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Tracking control of uncertain third order jerk equation Genesio-Tesi using adaptive backstepping

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ABSTRACT

This article presents the uncertain Genesio-Tesi, a third-order Jerk equation in the form of an ordinary differential equation, with the potential to exhibit chaos under certain conditions. The main focus of this article is to design a control function for the uncertain Genesio-Tesi, which has uncertain parameters with unknown values. The adaptive backstepping method designs the control function, demonstrating its ability to stabilize the system output towards a given trajectory using Lyapunov stability. To test the robustness of the proposed control method, simulations were conducted with various scenarios, including disturbances to the steady-state system. Simulation results show that the controller successfully drove the system output along a desired trajectory, whether constant or a function, and maintained system stability even with significant disturbances.

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1. INTRODUCTION

Mathematical models are formulated by translating natural phenomena into mathematical equations. The relationship between variables and their progression is reliant on parameters, which can be obtained through direct measurements or data processing. However, imprecise measurement techniques or measuring devices can lead to unsuitable parameter values, as can the methods used to estimate parameters through data processing. Even the slightest error in parameter values can significantly impact the accuracy of the mathematical model. Take, for example, the epidemic model used to analyze the spread of coronavirus disease (COVID-19) in [1]–[4]. The model employs a fixed incubation value obtained by dividing the incubation time by the mass in years. This value does not reflect the reality that the incubation period varies for each individual, as highlighted in [5]–[8]. It was known that the incubation period is not constant and changes over time or at certain intervals.

The Jerk equation is a third-order ordinary differential equation that can exhibit chaos, per the Pointcare-Bendixson theorem [9]. It is expressed as $x^{(3)}(t) = j(x^{(2)}(t), x^{(1)}(t), x(t))$, where $j(x)$ denotes the jerk term and $x^{(i)}(t) = \frac{d^i x(t)}{dt^i}$. The simplest form experiences chaos is $x^{(3)}(t) + Ax^{(2)}(t) - \left(x^{(1)}\right)^2(t) + x(t) = 0$, described in [10], [11]. Another form is the Genesio-Tesi equation, which includes a quadratic form in the first order. Its general form is expressed as $y^{(3)}(t) + cy^{(2)}(t) + by^{(1)}(t) + ay(t) - y^2(t) = 0$, as seen in [12], [13]. The Jerk equation is widely used in analyzing wave behavior in electronic circuits.

However, the resistor value determines its parameter value, which has a specific value and tolerance limit, resulting in uncertainty in the parameter value, as mentioned in [13], [14].

A reliable approach to designing controllers for nonlinear systems with uncertainty is the backstepping method. Despite its iterative nature, this technique provides impressive control performance due to its focus on system stability at each stage using Lyapunov stability criteria. Given its robustness, it is no surprise that the backstepping method is widely employed across various fields. Some studies, such as [15], [16], have successfully deployed the backstepping method to set up controllers for the DC-DC buck converter. The backstepping method has also proved helpful in motion controllers for unmanned aerial vehicles (UAVs) in [17]–[21] and ship motion maneuvers in [22]–[24]. Moreover, Hajji *et al.* [24] have implemented the backstepping method to design controllers for induction motor issues. The backstepping method has also tackled non-minimum phase nonlinear control problems in CSTR and paper-cutting machines, as demonstrated in [25]–[27]. The iterative procedure of the backstepping method is highly effective in stabilizing unstable internal dynamics, making it a valuable tool. Meanwhile, several previous studies have carried out the control design of the jerk equation and the Genesio-Tesi system. In study [28], the Jerk equation is controlled using a linear control based on feedback. In study [29], the stabilization of the Genesio-Tesi was carried out using the sliding-mode method. In study [30], the active compensation mechanism method is applied to Genesio-Tesi, which contains disturbance. In studies [31] and [32], the Genesio-Tesi was analyzed using a fractional model and the control design was carried out using the proportion-integral (PI) method and linear feedback control.

This article examines the dynamics of the Genesio-Tesi, which contains uncertain parameters, and the design of a controller to stabilize it. The system in this article differs from the system in study [14], which uses a system with uncertain parameters in the form of variations in system parameter values. The magnitude of this variation is assumed to be constant but the value and limits are unknown. As a result of the uncertain parameters, besides determining the controls that stabilize the system, parameter estimation is needed to determine the actual system conditions. Therefore, the controller design is carried out using an adaptive backstepping method that differs from the methods in studies [14], [29], [30]. Next, the effect of control parameters and estimator parameters on control performance is investigated. Control performance is calculated based on the absolute difference between the system output and the given trajectory, in this case the trajectory is a constant and a function. Statistical tests in the form of correlation and regression tests were given to determine the nature and weight of the effect of each parameter on control performance.

2. METHOD

This section presents the problems and method used in this study. The discussion begins with the introduction of the Genesio-Tesi equations followed by a local stability analysis. Finally, a control design for the Genesio-Tesi system with uncertain parameters using adaptive backstepping is presented.

2.1. Genesio-Tesi equation

Genesio and Tesi [4] introduced a third-order ordinary differential equation, which is part of the Jerk equation with quadratic terms in the form

$$y^{(3)}(t) + cy^{(2)}(t) + by^{(1)}(t) + ay(t) - f(y) = 0 \tag{1}$$

where $y^{(i)}(t) = \frac{d^i y(t)}{dt^i}$ for $i = 1,2,3$. Equation (1) can be converted into a system of differential equations by transformation $\{x_1(t)=y(t), x_2(t)=y^{(1)}(t), x_3(t)=y^{(2)}(t)\}$ to obtain system of differential equations in (2).

$$\begin{cases} \dot{x}_1(t) &= x_2(t) \\ \dot{x}_2(t) &= x_3(t) \\ \dot{x}_3(t) &= -ax_1(t) - bx_2(t) - cx_3(t) + f(x_1) \end{cases} \tag{2}$$

In Genesio-Tesi, the function $f(x_1)$ is a quadratic form, that is $f(x_1) = x_1^2(t)$ [12]. If the given values of (a, b, c) have a tolerance range of $(\delta_a, \delta_b, \delta_c)$ for each parameter, then (2) can be written as a Genesio-Tesi with uncertain parameters.

$$\begin{cases} \dot{x}_1(t) &= x_2(t) \\ \dot{x}_2(t) &= x_3(t) \\ \dot{x}_3(t) &= (p + \delta)^T \phi(x) + f(x_1) \end{cases} \tag{3}$$

where $p = \langle a, b, c\rangle$ is the parameter value whose value is positive and known, $\delta = \langle \delta_a, \delta_b, \delta_c\rangle$ is the variant value on parameter p whose value is unknown, and $\phi(x) = \langle x_1, x_2, x_3\rangle$ is the basis function.

2.2. Local stability analysis of uncertain Genesio-Tesi

The local stability of the Genesio-Tesi will be analyzed around the equilibrium point. The equilibrium point (x_1^*, x_2^*, x_3^*) is an ordered triple pair that satisfies $f(x_1^*, x_2^*, x_3^*) = 0$ where $f(x_1, x_2, x_3)$ is a vector-valued function on the right-hand side of (2) and (3). From (2), we have (4).

$$\begin{cases} x_2(t) &= 0 \\ x_3(t) &= 0 \\ -(a+\delta_a)x_1(t) - (b+\delta_b)x_2(t) - (c+\delta_c)x_3(t) + f(x_1) &= 0 \end{cases} \tag{4}$$

The first two equations in (4) show that the only values (x_2^*, x_3^*) that satisfy are $x_2^* = 0$ and $x_3^* = 0$. Substitute the values $(x_2^* = 0, x_3^* = 0)$ into the third equation to get $x_1^2(t) - (a+\delta_a)x_1(t) = 0$. The x_1^* values that satisfy are $x_1^* = 0$ and $x_1^* = a + \delta_a$. So, the equilibrium points of the Genesio-Tesi with uncertain parameters in (3) are $E_1 = (0,0,0)$ and $E_2 = (a+\delta_a, 0,0)$.

The second step is to determine the linear form of (3) with the Taylor Series around the equilibrium point while simultaneously determining the eigenvalues of the Jacobian matrix. The Jacobian matrix from (3) is in (5).

$$\mathrm{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -(a+\delta_a) + 2x_1 & -(b+\delta_b) & -(c+\delta_c) \end{bmatrix} \tag{5}$$

Substitute the $E_1 = (0,0,0)$ into matrix A in (5) to obtain A_1 as Jacobian matrix around E_1

$$\mathrm{A}_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -(a+\delta_a) & -(b+\delta_b) & -(c+\delta_c) \end{bmatrix}$$

The characteristic polynomial of A_1 is $C_{\mathrm{A}_1} = \lambda^3 + (c+\delta_c)\lambda^2 + (b+\delta_b)\lambda + (a+\delta_a)$ and the Routh's table of C_{A_1} is

$$\begin{array}{c|ll} \lambda^3 & 1 & (b+\delta_b) \\ \lambda^2 & (c+\delta_c) & (a+\delta_a) \\ \lambda & (b+\delta_b) - \frac{(a+\delta_a)}{(c+\delta_c)} & 0 \\ 1 & (a+\delta_a) & 0 \end{array} \tag{6}$$

The third row and first column of the Routh's table in (6) produce the system stability conditions around E_1, namely $(b+\delta_b)(c+\delta_c) > (a+\delta_a)$ and its solution depends on the value of each uncertain parameter and its sign. If the uncertain parameter value is omitted, then the stable condition is $bc > a$ and there is a condition where $bc = a$ will causes the entries in the first column of the Routh's table to be zero and causes symmetry in the system. We omit detailed analysis of parameter values and their variants causing instability around E_1 because there are too many unknown values.

Next is analyzing the local stability around E_2. Substitute the value $E_2 = (a+\delta_a, 0,0)$ into Jacobian matrix A in (5) to obtain Jacobian matrix around E_2.

$$\mathrm{A}_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ (a+\delta_a) & -(b+\delta_b) & -(c+\delta_c) \end{bmatrix}$$

The characteristic polynomial of A_2 is $C_{\mathrm{A}_2} = \lambda^3 + (c+\delta_c)\lambda^2 + (b+\delta_b)\lambda - (a+\delta_a)$ and its Routh's table is given in (7).

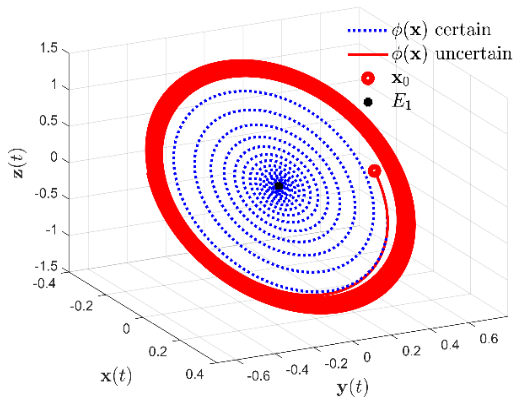
$$\begin{array}{c|ll} \lambda^3 & 1 & (b+\delta_b) \\ \lambda^2 & (c+\delta_c) & -(a+\delta_a) \\ \lambda & (b+\delta_b) + \frac{(a+\delta_a)}{(c+\delta_c)} & 0 \\ 1 & -(a+\delta_a) & 0 \end{array} \tag{7}$$

The Routh table determines system stability by analyzing sign changes in the first column of (7). An unstable linear system is indicated by sign change in the first column value, with the number of positive eigenvalues equaling the number of sign changes in the Routh's table. The (a, b, c) is positive and $(\delta_a, \delta_b\delta_c)$ is small, the fourth row of (7) produces a negative value. Consequently, the system is always unstable around the equilibrium point $E_2 = (a + \delta_a, 0,0)$. We provide an illustration to show the effect of the presence of uncertain parameters on stability. Table 1 shows the parameter values and their variations used for the simulation and their stability characteristics. Based on Table 1, the equilibrium points are $E_1 = (0,0,0)$ and $E_2 = (2,0,0)$. The equilibrium point E_1 is a stable equilibrium point because it satisfies $bc = 8 > a = 7$. However, this property changes with variations in parameter values. Using the varied values, we get $(b + \delta_b)(c + \delta_c) = 7.2 < (a + \delta_a) = 7.35$ which result in an unstable equilibrium point. Graphically, a comparison of the behavior of the two is shown in Figure 1.

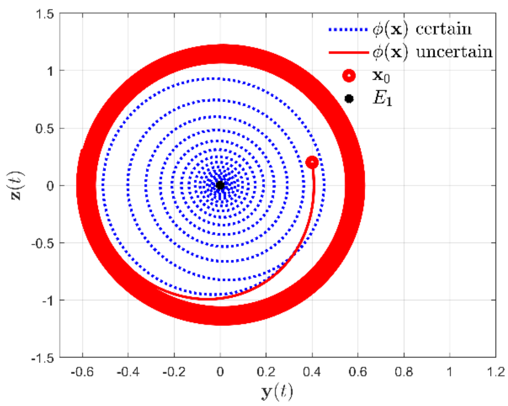
Figure 1(a) demonstrates the orbit behavior of both certain and uncertain systems in a 3D dimension. Starting from the initial point $x_0 = (0.1,0.4,0.2)$ in a system with parameter variations, the orbit gradually moves away from the equilibrium point in a circular motion. In contrast, the orbit of certain system moves asymptotically from the same initial point $x_0 = (0.1,0.4,0.2)$ towards the equilibrium point E_1. However, for a clearer understanding of the dynamics of an ever-expanding uncertain system, Figure 1(b) projects it onto the yz-plane. The thick red line in Figure 1(b) directly results from the system's orbit. This orbit is expanding gradually with minor increments, and this behavior is confirmed by Figure 2. Figure 2 shows the solution $x_1(t)$ against time. The $x_1(t)$ curve in the certain system (blue line) is asymptotically stable towards the origin, while the $x_1(t)$ curve with parameter variations continues to grow.

Table 1. Parameter values of the system and their variations

Parameter	Value	Variation
a	7	+5%
b	4	−5%
c	2	−5%



(a)



(b)

Figure 1. Comparison of the dynamics of the Genesio-Tesi solution in the presence of parameter variations (a) the curve in 3D coordinates (b) the curve projected on the yz-plane

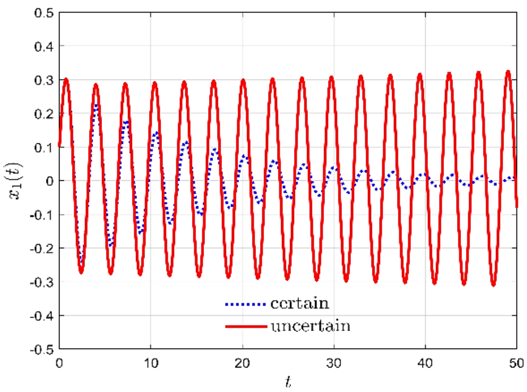


Figure 2. Comparison of $x_1(t)$ against time due to parameter variations that result in instability

2.3. Control design for uncertain Genesio-Tesi using adaptive backstepping

Consider the Genesio-Tesi with uncertain parameters in (3). Next, the control function $\mu(t)$ is introduced to the system corresponding to the dynamic equations containing uncertain parameters so that (3) can be written as (8).

$$\begin{cases} \dot{x}_1(t) &= x_2(t) \\ \dot{x}_2(t) &= x_3(t) \\ \dot{x}_3(t) &= (p+\delta)^T \phi(x) + f(x_1) + \mu(t) \end{cases} \tag{8}$$

The first step is determining the control function so that the state variable $x_1(t)$ goes to the trajectory $y_d(t)$ using virtual control $x_2(t)$. Virtual control is a state variable used as an additional input to ensure the Lyapunov stability in the backstepping method. We defined the difference between the output and the trajectory in (9).

$$e_1(t) = x_1(t) - y_d(t) \tag{9}$$

The derivative of the error equation in (9) with respect to t yields.

$$\dot{e}_1(t) = x_2(t) - \dot{y}_d(t)$$

Next, we define the Lyapunov function $V(e_1) = \frac{1}{2} e_1^2(t)$ and differentiate it with respect to t to produce.

$$\dot{V}(e_1) = e_1(t)[x_2(t) - \dot{y}_d(t)] \tag{10}$$

It is assumed that there is $r_1 \in \mathbb{R}^+$ which satisfies $\dot{V}(e_1) = -r_1 e_1^2(t)$ so that from (10) we obtain virtual control $x_2(t) - \dot{y}_d(t) = -r_1 e_1(t)$. From the virtual control $x_2(t)$, a new state variable is defined, $e_2(t) = x_2(t) - \dot{y}_d(t) + r_1 e_1(t)$, so that a new dynamic is obtained, namely $\dot{e}_2(t) = x_3(t) + r_1[e_2(t) - r_1 e_1(t)] - \ddot{y}_d(t)$.

The second step is stabilizing the state variable $e_2(t)$ using virtual control $x_3(t)$. Use the Lyapunov function $V(e_1, e_2) = V(e_1) + \frac{1}{2} e_2^2(t)$ and derive $V(e_1, e_2)$ with respect to t to yield.

$$\dot{V}(e_1, e_2) = -r_1 e_1^2(t) + e_2(t)[e_1(t) + x_3(t) + r_1(e_2(t) - r_1 e_1(t)) - \ddot{y}_d(t)] \tag{11}$$

The $x_3(t)$ is used as virtual control so that $\dot{V}(e_1, e_2) = -r_1 e_1^2(t) - r_2 e_2^2(t)$ is obtained for $r_1, r_2 \in \mathbb{R}^+$ and using (11) we get $e_1(t) + x_3(t) + r_1\left(e_2(t) - r_1 e_1(t)\right) - \ddot{y}_d(t) = -r_2 e_2(t)$. Based on virtual control $x_3(t)$, a new state variable is defined; $e_3(t) = x_3(t) + (1 - r_1^2)e_1(t) + (r_2 + r_1)e_2(t) - \ddot{y}_d(t)$ and using (8) we have the new dynamic that will be stabilised in (12).

$$\begin{aligned} \dot{e}_3(t) &= \mu(t) + f(\mathrm{e}) + (\mathrm{p} + \delta)^T\, \phi(\mathrm{e}, y_d, \dot{y}_d, \ddot{y}_d) - y_d^{(3)} \\ &+ (r_2 + r_1)(e_3 - r_2 e_2 - e_1) + (1 - r_1^2)(e_2 - r_1 e_1) \end{aligned} \tag{12}$$

The final step is stabilizing the final dynamic in (12) while minimizing the error between the uncertain parameters and their estimates. We define new Lyapunov function $V(e_1, e_2, e_3)$ by including the error term of uncertain parameter estimation in (13).

$$V(e_1, e_2, e_3) = V_{e_2} + \frac{1}{2} e_3^2(t) + \tilde{\delta}^T \Gamma \tilde{\delta} \tag{13}$$

where $\tilde{\delta} = \delta - \hat{\delta}$ is the difference between the uncertain parameters and its estimated value. Derivative $V(e_1, e_2, e_3)$ respect to t yields.

$$\dot{V}(e_1, e_2, e_3) = -r_1 e_1^2(t) - r_2 e_2^2(t) + e_3(t)[e_2(t) + \dot{e}_3(t)] + \tilde{\delta}^T \Gamma \frac{d\tilde{\delta}}{dt} \tag{14}$$

Define control $\mu(t)$ in (15).

$$\begin{aligned} \mu(t) &= y_d^{(3)} - f(\mathrm{e}) - \left(\mathrm{p} + \hat{\delta}\right)^T \phi(\mathrm{e}, y_d, \dot{y}_d, \ddot{y}_d) \\ &+ [r_1(1 - r_1^2) + r_1 + r_2]e_1 + [r_1^2 + r_1 r_2 + r_2^2 - 2]e_2 - [r_1 + r_2 + r_3]e_3 \end{aligned} \tag{15}$$

Then substitute the control $\mu(t)$ into (12) and (14) to produce (16).

$$\dot{V}(e_1,e_2,e_3) = -r_1e_1^2(t) - r_2e_2^2(t) - r_3e_3^2 + e_3(t)\tilde{\delta}^T\phi(\mathrm{e},y_d,\dot{y}_d,\ddot{y}_d) + \tilde{\delta}^T\Gamma\frac{d\tilde{\delta}}{dt} \quad (16)$$

We can get $\dot{V}(e_1,e_2,e_3) < 0$ for every $t \geq 0$ in (16) by making zeros terms containing $\tilde{\delta}$ and from the fact that $\tilde{\delta} \neq 0$ then we obtain $\frac{d\tilde{\delta}}{dt} = -e_3(t)\Gamma^{-1}\phi(\mathrm{e},y_d,\dot{y}_d,\ddot{y}_d)$. The $\hat{\delta}$ is used in (15) and using relation $\tilde{\delta} = \delta - \hat{\delta}$ we have the dynamic for parameter estimation in (17).

$$\frac{d\hat{\delta}}{dt} = e_3(t)\Gamma^{-1}\phi(\mathrm{e},y_d,\dot{y}_d,\ddot{y}_d) \quad (17)$$

3. RESULTS AND DISCUSSION

This section showcases the implementation of the control design for Genesio-Tesi equations with uncertain parameters in various scenarios. To evaluate the control's effectiveness, we conducted a simulation using the data provided in Table 2. The first scenario involves a single uncertain parameter. The primary goal is to guarantee that the outcome adheres to the constant trajectory $y_d(t) = 2$ and the function trajectory $y_d(t) = 2\sin(t) + \frac{1}{2}$. Figures 3, 4, and 5 present the simulation results obtained by varying the control parameters $\{r_1, r_2, r_3\}$ in order to assess its performance. The control function's effectiveness in directing the output towards the function trajectory is unequivocally demonstrated in Figures 3(a) and 4(a). Furthermore, Figure 5(a) serves as clear evidence of how the control function effectively directs the output towards a constant trajectory. However, part (b) of the figures clearly reveals that not all parameter estimates can accurately capture uncertain parameter values. Specifically, Figures 3(b) and 4(b) unambiguously show that the parameter estimates correspond to the uncertain parameters employed in the simulation. Conversely, none of the simulation parameter combinations in Figure 5(b) yielded the correct parameter estimates.

The simulation results show that the control parameter's value influences the system's output convergence speed. Therefore, a simulation is carried out by varying the parameter values $\{r_1, r_2, r_3, \Gamma\}$ and testing the relationship between these values and the control performance. For this simulation, the function trajectory $y_d(t) = 2\sin(t) + \frac{1}{2}$ is used and time interval $t \in [0,5]$ is partitioned by $n = 5 \times 10^3$ partitions. Variations in the values used are $r_1, r_2, r_3 = \{1,3,5,7,9\}$ and $\Gamma = \{0.01, 0.1, 1, 10, 100\}$. The amount of data generated is $5^4 = 625$ data. The correlation test was carried out using the Spearman method, and the regression test used was multiple linear regression. The simulation results are shown in Table 3. Table 3 shows that for each case of definite parameters, $\{r_1, r_2, r_3\}$ values are negatively correlated with control performance. Thus, we can reduce the value of $\{r_1, r_2, r_3\}$ to improve control performance. The weight of each value $\{r_1, r_2, r_3\}$ on control performance in each case is almost identical. In contrast to $\{r_1, r_2, r_3\}$, the value of Γ positively affects control performance even though it is tiny. With a correlation value below 0.3, there is no correlation Γ on control performance, which is also seen in the very small regression coefficient.

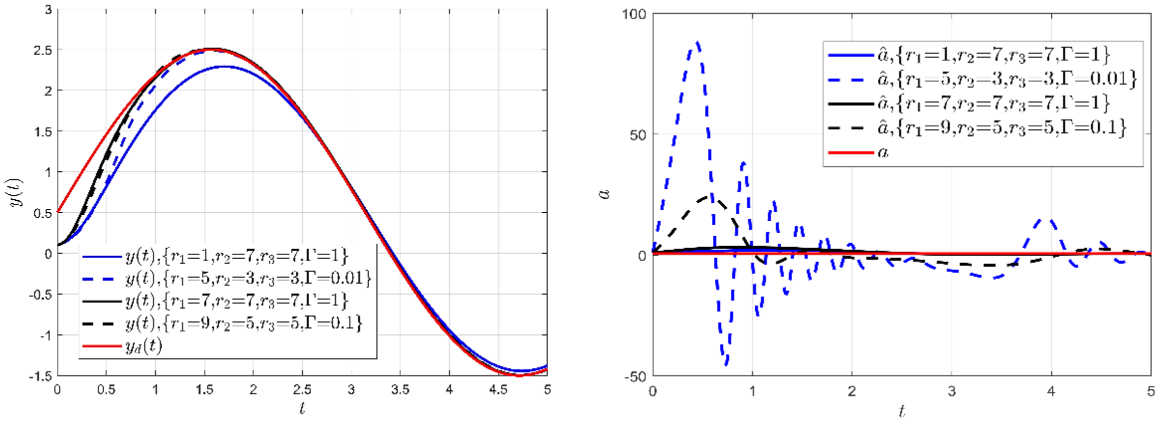
The second scenario is to control a system where all parameters contain uncertain values. Because of three uncertain parameters, we need an estimator positive definite matrix $\Gamma \in \mathbb{R}^{3\times 3}$ with infinite forms. A positive definite matrix is one in which all the eigenvalues are positive. We choose the diagonal matrix and the upper triangular matrix for simplicity because the eigenvalues can be obtained from the diagonal entries.

Choose matrices $\Gamma_1 = \begin{bmatrix} 1106 & 0 & 0 \\ 0 & 10070 & 0 \\ 0 & 0 & 13926 \end{bmatrix}$ and $\Gamma_2 = \begin{bmatrix} 1106 & 2825 & 3361 \\ 0 & 10070 & 8699 \\ 0 & 0 & 13926 \end{bmatrix}$
with eigenvalues $\lambda = \{1106, 10070, 13926\}$.

The simulation results are shown in Figure 6. Figure 6(a) highlights the system's outputs that have followed a constant trajectory despite varying convergence rates. On the other hand, Figure 6(b) showcases the system's output following a functional trajectory. These figures show that the system's output quickly converges faster on the functional trajectory than on the constant trajectory. This is particularly evident at $t \approx 2$, where all system outputs on the functional trajectory are close to the given trajectory.

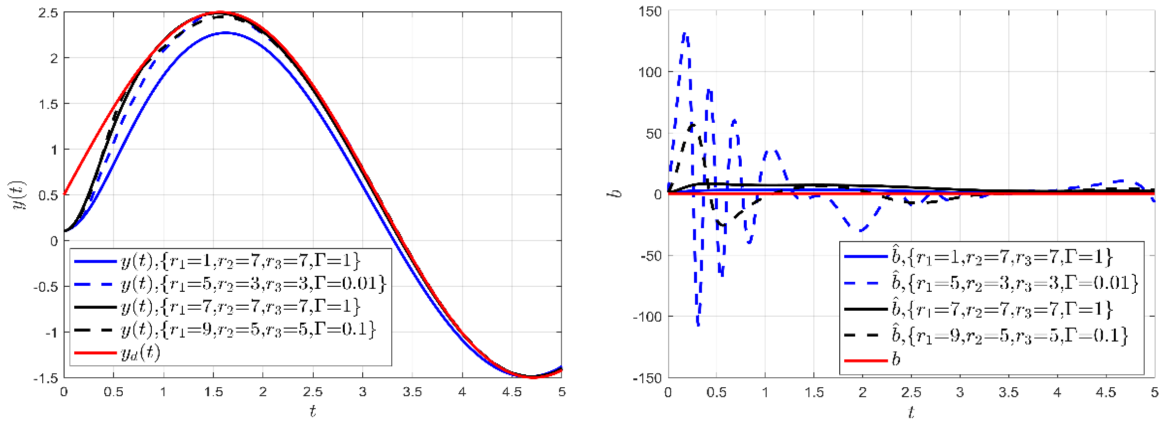
Table 2. Parameter and adjustment for simulations

Parameter	Value	Description
$\{a, b, c\}$	$\{5,3,0.5\}$	Certain parameter for the model
$\{\delta_a, \delta_b, \delta_c\}$	$\{0.6,0.1,0.5\}$	Uncertain parameter for the model
t	$[0,5]$	Time interval
x_0	$\{0.1,0.4,0.2\}$	Initial point for state
δ_0	$\{0,0,0\}$	Initial point for estimation



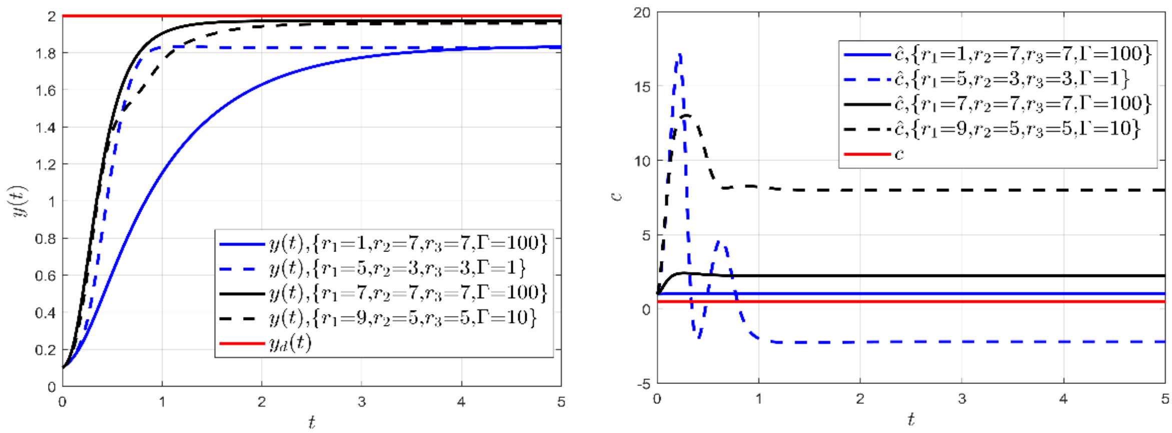
(a) (b)

Figure 3. The behavior of the Genesio-Tesi with uncertain parameters on parameter a (a) system's output with function trajectory $y_d(t) = 2\sin(t) + \frac{1}{2}$ and (b) estimation parameter $\hat{a}$



(a) (b)

Figure 4. The behavior of the Genesio-Tesi with uncertain parameters on parameter b (a) system's output with function trajectory $y_d(t) = 2\sin(t) + \frac{1}{2}$ and (b) estimation parameter $\hat{b}$



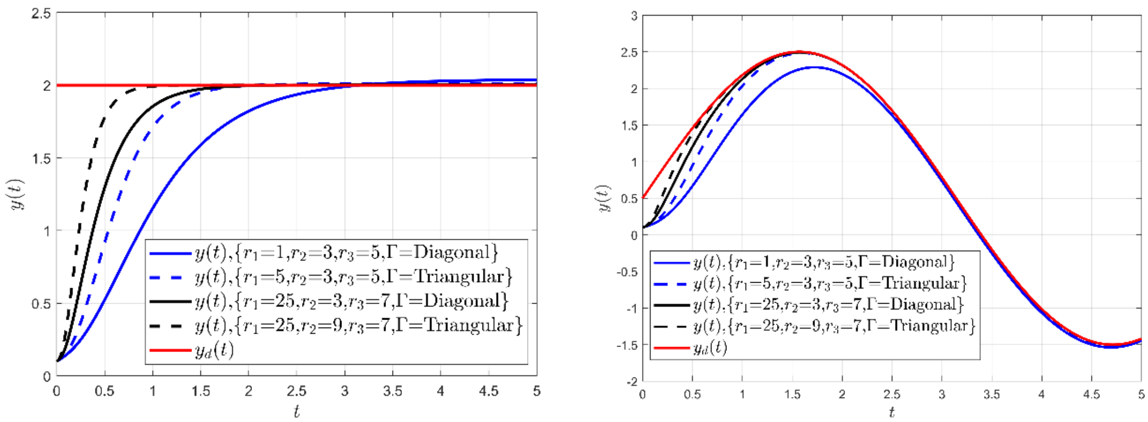
(a) (b)

Figure 5. The behavior of the Genesio-Tesi with uncertain parameters on parameter c (a) system's output with constant trajectory $y_d(t) = 2$ and (b) estimation parameter $\hat{c}$

Table 3. Correlation and regression test results for the function trajectory with a variety of parameters

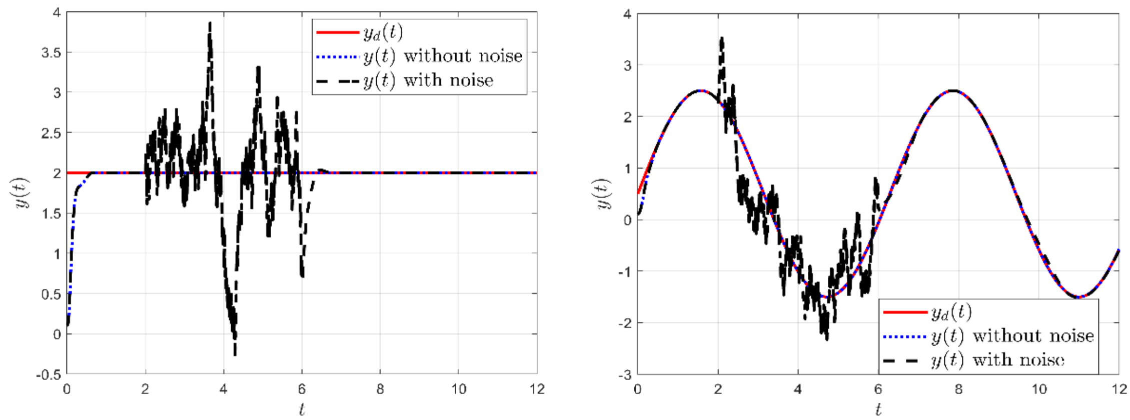
Uncertain Parameter	Correlation				Regression			
	r_1	r_2	r_3	Γ	r_1	r_2	r_3	Γ
δ_a	-0.54903	-0.50972	-0.46761	0.22483	0.02742	0.03975	0.04305	0.00517
δ_b	-0.53260	-0.49038	-0.45395	0.29462	0.02585	0.04586	0.04409	0.00684
δ_c	-0.54778	-0.54778	-0.48781	0.16511	0.03328	0.05773	0.05004	0.00068

The last scenario is to test the robustness of the proposed control method. The disturbance is given to the system while has reached a steady-state point. The disturbance is given to each variable $\{x_1(t), x_2(t), x_3(t)\}$ with the noise $d(t) = 100 \times \text{rand}()$ for $t \in [2,6]$. The control parameters used are $\{r_1 = 25, r_2 = 9, r_3 = 15\}$ and the matrix estimator is $\Gamma = \text{diag}([0.01\ 1\ 10])$. The simulation results are shown in Figure 7. Figures 7(a) and 7(b) show that at $t \in [2,6]$, there are irregular spikes in the curve. The disturbance function is causing a disruption in the system's output, leading to a significant surge and deviation from the intended track. However, the motion of the perturbed system in Figure 7(b) undoubtedly aligns with the intended trajectory pattern with great accuracy. After $t \geq 6$, the disturbance is removed. The proposed control function can bring back the output to a given trajectory. This shows that the control function is very good at stabilizing the system. The system's disturbance cannot be eliminated because the control design does not involve aspects of the disturbance function. However, the control function can quickly restore system output when the disturbance is removed.



(a) (b)

Figure 6. Comparison of system dynamics with several variations of control parameters (a) constant trajectory $y_d(t) = 2$ and (b) function trajectory $y_d(t) = 2\sin(t) + \frac{1}{2}$



(a) (b)

Figure 7. Comparison of system dynamics with disturbances at $t \in [2,6]$ (a) constant trajectory $y_d(t) = 2$ and (b) function trajectory $y_d(t) = 2\sin(t) + \frac{1}{2}$

4. CONCLUSION

This article presents a control design for the Genesio-Tesi with uncertain parameters caused by variations in model parameter values. The control design approach uses the backstepping method, based on Lyapunov stability, which has proven to make the system globally asymptotically stable. Simulation results demonstrate that the controller can rapidly drive the output into the desired trajectory. Based on statistical test results, it is clear that the backstepping control parameter significantly enhances control performance. However, the parameter values for parameter estimation are insufficient to support control performance improvement. During the robustness test, an unknown disturbance is introduced to the steady-state of the system for a specific period of time. This causes the system output to become disrupted, as the control design does not account for the disturbance. However, once the disturbance is removed, the controller is able to restore the output to its original state. Identifying the optimal estimator matrix is crucial to effectively reduce errors between uncertain parameters and their estimates while maintaining superior control performance. Further research will be aimed at determining the suitable estimator matrix.

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AUTHOR CONTRIBUTIONS STATEMENT

This journal uses the Contributor Roles Taxonomy (CRediT) to recognize individual author contributions, reduce authorship disputes, and facilitate collaboration.

Name of Author	C	M	So	Va	Fo	I	R	D	O	E	Vi	Su	P	Fu
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C : **C**onceptualization
M : **M**ethodology
So : **So**ftware
Va : **Va**lidation
Fo : **Fo**rmal analysis
I : **I**nvestigation
R : **R**esources
D : **D**ata Curation
O : Writing - **O**riginal Draft
E : Writing - Review & **E**diting
Vi : **Vi**sualization
Su : **Su**pervision
P : **P**roject administration
Fu : **Fu**nding acquisition

CONFLICT OF INTEREST STATEMENT

Authors state no conflict of interest.

DATA AVAILABILITY

The authors confirm that the data supporting the findings of this study are available within the article.

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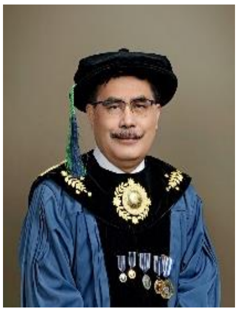

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