

Model reference adaptive control of networked systems with state and input delays

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ABSTRACT

Adaptive control strategies have been developed in response to more advanced complex systems and to deal with uncertain systems while maintaining the desired conditions. This paper addresses the networked unknown and unstable heterogeneous systems following a stable reference (leader), which is related to network synchronization. We deliver two different scenarios; each agent both fully communicates to the leader and shares communication among neighborhood agents and the leader. The communication among agents and the leader are weighted using Laplacian-like matrix and the model weight matrix in turn. Also, the state and input delays are induced to the systems to capture the real limited communication while the prediction of the reference signals and the augmented systems are proposed to deal with them. Moreover, the rigorous mathematical foundations of two adaptive laws, the stability analysis, the threshold of network, and the communication network are thoroughly presented. Also, the numerical illustrations of the two scenarios are given to show the effectiveness of the proposed method in the networked system. The results show that for both scenarios working on the required setting, the perfect tracking to the leader is guaranteed. Beyond that, the future research would implement the distributed adaptive control-oriented learning of networked system under some faults.

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1. INTRODUCTION

With the more advanced complex systems, the developments of control methods have been intriguing from the perspective of classical, modern and learning-oriented algorithms. More specifically, to deal with uncertain systems while maintaining the desired conditions, the interesting methodology leads to adaptive control [1]. The challenges to this adaptive control problem have been always available, such as the stability analysis on model-free [2], [3], the time-varying parameterization [4] with asymptotic tracking [5], the disturbance rejection under time-varying frequencies [6], the linear matrix inequality (LMI) certification with some frequency-limited [7], the scaling estimation [8]–[10] to the non-minimum phase systems for microgrid [11]. Furthermore, some choose converting to classical problem [12] for approximation, while iteration-based adaptive control is applied when the state estimation is avoided [13]. However, while those issues have been mostly studied, the problem with the unknown distributive “not-lumped” delays have always existed in the model reference adaptive control (MRAC) methods [14]–[17]. To capture this issue, the dynamic switches [18], [19] and

multivariable systems with uncertain time delays [20]–[22], accommodating time-varying control bounds and exogenous noise in the systems have become the options. Note that, there are some famed methods used in the references to design the MRAC, such as the Lyapunov [23], [24], the Massachusetts Institute of Technology (MIT) rule [25] and the adaptation approach [26]. This paper applies the same method as [26] to handle the delays and addresses the delayed system to deal with the real application due to limited signal communication.

Furthermore, the direct MRAC adaptation-based control is implemented for linear systems with input and state delays and the construction method is relied on the concept of reference trajectory prediction, and on the formulation of an augmented system using the Lyapunov–Krasovskii based updates. Here, we develop the problem into networked systems [27]–[29] to see how the system behaves considering the lumped known delays with completely unknown and unstable multi-agent system following a stable reference. It presents two scenarios: one where each agent fully communicates with the leader and another where agents communicate with both the leader and their neighbors. Communication weights are assigned using a Laplacian-like matrix and a model weight matrix. The study also incorporates state and input delays to reflect real-world limited communication, employing reference signal prediction and augmented systems to manage these delays.

Beyond that, for the future research, we will focus on the network of adaptive control-oriented learning according to the recent surveys from [30], [31], and [32], promoting the breakthrough and the robust properties of higher level intelligent systems with nonlinearities. Also, after analyzing the performance of the system and sensitivity of the delays, the distributed control of nonlinear system or even the robust adaptive fault-tolerant control of networked systems with various complex environments [33]–[39] have been the near future research. In addition, we would like to apply the distributed adaptive control-oriented learning of networked system under some faults [40], [41]. Finally, the structure in this paper is initiated from the problem statement of the adaptive control of networked systems considering the delays in section 2. Moreover, the adaptation approach used and the stability analysis are elaborated in section 3. While the numerical examples from two different networks are given in section 4 to see how adaptation approach performs against the delays on inputs and states. The conclusion and the possible future research are explained in section 5.

Notations. \mathbb{R}^p represents the p -dimensional Euclidean space and I_p defines the identity matrix of size $\mathbb{R}^{p \times p}$ while $P = \text{diag}\{p_i\}$ denotes the diagonal matrix with entries $p_i, \forall i$. Moreover, $\mathbf{1}_p = [1, \dots, 1]^T$ is the vector of all ones in \mathbb{R}^p . The operator \otimes and \odot denotes the Kronecker product and the Hadamard product whereas operator $\text{tr}[P]$ defines the trace of matrix P . Furthermore, operator $|P|$, $\|P\|_2$, and $\|P\|_F$ are the absolute value of the element-wise, the Euclidean norm, and the Frobenius norm of matrix P .

2. ADAPTIVE CONTROL OF NETWORKED SYSTEMS

Let us introduce the networked linear uncertain systems of ℓ agents and a model, subscript m , with state τ_x and input delays τ_u , with $\tau_x \leq \tau_u$, as depicted in Figures 1(a)–1(c). The subsystems of ℓ agents are written as (1),

$$\dot{x}_i(t) = A_i x_i(t) - A_i^\zeta x_i(t - \tau_x) + B_i u_i(t - \tau_u) \quad (1)$$

where $x_i \in \mathbb{R}^n$ and $u_i \in \mathbb{R}^p$ denote the states and the inputs, for $i = 1, \dots, \ell$. The matrices of $A_i \in \mathbb{R}^{n \times n}$, $B_i \in \mathbb{R}^{n \times p}$, and $A_i^\zeta \in A_i$ are the uncertain constant real. The model reference is described as (2),

$$\dot{x}_m(t) = A_m x_m(t) + B_m r(t - \tau_u) \quad (2)$$

in which $x_m \in \mathbb{R}^n$ and $r \in \mathbb{R}^p$ define the state and the reference signal of the model with constant matrices of $A_m \in \mathbb{R}^{n \times n}$ and $B_m \in \mathbb{R}^{n \times p}$. The goal is to adjust the local control u_i such that the behaviour of the agents in (1) approaches the output of the model in (2). Note that, we define the n agents and a model via weighted digraph $\mathcal{G} := \{\mathcal{V} = \{1, \dots, \ell\} \cup m, \mathcal{E}, w_{ij}\}$ where \mathcal{V} , \mathcal{E} , and $w_{ij} \in \mathcal{E}$ show the set of agents, directed edges, and the weight function respectively. We express the induced subgraph on ℓ agents as \mathcal{G}_ℓ and the model as \mathcal{G}_m . The incoming arrows for i -th agent imply the measurement from the neighbors j with weight w_{ij} . Furthermore, the network is balanced where $w_i = \sum_j w_{ij} = 1$, so that the degree for the agents is $\mathbb{D} := \text{diag}\{d_1, \dots, d_\ell\} = I_\ell$. The state errors between \mathcal{G}_ℓ and \mathcal{G}_m are denoted as linear operation of $\bar{x} = [x_1^T, \dots, x_\ell^T]^T$ multiplied by the Laplacian-like matrix of \mathcal{G}_ℓ , written as $\mathbb{L}_\ell := \mathbb{D} - \mathbb{A}_\ell$, and subtracted by the model \mathbb{A}_m . Here, \mathbb{A}_ℓ shows the adjacency matrix of \mathcal{G}_ℓ whereas $\mathbb{A}_m \in \mathbb{R}^{n \times n}$ represent the diagonal matrix containing the weights from the model to the connected agents only. The state error for system i , written as e_i , the Laplacian-like matrix \mathbb{L}_ℓ

and the model weight \mathbb{A}_m are formulated as (3),

$$e_i := [(\mathbb{L}_\ell \otimes I_n)\bar{x} - (\mathbb{A}_m \otimes I_n)\bar{x}_m]_i \rightarrow \mathbb{L}_n = \begin{bmatrix} w_1 & \cdots & w_{1\ell} \\ \vdots & \ddots & \vdots \\ w_{\ell 1} & \cdots & w_\ell \end{bmatrix}, \mathbb{A}_m = \begin{bmatrix} w_{1m} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & w_{\ell m} \end{bmatrix} \quad (3)$$

and the goal is to ensure $\lim_{t \rightarrow \infty} \bar{e} \rightarrow 0$ where $\bar{e} = [e_1^\top, \dots, e_\ell^\top]^\top$. Finally, we end the problem statement by describing the interconnected dynamical systems of (1) of the form,

$$\dot{\bar{x}}(t) = A\bar{x}(t) + A^\zeta \bar{x}(t - \tau_x) + B\bar{u}(t - \tau_u) \quad (4)$$

where $\bar{x} = [x_1^\top, \dots, x_\ell^\top]^\top \in \mathbb{R}^{\bar{n}}$ with $\bar{n} = n \times \ell$ defines the set of the states, while $\bar{u} = [u_1, \dots, u_\ell]^\top \in \mathbb{R}^\ell$. The matrices of $A := \text{diag}\{A_1, \dots, A_\ell\} \in \mathbb{R}^{\bar{n} \times \bar{n}}$ and $B := \text{diag}\{B_1, \dots, B_\ell\} \in \mathbb{R}^{\bar{n} \times \ell}$ are diagonal blocks of \mathcal{G}_ℓ . Also, we expand the model in (2) into diagonal block matrix of A_m and B_m of the form,

$$\dot{\bar{x}}_m(t) = A_m \bar{x}_m(t) + B_m \bar{r}(t - \tau_u) \quad (5)$$

where $\bar{x}_m = 1_\ell \otimes x_m$, $\bar{r} = 1_\ell \otimes r$, $A_m = I_\ell \otimes A_m$, and $B_m = I_\ell \otimes B_m$ with the same dimension as (4). Note that, the interactions among agents \mathcal{G}_ℓ and the model \mathcal{G}_m are constructed based on \mathbb{L}_ℓ and \mathbb{A}_m .

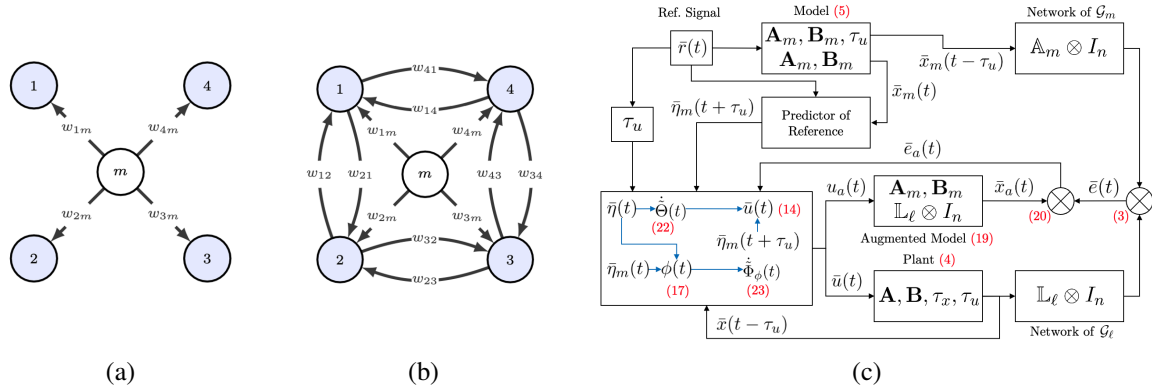


Figure 1. The two network scenarios shown in (a) example 1, (b) example 2, and the scheme in (c) defines the block diagram of the proposed method

Moreover, to reach the tracking problem for the agents, the local control scenario $\bar{u} = [u_1, \dots, u_\ell]$ is designed as follows in which Θ and $(\Phi_x, \Phi_\zeta, \Phi_r)$ are used interchangeably,

$$\bar{u}(t - \tau_u) = \Theta^\top(t) \bar{\eta}(t) \quad (6a)$$

$$= \begin{bmatrix} \left[\theta_x^{1\top}(t), \theta_\zeta^{1\top}(t), \theta_r^{1\top}(t) \right] & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \left[\theta_x^{\ell\top}(t), \theta_\zeta^{\ell\top}(t), \theta_r^{\ell\top}(t) \right] \end{bmatrix} \begin{bmatrix} [x_1^\top(t), x_1^\top(t - \tau_x), r(t - \tau_u)]^\top \\ \vdots \\ [x_\ell^\top(t), x_\ell^\top(t - \tau_x), r(t - \tau_u)]^\top \end{bmatrix} \quad (6b)$$

$$= \Phi_x^\top(t) \bar{x}(t) + \Phi_\zeta^\top(t) \bar{x}(t - \tau_x) + \Phi_r(t) \bar{r}(t - \tau_u) \quad (6c)$$

where $\Phi_x = \text{diag}\{\theta_x^1, \dots, \theta_x^\ell\}$, $\Phi_\zeta = \text{diag}\{\theta_\zeta^1, \dots, \theta_\zeta^\ell\}$ and $\Phi_r = \text{diag}\{\theta_r^1, \dots, \theta_r^\ell\}$. By substituting (6c) into (4) and if the optimal parameterization is achieved, where the states of \mathcal{G}_ℓ is on par with that of \mathcal{G}_m denoted as $\bar{x} \equiv \bar{x}_m$, then there exist the following balanced equations,

$$A + B\Phi_x^{*\top} - A_m = 0, \quad A_\zeta + B\Phi_\zeta^{*\top} = 0, \quad B\Phi_r^* - B_m = 0, \quad (7)$$

and $\lim_{t \rightarrow \infty} \bar{e}(t) = (\mathbb{L}_\ell \otimes I_n)\bar{x} - (\mathbb{A}_m \otimes I_n)\bar{x}_m \rightarrow 0$ is fulfilled. Finally, the assumption and remarks are required to guarantee the tracking and the consensus of the network \mathcal{G}_ℓ following the model \mathcal{G}_m .

Assumption 1 (Dynamics) : The dynamics of (4) is unknown but stable and the delays (τ_x, τ_u) are known. Also, we assume the signs of θ_r^* in $\Theta_i, \forall i = 1 \dots, \ell$ are known. The persistent excitation of exogenous noises is less than that of $\bar{r}(t)$ and $\bar{u}(t)$, also $\exists \Phi_x^*, \Phi_\zeta^*, \Phi_r^*$ for arbitrary \mathbb{L}_ℓ and \mathbb{A}_m such that Remark 1 and 2 hold.

Remark 1 (Threshold of network) : The eigenvalues of the Laplacian-like matrix \mathbb{L}_ℓ and the model weight \mathbb{A}_m are greater than or equal to a constant $\vartheta = 0.1$, denoted as $\lambda_i^\ell \geq \vartheta$ and $\lambda_i^m \geq \vartheta$ respectively. Note that this threshold ϑ is user-design and depends on the complexity of the networks.

Remark 2 (Communication network) : The network is balanced $[(\mathbb{L}_\ell - \mathbb{A}_m) \otimes I_n] \mathbf{1}_{\bar{n}} = 0$. We design such that there is always a directed path from the model \mathcal{G}_m to each agent in \mathcal{G}_ℓ as shown in Figures 1(a) and 1(b). If this is violated, then $\exists \lambda_i^m = 0 < \vartheta$ for some i and the design of the augmented model in (15) should be modified.

3. ADAPTATION ALGORITHMS AND STABILITY

We consider the closed form of the tracking error $\bar{e} := (\mathbb{L}_\ell \otimes I_n)\bar{x} - (\mathbb{A}_m \otimes I_n)\bar{x}_m$ to generate the adaptive laws and to analyze the stability. Recalling (4), (5) and (7), we have the following formula,

$$\begin{aligned} \dot{\bar{e}}(t) &= (\mathbb{L}_\ell \otimes I_n)\dot{\bar{x}} - (\mathbb{A}_m \otimes I_n)\dot{\bar{x}}_m \\ &= (\mathbb{L}_\ell \otimes I_n) [A\bar{x}(t) + A^\zeta \bar{x}(t - \tau_x) + B\bar{u}(t - \tau_u)] - (\mathbb{A}_m \otimes I_n) [A_m \bar{x}_m(t) + B_m \bar{r}(t - \tau_u)] \end{aligned} \quad (8a)$$

$$\begin{aligned} &= [(\mathbb{L}_\ell \otimes I_n)A_m - (\mathbb{L}_\ell \otimes I_n)B\Phi_x^{*\top}(t)] \bar{x}(t) - (\mathbb{L}_\ell \otimes I_n)B\Phi_\zeta^{*\top}(t)\bar{x}(t - \tau_x) \\ &\quad + (\mathbb{L}_\ell \otimes I_n)B\bar{u}(t - \tau_u) - (\mathbb{L}_\ell \otimes I_n)A_m \bar{x}(t) + A_m \bar{e}(t) - (\mathbb{L}_\ell \otimes I_n)B\Phi_r^*(t)\bar{r}(t - \tau_u) \end{aligned} \quad (8b)$$

$$= A_m \bar{e}(t) - (\mathbb{L}_\ell \otimes I_n)B \left[[\Phi_x^{*\top}(t)\bar{x}(t) + \Phi_\zeta^{*\top}(t)\bar{x}(t - \tau_x) + \Phi_r^*(t)\bar{r}(t - \tau_u)] + \bar{u}(t - \tau_u) \right] \quad (8c)$$

and by introducing a parameter error $\tilde{\Theta}$ where $\Theta := \Theta^* + \tilde{\Theta}$, the formula in (8c) based on (6c) is denoted as,

$$\begin{aligned} \dot{\bar{e}}(t) &= A_m \bar{e}(t) - (\mathbb{L}_\ell \otimes I_n)B\Theta^{*\top}(t)\bar{\eta}(t) + (\mathbb{L}_\ell \otimes I_n)B\bar{u}(t - \tau_u) \\ &= A_m \bar{e}(t) + (\mathbb{L}_\ell \otimes I_n)B \left[\tilde{\Theta}^\top(t) - \Theta^\top(t) \right] \bar{\eta}(t) + (\mathbb{L}_\ell \otimes I_n)B\bar{u}(t - \tau_u). \end{aligned} \quad (9)$$

However, this input-delay $\bar{u}(t - \tau_u)$ should be adjusted for the sake of eliminating the required prediction to the control signal of $\bar{u}(t + \tau_u|t)$. To tackle this issue, the time-varying function of inputs is designed to come from the model \mathcal{G}_m , defined as $\bar{\eta}_m(t)$, which is more easily to handle, such that,

$$\bar{u}(t) = \Theta^\top(t)\bar{\eta}_m(t + \tau_u|t) \quad (10)$$

where for the delay-term $\bar{u}(t - \tau_u)$, it becomes the following,

$$\begin{aligned} \bar{u}(t - \tau_u) &= \Theta^\top(t - \tau_u)\bar{\eta}_m(t) \\ &= \Phi_x^\top(t - \tau_u)\bar{x}_m(t) + \Phi_\zeta^\top(t - \tau_u)\bar{x}_m(t - \tau_x) + \Phi_r(t - \tau_u)\bar{r}(t - \tau_u). \end{aligned} \quad (11)$$

Therefore, the error equation in (9) is expressed as follows,

$$\begin{aligned} \dot{\bar{e}}(t) &= A_m \bar{e}(t) + (\mathbb{L}_\ell \otimes I_n)B \left[\tilde{\Theta}^\top(t) - \Theta^\top(t) \right] \bar{\eta}(t) + (\mathbb{L}_\ell \otimes I_n)B\Theta^\top(t - \tau_u)\bar{\eta}_m(t) \\ &= A_m \bar{e}(t) + (\mathbb{L}_\ell \otimes I_n)B \left[\tilde{\Theta}^\top(t)\bar{\eta}(t) - \phi(t) \right] \end{aligned} \quad (12)$$

where the delay-term $\phi(t)$ is the inputs difference from two time-varying functions of $\bar{\eta}$ and $\bar{\eta}_m$ as (17),

$$\phi(t) \in \mathbb{R}^\ell = \Theta^\top(t)\bar{\eta}(t) - \Theta^\top(t - \tau_u)\bar{\eta}_m(t). \quad (13)$$

Now, let us consider the auxiliary gain Φ_ϕ where $\Phi_\phi = \text{diag}\{\theta_\phi^1, \dots, \theta_\phi^\ell\}$ such that this $B = B_m \Phi_\phi^*$ holds. The idea is to make the standard error in (12), therefore

$$\begin{aligned} \dot{\bar{e}}(t) &= A_m \bar{e}(t) + (\mathbb{L}_\ell \otimes I_n)B\tilde{\Theta}^\top(t)\bar{\eta}(t) - (\mathbb{L}_\ell \otimes I_n)B_m \Phi_\phi^*(t)\phi(t) \\ &= A_m \bar{e}(t) + (\mathbb{L}_\ell \otimes I_n)B\tilde{\Theta}^\top(t)\bar{\eta}(t) + (\mathbb{L}_\ell \otimes I_n)B_m \tilde{\Phi}_\phi(t)\phi(t) - (\mathbb{L}_\ell \otimes I_n)B_m \Phi_\phi(t)\phi(t) \end{aligned} \quad (14)$$

in which this Φ_ϕ should be adjusted over time where $\bar{u}_a = \Phi_\phi \phi$ and to cancel the Φ_ϕ term in (14), the auxiliary model is required as written in (15). Here is some additional comments on Remark 2, if $\exists \lambda_i^m = 0 < \vartheta$, then $\exists \lambda_i^\ell = 0$, then $\exists u_{a_i} = 0$, then $\exists \theta_\phi^{*i} = 0$, violating $B_i = B_m \theta_\phi^{*i}$ for some i .

$$\dot{\bar{x}}_a = A_m \bar{x}_a(t) + (\mathbb{L}_\ell \otimes I_n) B_m \Phi_\phi(t) \phi(t) \quad (15)$$

This also means for some i , the unknown B is used in (19) instead of B_m , which is impossible. This forces the existence of the directed path in Remark 2, considering ϑ in Remark 1. Then, the augmented error is defined as,

$$\bar{e}_a(t) = \bar{e}(t) + \bar{x}_a(t) \quad (16)$$

where by adding (14) and (15), we have the standard error equation in adaptive control of networked system,

$$\dot{\bar{e}}_a(t) = A_m \bar{e}_a(t) + (\mathbb{L}_\ell \otimes I_n) B \tilde{\Theta}^\top(t) \bar{\eta}(t) + (\mathbb{L}_\ell \otimes I_n) B_m \tilde{\Phi}_\phi(t) \phi(t) \quad (17)$$

and this leads to natural form of adjustable parameterization. To achieve $\lim_{t \rightarrow \infty} \bar{e}_a = 0$ and $\lim_{t \rightarrow \infty} \bar{x}_a = 0$, the update schemes of $\tilde{\Theta}$ and $\tilde{\Phi}_\phi$ are chosen as (22) and (23),

$$\dot{\tilde{\Theta}}^\top = \dot{\Theta}^\top = -\text{sign}(\Phi_r^*) \Gamma_\theta B_m^\top (\mathbb{L}_\ell \otimes I_n)^\top P \bar{e}_a(t) \bar{\eta}^\top(t) \quad (18)$$

$$\dot{\tilde{\Phi}}_\phi = \dot{\Phi}_\phi = -\Gamma_\phi B_m^\top (\mathbb{L}_\ell \otimes I_n)^\top P \bar{e}_a(t) \phi(t) \quad (19)$$

where $\Gamma_\theta, \Gamma_\phi \in \mathbb{R}^{\ell \times \ell}$ and $P = P^\top \succ 0 \in \mathbb{R}^{\bar{n} \times \bar{n}}$ satisfies the Lyapunov function $\mathbf{A}_m^\top P + P \mathbf{A}_m = -(Q + Q_a)$ for positive definite matrices of $Q = Q^\top \succ 0$ and $Q_a = Q_a^\top \succ 0$. The stability of the networked system given (17) is presented by the opting the following Lyapunov function candidate $V(\bar{e}_a, \tilde{\Theta}, \tilde{\Phi}_\phi)$,

$$V(\bar{e}_a, \tilde{\Theta}, \tilde{\Phi}_\phi) = \bar{e}_a^\top(t) P \bar{e}_a(t) + \int_{-\tau_x}^0 \bar{e}_a^\top(\sigma) Q_a \bar{e}_a(\sigma) d\sigma \quad (20a)$$

$$+ \text{tr} \left[\tilde{\Theta}(t) \Gamma_\theta^{-1} |\Phi_r^{*-1}| \tilde{\Theta}^\top(t) \right] + \int_{-\tau_1}^0 \text{tr} \left[\tilde{\Theta}(\sigma) \Gamma_\theta^{-1} |\Phi_r^{*-1}| \tilde{\Theta}^\top(\sigma) \right] d\sigma \quad (20b)$$

$$+ \text{tr} \left[\tilde{\Phi}_\phi(t) \Gamma_\phi^{-1} \tilde{\Phi}_\phi^\top(t) \right] + \int_{-\tau_2}^0 \text{tr} \left[\tilde{\Phi}_\phi(\sigma) \Gamma_\phi^{-1} \tilde{\Phi}_\phi^\top(\sigma) \right] d\sigma \quad (20c)$$

where the derivative along the trajectory is guaranteed to be negative definite considering $\text{tr}[ab^\top] = b^\top a$,

$$\begin{aligned} \dot{V} &= \bar{e}_a^\top(t) [A_m^\top P + P A_m] \bar{e}_a(t) + 2\bar{e}_a^\top(t) P (\mathbb{L}_\ell \otimes I_n) B \tilde{\Theta}^\top(t) \bar{\eta}(t) \\ &\quad + 2\bar{e}_a^\top(t) P (\mathbb{L}_\ell \otimes I_n) B_m \tilde{\Phi}_\phi(t) \phi(t) - \bar{e}_a^\top(t - \tau_x) Q_a \bar{e}_a(t - \tau_x) \\ &\quad + 2 \text{tr} \left[\dot{\tilde{\Theta}}(t) \Gamma_\theta^{-1} |\Phi_r^{*-1}| \tilde{\Theta}^\top(t) \right] - \text{tr} \left[\left(\tilde{\Theta}(t) + \tilde{\Theta}(t - \tau_1) \right) \Gamma_\theta^{-1} |\Phi_r^{*-1}| \left(\tilde{\Theta}(t) + \tilde{\Theta}(t - \tau_1) \right)^\top \right] \\ &\quad + 2 \text{tr} \left[\dot{\tilde{\Phi}}_\phi(t) \Gamma_\phi^{-1} \tilde{\Phi}_\phi^\top(t) \right] - \text{tr} \left[\left(\tilde{\Phi}_\phi(t) + \tilde{\Phi}_\phi(t - \tau_2) \right) \Gamma_\phi^{-1} \left(\tilde{\Phi}_\phi(t) + \tilde{\Phi}_\phi(t - \tau_2) \right)^\top \right] \\ &= -\bar{e}_a^\top(t) Q \bar{e}_a(t) - \text{tr} \left[\left(\tilde{\Phi}_\phi(t) + \tilde{\Phi}_\phi(t - \tau_2) \right) \Gamma_\phi^{-1} \left(\tilde{\Phi}_\phi(t) + \tilde{\Phi}_\phi(t - \tau_2) \right)^\top \right] \\ &\quad - \text{tr} \left[\left(\tilde{\Theta}(t) + \tilde{\Theta}(t - \tau_1) \right) \Gamma_\theta^{-1} |\Phi_r^{*-1}| \left(\tilde{\Theta}(t) + \tilde{\Theta}(t - \tau_1) \right)^\top \right] - \bar{e}_a^\top(t - \tau_x) Q_a \bar{e}_a(t - \tau_x) \\ &\quad + 2\bar{e}_a^\top(t) P (\mathbb{L}_\ell \otimes I_n) B \tilde{\Theta}^\top(t) \bar{\eta}(t) - 2 \text{tr} \left[\bar{\eta}(t) \bar{e}_a^\top(t) P (\mathbb{L}_\ell \otimes I_n) B_m \Gamma_\theta \Gamma_\theta^{-1} |\Phi_r^{*-1}| \tilde{\Theta}^\top(t) \right] \quad (21a) \\ &\quad + 2\bar{e}_a^\top(t) P (\mathbb{L}_\ell \otimes I_n) B_m \tilde{\Phi}_\phi(t) \phi(t) - 2 \text{tr} \left[\phi(t) \bar{e}_a^\top(t) P (\mathbb{L}_\ell \otimes I_n) B_m \Gamma_\phi \Gamma_\phi^{-1} \tilde{\Phi}_\phi^\top(t) \right] \quad (21b) \\ &< 0. \quad (21c) \end{aligned}$$

For the chosen $\tau_1 \leq \tau_x$ and $\tau_2 \leq \tau_u$ and the designed $\tilde{\Theta}$ and $\tilde{\Phi}_\phi$ as (18) and (19) in turn. To end, we deliver the sufficient condition for stability in the following remark and the theorem covering the proposed idea.

Remark 3 (Stability) : The non-integral parts in (20a)-(20c) are adequate for stability and tracking, since

$$V_d = \bar{e}_a^\top(t) P \bar{e}_a(t) + \text{tr} \left[\tilde{\Theta}(t) \Gamma_\theta^{-1} |\Phi_r^{*-1}| \tilde{\Theta}^\top(t) \right] + \text{tr} \left[\tilde{\Phi}_\phi(t) \Gamma_\phi^{-1} \tilde{\Phi}_\phi^\top(t) \right] \quad (22)$$

and the derivative of V_d results in $\dot{V}_d = -\bar{e}_a^\top(t) Q \bar{e}_a(t) < 0$ due to the cancellation of (21a) and (21b).

Theorem 1 : Consider the networked delayed-system (4) of \mathcal{G}_ℓ and (5) of \mathcal{G}_m with Laplacian-like matrix $(\mathbb{L}_\ell \otimes I_n)$ and model weight $(\mathbb{A}_m \otimes I_n)$ satisfying Remark 1 and 2. The pairs of (\mathbf{A}, \mathbf{B}) is stabilizable satisfying Assumption 1 and let control signal be $\bar{u}(t) := \Theta^\top(t) \bar{\eta}_m(t + \tau_u | t)$ as in (10) where $\eta_{m_i} : \mathbb{R}^+ \rightarrow \mathbb{R}^q$, $\bar{\eta}_m = [\eta_{m_1}^\top, \dots, \eta_{m_\ell}^\top]^\top$ be the measured time-varying functions and $\Theta \in \mathbb{R}^{\bar{q} \times \ell} = \text{diag}\{\Theta^1, \dots, \Theta^\ell\}$ with $[\theta_x^{i\top}, \theta_\zeta^{i\top}, \theta_r^{i\top}]^\top$, $\bar{q} = q \times \ell$ be the adaptive term such that there exists the augmented model $\bar{x}_a(t)$ in (15) and the augmented error $\bar{e}_a(t)$ in (17), then the adaptive terms of,

$$\begin{aligned} \dot{\Theta}^\top &= \dot{\Theta}^\top = -\text{sign}(\Phi_r^*) \Gamma_\theta \mathbf{B}_m^\top (\mathbb{L}_\ell \otimes I_n^\top) P \bar{e}_a(t) \bar{\eta}^\top(t) \\ \dot{\Phi}_\phi &= \dot{\Phi}_\phi = -\Gamma_\phi \mathbf{B}_m^\top (\mathbb{L}_\ell \otimes I_n)^\top P \bar{e}_a(t) \phi(t) \end{aligned} \quad (23)$$

guarantee the stability and the tracking in which the equilibrium (\bar{x}, Θ) is uniformly stable.

We summarize the proposed method graphically in Figure 1(c) in which the idea is to make the agents in (5) affected by the network \mathbb{L}_ℓ follow the leader in (4) influenced by \mathbb{A}_m . To deal with the delays, we propose the predictor signals from the leader and the augmented systems where the updated laws are presented in (23). Note that, if the two remarks and the given assumption hold, then the theorem given above is guaranteed.

4. NUMERICAL RESULTS AND FINDINGS

In this section, we consider the dynamics of a model and the unknown $\ell := 4$ agents with known delays $\tau_x = 3$ s and $\tau_u = 5$ s where the networks are shown in Figures 1(a) and 1(b) as follows,

$$\begin{aligned} \dot{\bar{x}}_m(t) &= \left(I_4 \otimes \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \right) \bar{x}_m(t) + \left(I_4 \otimes \begin{bmatrix} 0 \\ -2 \end{bmatrix} \right) \bar{r}(t - \tau_u) \\ \dot{\bar{x}}(t) &= \text{diag} \left\{ \begin{bmatrix} 0 & 1 \\ -2 - i & -1 - i \end{bmatrix} \right\} \bar{x}(t) + \text{diag} \left\{ \begin{bmatrix} 0 & 0 \\ \frac{2+i}{10} & \frac{2+i}{20} \end{bmatrix} \right\} \bar{x}(t - \tau_x) + \text{diag} \left\{ \begin{bmatrix} 0 \\ 2+i \end{bmatrix} \right\} \bar{u}(t - \tau_u) \end{aligned}$$

for all $i = 1, \dots, 4$. The degree matrix $\mathbb{D} = I_4$ for both examples and the Laplacian-like matrix $\{\mathbb{L}_\ell | \mathbb{L}_\ell = \mathbb{D} - \mathbb{A}_\ell\}$ are defined as follows, where the model weight is chosen such that $[(\mathbb{L}_\ell - \mathbb{A}_m) \otimes I_n] \mathbf{1}_{\bar{n}} = 0$,

$$\mathbb{L}_\ell = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \otimes I_n, \quad \mathbb{A}_m = I_4 \otimes I_n \quad \left| \quad \mathbb{L}_\ell = \begin{bmatrix} 1 & -\gamma & 0 & -\gamma \\ -\gamma & 1 & -\gamma & 0 \\ 0 & -\gamma & 1 & -\gamma \\ -\gamma & 0 & -\gamma & 1 \end{bmatrix} \otimes I_n, \quad \mathbb{A}_m = 0.4 I_4 \otimes I_n$$

for example 1 and 2 in turn where $\gamma = 0.3$. Here, we design $\Gamma_\theta = \Gamma_\phi = I_4$, $Q + Q_a = I_4 \otimes 0.1 I_n$ while $P = I_4 \otimes ([p_1, p_2]/100)$ where $p_1 = [25 \ 5]^\top$, $p_2 = [5 \ 5]^\top$ satisfying $A_m^\top P + P A_m = -(Q + Q_a)$. Moreover, we design the initial conditions $\Theta(0) = -0.001 \text{diag}\{12.5, 10, 7.5, 5\} \otimes \mathbf{1}_q$, $\Phi_\phi(0) = -0.1 \text{diag}\{4, 3, 2, 1\}$.

The simulation is disturbed by the bounded noise $v_i(t), v_m(t) \in \Omega$ satisfying Assumption 1. It is obvious for example 1 in Figure 1(a), since all agents are fully connected to the model $w_{i\ell} = 1, \forall i$, then the results converge faster as portrayed in Figures 2(a)-2(b). Regarding example 2 depicted in Figure 1(b), it requires more communication among the linked agents given by $\gamma = 0.3$ which are unstable, and the leader, yielding the slow convergences as shown in Figures 2(c)-2(d). However, due to the stable updated gains approaching the optimal values, the perfect tracking is achieved.

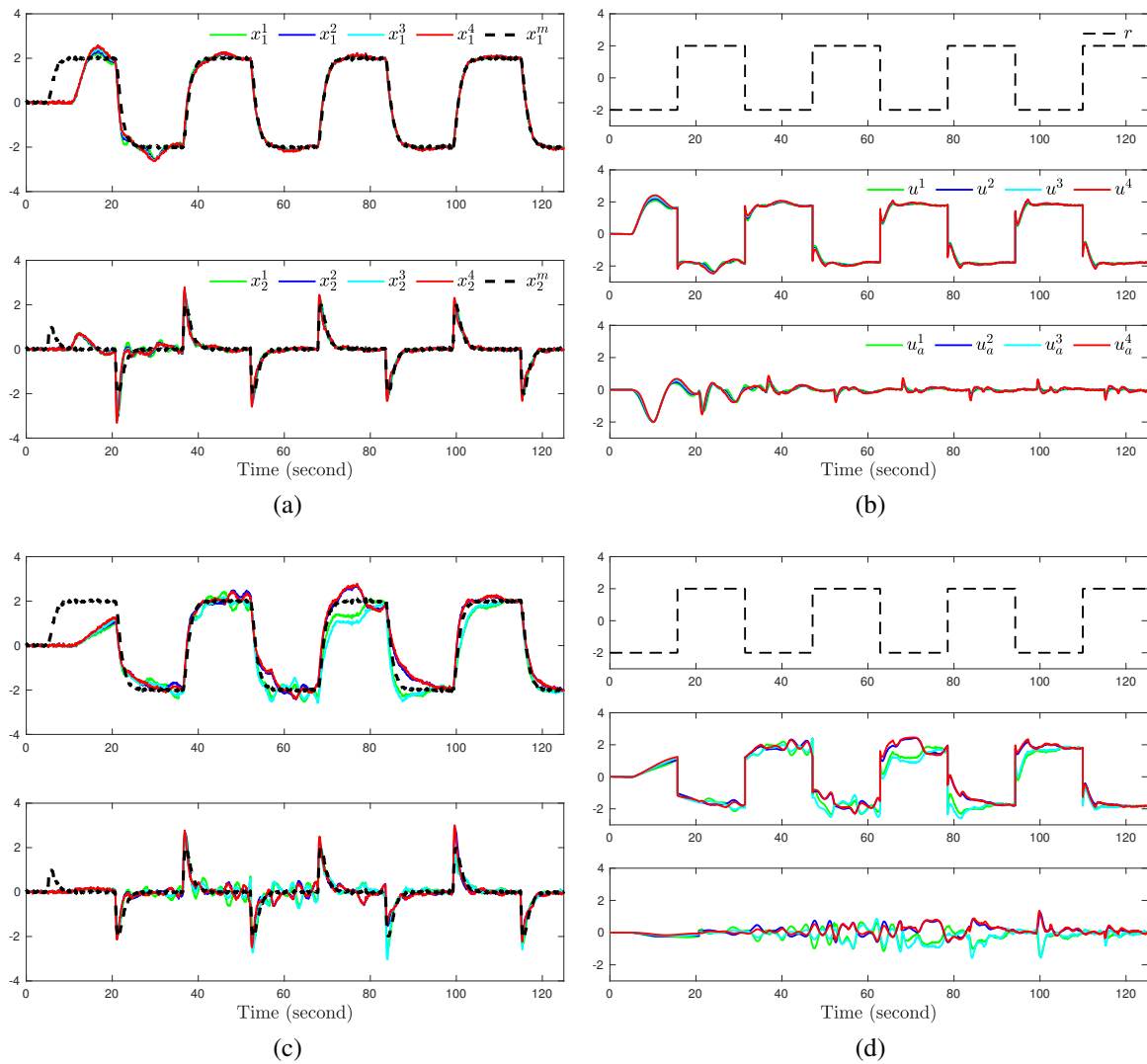


Figure 2. The results converge faster and slow: (a) tracking of states $x^i(t)$ for example 1 shown in Figure 1(a); (b) $\bar{r}(t)$, $\bar{u}(t)$, $\bar{u}_a(t)$ of example 1; (c) Tracking of states $x^i(t)$ for example 2 depicted in Figure 1(b); and (d) $\bar{r}(t)$, $\bar{u}(t)$, $\bar{u}_a(t)$ of example 2

5. CONCLUSION

The mathematical derivation of the unknown heterogeneous systems following a leader has been delivered, including the stability and some remarks. The threshold and the communication network have been given, to show the boundary of the proposed problem. The results show that the proposed method could handle the state and input lumped delays among the agents and the leader in the networked system to reach certain consensus from the leader. The numerical simulation of two different networks is provided, yielding the perfect tracking. The near future research would apply the distributed adaptive control-oriented learning of networked system under some faults.

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


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