replace more than 20-30% of diesel capacity. The use of energy storage can extend the period of "clean" electricity use during the day and further reduce diesel consumption, as well as improve overall system reliability. As the cost of energy storage decreases in the future, it will increasingly be used in isolated power systems, reducing dependence on imported energy resources [7]–[10].

The problem of optimal load management in modern distribution systems is becoming particularly relevant in the era of Smart grid and Microgrid. In particular, this is due to the fact that the operation of autonomous power systems with a high degree of renewable energy sources can create significant system balancing problems. As a consequence, from the point of view of cost minimization, decentralized integration of renewable energy sources on the basis of Smart grid and Microgrid smart energy systems is currently seen as the most promising way to improve the stability and reliability of such systems [7]–[10].

Existing research in this area can be divided into several categories. A significant part of the work is aimed at development, optimization and integration of individual elements or devices of the power system such as multi terminal interlinking converters (ILCs) [11], [12], maximum power point tracker controllers [13]–[15], different other types of battery controllers [14], controller of six pulse three phase rectifier [15], maximum power point tracking [16], generators [17]. Another area of research is the optimization of placement from the modes of work of various elements, for example, reactive power compensation units [18]. The resulting optimization problems are often solved using population algorithms. In 2023, Myintzu *et al.* [19] presents a technique to allocate a shunt capacitor using the particle swarm optimization algorithm. A method of the optimal allocation of an energy storage system using the wild geese algorithm is proposed in [20]. Metaheuristic optimization algorithms are also used to solve optimization problems at a higher level, for example an economic dispatch problem in microgrid system [21] or a designing power system stabilizer [22].

Synchronization, ensuring optimally balanced operation of such hybrid generation facilities integrated into microgrids, is a complex engineering task requiring the use of appropriate automation and software. Improvement of these automation tools and software, increasing the efficiency of interaction of all elements of microgrids, is an important task of their further development. A number of works are devoted to the problems of optimizing the operation of generating stations of power supply systems using renewable energy sources, as well as to the technical and economic assessment of power supply to autonomous consumers. They mainly either provide economic justification of the efficiency of connection to the centralized power supply or consider the possibility of using local sources of small-scale energy [23]–[30].

The authors of many works on similar topics, using mathematical modeling methods, propose to create a technical and economic model to analyze the feasibility of large energy complementarity between different stations: to solve the problem of uninterrupted power supply at photovoltaic power plants at night, the authors [29], [30] propose to use the theory of complementarity of hydro and solar energy, which allows to solve the problem of unstable solar energy generation in the dark and at night, when solar insolation is less than the average daily. To reduce risks, increase the reliability and stability of such power systems, they are additionally equipped with energy storage devices [31]–[34]. As for the complementarity between wind and HPPs, everything depends on the average wind speed, so due to the instability of wind resources, the authors [35], [36] combined pumped storage power plants with wind power plants, striving for an optimal mode of complementary operation and maximizing profits [35].

In this paper, the authors consider the optimization of the operation of a hybrid autonomous system: micro hydropower plant, energy storage system and wind power plant in conditions of severe water shortage.

Optimization is performed in order to maintain a stable level of power generation and is also carried out by optimal load management of the following power objects: a group of private houses, an apartment building, a farm, a small industrial enterprise with predominant consumption of active power. The load power is conditionally divided into two components: strictly defined and variable, i.e., variable during the optimization process. At the same time, power consumption schedules have a daily maximum and vary depending on the day of the week.

2. METHOD

2.1. Accepted assumptions and initial data

The autonomous power system considered in the work is presented in a simplified form in Figure 1. Power energy generation is produced by a wind turbine and micro HPPs, there is also an energy storage system (ESS) device in the power system containing the energy storage itself and a converter connecting it to the grid. For the sake of certainty, will be assumed that a battery of accumulators or supercapacitors acts as an energy storage device [36], which, however, is not mandatory in this study.

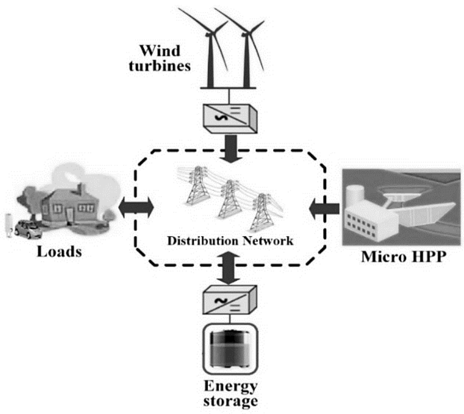

Figure 1. The model of the considered autonomous power system

When creating a mathematical model for optimal load management of an autonomous power system, the following assumptions are taken as a basis:

- The consumption schedule $P_{load}(t)$ is heterogeneous in time both during the day and for different days of the optimization interval, which is assumed to be equal to 168 hours–one week. A typical view of the dependence of $P_{load}(t)$ during the day is shown in Figure 2, where the average experimental curve of the energy consumption of the village in winter is presented;
- The power of the load can be conditionally represented as the sum of strictly specified $P_{const}(t)$ and variable $P_{var}(t)$ parts, of which only the latter can change during optimization. Obviously: $P_{load}(t) = P_{const}(t) + P_{var}(t)$;
- Since the total amount of power energy $W_{load} = \int_0^T P_{load}(t)dt$ consumed during the optimization interval T is known, respectively, the average power consumption $P_{ave} = \frac{W_{load}}{T}$ and, in addition, the value of energy consumed by the variable part of the load $W_{load,var} = \int_0^T P_{var}(t)dt$ can be set;
- The initial $W_{bat}(0)$ and the final value of energy $W_{bat}(T) = W_{bat}(0) + \int_0^T P_{bat}(t)dt$ of the ESS are the same, from which it follows that $\int_0^T P_{bat}(t)dt = 0$;
- The best mode of power energy generation for micro HPPs is the mode in which the minimum amount of water is consumed in the absence of a power shortage in the load nodes;
- The power generated by the wind turbine into the network $P_{wind}(t)$ is set based on the average statistical values for the winter period of the corresponding area. The average value of this power can be defined as $P_{ave,wind} = \frac{1}{T}\int_0^T P_{wind}(t)dt$;
- Losses in the elements of the autonomous power system under consideration are not taken into account, this simplifying assumption can be easily removed and accepted here so as not to overload the presentation with details.

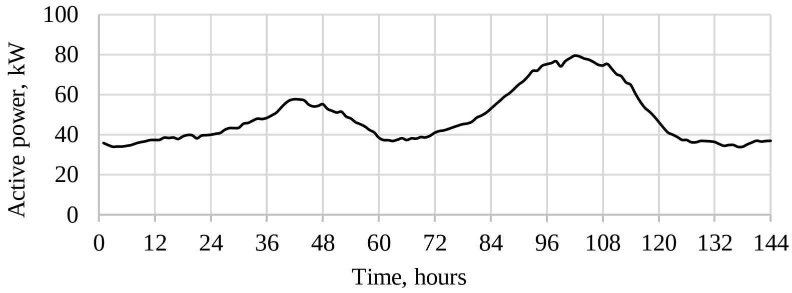

Figure 2. Experimental dependence $P_{load}(t)$ of village consumption in winter

2.2. Problem formulation

The goal of optimizing the local power system can be formulated as follows. It is necessary to control the variable part of the load $P_{var}(t)$ and the energy storage $P_{bat}(t)$ in such a way that two conditions are met simultaneously:

- The limit mode of power generation of micro HPPs takes place when their generating capacity does not change over time: $P_g(t) = P_{g,0} = const$. This condition corresponds to the situation of the winter period, when the power energy produced by micro HPPs is in short supply, and consumption is always the maximum possible, that is, uneven load means underutilization of available water;
- With a given power energy consumption W_{load}, the installed capacities of the wind turbine $P_{wind,max}$, ESS $P_{bat,max}$ and ESS capacity $W_{bat,max}$ should be minimal, this condition ensures the minimum cost of the entire system.

The formulated optimization goal can be written as (1).

$$\begin{cases} P_{wind,\max} \to \min, \\ P_{bat,\max} \to \min, \\ W_{bat,\max} \to \min, \\ \Delta = \delta\left(P_{g,ave}\right), \\ P_{wind,\max} \geq 0,\ P_{bat,\max} \geq 0, W_{bat,\max} \geq 0. \end{cases} \quad (1)$$

Here the variation δ is defined as (2).

$$\delta = \max_{t\in[0,T]} \frac{|P_{g,ave} - P_g(t)|}{P_{g,ave}},\ P_{g,ave} = \frac{1}{T}\int_0^T P_g(t)dt, \quad (2)$$

and Δ, the value of this variation specified in the equality type constraint.

The solution of this nonlinear multi-objective optimization problem (1) should be performed under a constraint $\Delta = \delta(P_{g,ave})$, the calculation of which significantly exceeds the complexity of the calculation of the minimized functions. Therefore, let's consider this restriction itself and the way it is calculated in more detail. The equality of the variation Δ to zero corresponds to the fact that the micro HPP operates in an optimal mode. However, in this case, the values$P_{wind,max}$, $P_{bat,max}$, $W_{bat,max}$ can be large and the whole system as a whole will be very expensive. Reducing the installed capacity of the wind turbine $P_{wind,max}$, ESS $P_{bat,max}$, as well as the ESS $W_{bat,max}$ of batteries reduces the cost of the system, but increasingly loads micro hydropower plants and, starting with some values, leads to the inability to maintain power energy generation P_g at a constant level and, consequently, the appearance of non-zero values of variation$\delta(P_{g,ave})$.

Of considerable interest is the solution of the multi-purpose task at various specified levels of variation Δ. The higher this level, the lower the cost of the entire system, but at the same time, the less efficient the micro hydropower plant works. The compromise between these mutually contradictory criteria depends on the specific situation (technical capabilities available for the implementation of the project, long-term plans for the development of this local network, the planned amount of funding) of the customer. Let's move on to the method of calculating the constraint $\Delta = \delta(P_{g,ave})$. In fact, it is necessary to specify an algorithm for calculating the optimal $P_g(t)$. Since with a known $P_g(t)$, calculating the constraint is not a problem and in this case the optimality is understood in the sense that using all the capabilities of the local power system, namely: control of the variable part of the load $P_{var}(t)$, control of the ESS $P_{bat}(t)$ and using the energy $P_{wind}(t)$ received from the wind turbine $P_{wind}(t)$ such values $P_{var}(t)$, $P_{bat}(t)$, $P_g(t)$ must be found at which the smallest deviation $P_g(t)$ from its average value $P_{g,ave}$ is possible for a given time interval $[0, T]$.

The complexity of task (1) can be somewhat reduced if the ratio of the cost of a kilowatt of installed power of a S_{wind} wind turbine and an S_{bat} ESS is known. Let $\frac{S_{wind}}{S_{bat}=m}$, then, due to the fact that $P_{wind,max}$ and $P_{bat,max}$ are the installed capacities of the wind turbine and ESS, respectively, and the meaning of the task (1), to find the cheapest optimal configuration of the local power system, we can proceed to (3).

$$\begin{cases} P_{wind,\max} + m \cdot P_{bat,\max} \to \min, \\ W_{bat,\max} \to \min, \\ \delta\left(P_{g,ave}\right) = \max\limits_{t\in[0,T]}\left(\left|P_{g,ave} - P_g(t)\right| / P_{g,ave}\right) = \Delta, \\ P_{wind,\max} \geq 0,\ P_{bat,\max} \geq 0,\ W_{bat,\max} \geq 0. \end{cases} \tag{3}$$

It is a nonlinear multi-objective problem, therefore the solution of (3) is the Pareto set (set of all Pareto efficient situations).

3. RESULTS AND DISCUSSION

Thus, "inside" (high-level) the nonlinear multi-objective problem formulated above, it is a multiple solution to a simpler problem needed: determining the optimal one $P_g(t)$ and calculating using it: $\delta\left(P_{g,ave}\right)$. Let us now consider the formulation of the "internal" (low-level) problem. The initial data for the task are:

– Dependencies $P_{const}(t)$ and $P_{wind}(t)$, their characteristic form is shown in Figure 2;
– The total amount of power energy W_{load} consumed during T and the total amount of energy $W_{load,var}$ consumed by the variable part of the load.

The statement of the problem also includes a restriction on the variables of the task and the relationships between them. The power balance [23] in the considered network is expressed by the ratio:

$$\forall_{t\in[0,T]}:\ \underbrace{P_{const}(t) + P_{var}(t)}_{P_{load}(t)} = P_g(t) + P_{wind}(t) + P_{bat}(t). \tag{4}$$

The load must consume energy W_{load} during the time T. In this case, the part of the energy of the load that is consumed by the variable part of the load $W_{load,var}$ in accordance with the assumptions adopted above is also given in (5):

$$W_{load} = \int_0^T P_{load}(t)dt\,,\ W_{load,var} = \int_0^T P_{var}(t)dt. \tag{5}$$

given as the initial data, and the value of is actually determined. For the ESS:

$$\forall_{t\in[0,T]}:\quad -Pbat_{bat,max}{}_{bat,max} \tag{6}$$

$$\forall_{t\in[0,T]}:\quad 0 \leq W_{bat}(t) \leq W_{bat,max} \tag{7}$$

The latter inequality will continue to be used in another equivalent form.

$$\forall_{t\in[0,T]}:\quad 0 \leq \int_0^t P_{bat}(t)dt \leq W_{bat,max} \tag{8}$$

due to the fact that $W_{bat}(T) = W_{bat}(0)$:

$$\int_0^t P_{bat}(t)dt = 0. \tag{9}$$

The variable component $P_{var}(t)$ of the power of the load must be non-negative, as well as the power $P_g(t)$ produced by the micro HPP is positive and cannot exceed the total installed capacity of generators.

$$\forall_{t\in[0,T]}: \quad 0 \le P_{var}(t) \le Pg_{g,max}{}_{var,max.} \tag{10}$$

3.1. Transition to discrete time

Let's introduce into consideration the vectors $P_{var}, P_{bat}, P_g, P_{wind}, P_{load}, P_{const}$, the elements of which are the values, respectively, of the values of $P_{var}(t)$, $P_{bat}(t)$, $P_{net}(t)$, $P_{wind}(t)$, $P_{load}(t)$, $P_{const}(t)$ at discrete moments of time t_k: $\{t_1 = 0;\ t_k = t_{k-1} + h;\ t_N = T\}$, where *h* = 10 minutes is the observation step in the calculations. Then P_{var}, P_{bat} and similarly for all introduced vectors:

$$P_{var} = \left[P_{var,1}, P_{var,2}, \dots, P_{var,N}\right]^T,\ P_{bat} = \left[P_{bat,1}, P_{bat,2}, \dots, P_{bat,N}\right]^T. \tag{11}$$

In (11), *T* is the transposition symbol and, for the sake of brevity, the notation $P_{var,k} = P_{var}(t_k)$, $P_{bat,k} = P_{bat}(t_k)$ is adopted, which will then be used for all vectors. the unknown vector quantities of the task are the vectors P_{var}, P_{bat}, P_g. Let's combine them into one vector of unknown *Y*:

$$Y = \begin{bmatrix} P_{var} \\ P_{bat} \\ P_g \end{bmatrix} \tag{12}$$

Let's write now the ratios (3)–(10), using the introduced constraint vectors, starting with equality type constraints (4), (5), (9). The ratio (4) will be rewritten in such a way that only known quantities are in the right part:

$$\forall_{t\in[0,T]}:\ P_{var}(t) + P_{bat}(t) + P_g(t) = P_{const}(t) - P_{wind}(t).$$

from where for vectors:

$$[E, \quad E, \quad E] \cdot \begin{bmatrix} P_{var} \\ P_{bat} \\ P_g \end{bmatrix} = [P_{const} - P_{wind}], \tag{13}$$

where *E* is the unit matrix. Let's rewrite (5) in such a way that only known quantities are in the right part:

$$\int_0^T \left(P_g(t) + P_{bat}(t)\right)dt = W_{load} - h\sum_{k=1}^{k=1008} P_{wind,k}.$$

Replacing integrals with sums, the following equation for the elements of vectors will be obtained:

$$h\sum_{k=1}^{k=1008} \left(P_{g,k} + P_{bat,k}\right) = W_{load} - h\sum_{k=1}^{k=1008} P_{wind,k}.$$

The second equality in (5) can also be written for elements of vectors in the form:

$$W_{load,var} = h\sum_{k=1}^{k=1008} P_{var,k}.$$

for further application, it is convenient to present the latter relations in matrix form:

$$\begin{bmatrix} 1, & 0, & 0 \\ 0, & 1, & 1 \end{bmatrix} \cdot \begin{bmatrix} P_{var} \\ P_{bat} \\ P_g \end{bmatrix} = \begin{bmatrix} h\sum_{k=1}^{k=1008} P_{var,k} \\ \frac{W_{load}}{h} - \sum_{k=1}^{k=1008} P_{wind,k} \end{bmatrix}, \tag{14}$$

In the ratio (13), *h* = 10 minutes is the time step of the grid (respectively, one week = 168 hours or 1008 ten-minute intervals), 0 is the row of the matrix containing 1008 zeros, 1 is the row of the matrix containing 1008 units. Similarly, to (14), (9) for the components of vectors will have the form:

$$h\sum_{k=1}^{k=1008} P_{bat,k} = 0.$$

or in matrix form:

$$[0, \quad 1, \quad 0]\cdot\begin{bmatrix}P_{var}\\P_{bat}\\P_g\end{bmatrix} = 0. \tag{15}$$

3.2. Reducing the procedure for calculating constraints to a linear programming problem

Let's move on to the constraints of the type of inequalities (5), (7) and (9). Constraints (5) and (9) set the boundaries of the change of the unknowns of the task. The corresponding inequalities for the elements of vectors have the form:

$$-Pbat, k\overline{1{,}1008}_{bat,max_{bat,max}} \tag{16}$$

$$0 \le P_{var,k} \le Pg, k\overline{1{,}1008}_{g,max_{var,max}} \tag{17}$$

Constraint (8), when passing to vectors, generates 1008 double inequalities:

$$0 \le h\sum_{k=1}^{k=S} P_{bat,k} \le W\overline{1{,}1008}_{bat,max}$$

the matrix form of these inequalities has the form:

$$\begin{bmatrix}N & D & N\\N & -D & N\end{bmatrix}\cdot\begin{bmatrix}P_{var}\\P_{bat}\\P_g\end{bmatrix} \le \frac{W_{bat,max}}{h\begin{bmatrix}1^T\\0^T\end{bmatrix}\underbrace{\begin{bmatrix}1 & 0 & \cdots & 0\\1 & 1 & \cdots & 0\\\vdots & \vdots & \ddots & \vdots\\1 & 1 & \cdots & 1\end{bmatrix}}_{1008}} \tag{18}$$

in the ratio (18), N is a square matrix of size 1008×1008, all elements of which are zeros.

Thus, the formation of constraints of the type of equalities and inequalities is completed and you can proceed to writing the objective function. As the objective function, as it was defined in the formulation of the problem, some functional $F(P_{var},\ P_{bat},\ P_g)$ should act that characterizes the heterogeneity of power energy generation at micro HPPs. The next task is connected with looking for a minimum of this functional. let's introduce into consideration two vectors A and B, whose components α_k and β_k are non-negative:

$$A = [\alpha_1, \alpha_2, \dots, \alpha_{1008}]^T,\ B = [\beta_1, \beta_2, \dots, \beta_{1008}]^T,\ \alpha_k \ge 0,\ \beta_k \ge 0,\ k = \overline{1{,}1008}. \tag{19}$$

using $P_{ave} = \frac{W_{load}}{T}$, the average power consumed by the load over the time interval T, $P_{ave,wind} = \frac{1}{T}\int_0^T P_{wind}(t)dt$, the average power supplied to the network by the wind turbine and considering that $\int_0^T P_{bat}(t)dt = 0$ the inequalities for the power consumed from the network can be written as follows:

$$(P_{ave} - P_{ave,wind}) - \alpha_k \le P_{g,k} \le (P_{ave} - P_{ave,wind}) + \beta_k,\ k = \overline{1{,}1008},$$

which, with a tendency to zero α_k and β_k, guarantee the constancy of or, what is the same thing, the equality of all among themselves. The matrix form of the notation of the last inequalities has the form:

$$\begin{bmatrix}N & N & -E & -E & N\\N & N & E & N & -E\end{bmatrix}\cdot\begin{bmatrix}P_{var}\\P_{bat}\\P_g\\A\\B\end{bmatrix} \le (P_{ave} - P_{ave,wind})\cdot\begin{bmatrix}1^T\\0^T\end{bmatrix}, \tag{20}$$

Thus, a linear functional can be taken as a minimized functional:

$$F(P_{var},\ P_{bat},\ P_g) = \sum_{k=1}^{k=1008}(\alpha_k + \beta_k) \xrightarrow[P_{var},\ P_{bat},\ P_g]{} min. \quad (21)$$

The problem (21) of minimizing a linear functional with linear constraints of the type of the above equations and inequalities is classified as a linear programming problem. It should be noted additionally that the result of the above classification is extremely important. It is explained by the fact that the methods of solving this problem are well studied, moreover, the existence of its only solution providing the global minimum of the functional *F* is proved. Let's bring the problem to the standard form used, for example, in the MATLAB package:

$$F(X) = \langle C, X\rangle \xrightarrow[X]{} min,$$
$$\begin{cases} A_{eq}X = b_{eq}, \\ A_{ineq}X \le b_{ineq}, \\ Xmax_{min}\{ \end{cases}$$

In the expression of the minimized functional, brackets ⟨·,·⟩ denote the scalar product of vectors. The vector X of unknowns and the vector of coefficients C from the task have the form:

$$X = [P_{var},\quad P_{bat},\quad P_g,\quad A,\quad B]^T,$$
$$C = [0,\quad 0,\quad 1,\quad 1,\quad 1]^T.$$

In result of combination of the relations (13)–(15) into one matrix equality, a matrix A_{eq} and a vector b_{eq} are formed:

$$\underbrace{\begin{bmatrix} E & E & E & N & N \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}}_{A_{eq}} \cdot \begin{bmatrix} P_{var} \\ P_{bat} \\ P_g \\ A \\ B \end{bmatrix} \le \underbrace{\begin{bmatrix} P_{const} - P_{wind} \\ h\sum_{k=1}^{k=1008} P_{var,k} \\ \frac{W_{load}}{h} - \sum_{k=1}^{k=1008} P_{wind,k} \\ 0 \end{bmatrix}}_{b_{eq}}. \quad (22)$$

by combining the relations (18) and (20) into one matrix inequality, the matrix A_{ineq} and vector b_{ineq} will be formed:

$$\underbrace{\begin{bmatrix} N & N & -E & -E & N \\ N & N & E & N & -E \\ N & D & N & N & N \\ N & -D & N & N & N \end{bmatrix}}_{A_{ineq}} \cdot \begin{bmatrix} P_{var} \\ P_{bat} \\ P_g \\ A \\ B \end{bmatrix} \le \underbrace{\begin{bmatrix} -\left(P_{ave} - P_{ave,wind}\right) \cdot 1^T \\ \left(P_{ave} - P_{ave,wind}\right) \cdot 1^T \\ \frac{W_{bat,max}}{h^T} \\ 0^T \end{bmatrix}}_{b_{ineq}} [] \quad (23)$$

Equalities (16), (17), and (19) make it possible to form vectors X_{min} and X_{max}:

$$X_{min} = \left[0, -P_{bat,max}\, 1, 0, 0, 0\right],$$
$$X_{max} = \left[P_{var,max}\, 1, P_{bat,max}\, 1, P_{g,max}\, 1, \left(P_{ave} - P_{ave,wind}\right)1, (P_{g,max} - (P_{ave} - P_{ave,wind}))1\right]$$

Thus, the internal issue of determining the optimal P_g, and accordingly $P_g(t)$, is reduced to the standard form for linear programming tasks and the task (1) is fully posed.

4. CONCLUSION

The mathematical formulation of the problem is formulated of optimal load management for an autonomous power system with a micro HPP, a wind power turbine, and an energy storage system in conditions of water deficit. The difference between the proposed formulation of the problem is the use of the stability of the generation of micro-HPPs as one of the criteria for optimality and minimum costs for a wind power turbine, and an energy storage system as the second criterion. The resulting nonlinear multi-criteria problem with a complex system of constraints is supposed to be solved using stochastic metaheuristic optimization methods such as the Genetic algorithm or Swarm Intelligence algorithms, which are able to effectively solve optimization problems of technical systems with nonlinear constraints. The subtask of determining the optimal micro HPP generation schedule for the given parameters of the power system is considered separately. By taking into account the peculiarities of technological processes in the system and the transition to a discrete time step, it was possible to convert this subtask into a linear programming problem without significant losses in accuracy. In turn, this makes it possible to apply a deterministic solution algorithm, such as the simplex method, and is guaranteed to find the global optimum of the problem. The next stage of the study will be the approbation of the developed model on the data of a real power system (Gorno-Badakhshan Autonomous Oblast of the Republic of Tajikistan) with the selection and adaptation of the high-level stochastic optimization algorithm to solve a multi-criteria problem, as well as the calculation of the economic effect of the proposed approach for the power system. To do this, a software implementation will be created, and computational experiments will be conducted on real data.

The proposed method of rational distribution of disconnected power among consumers during the elimination of emergency situations in the power system allows for an objective assessment of the capabilities of each consumer and can serve as a basis for introducing market relations, for example, when developing tariffs differentiated by reliability. The proposed method is based on original mathematical models of production systems of industrial electricity consumers.

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