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Mixed H_2/H_∞ robust controllers in aircraft control problem

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ABSTRACT

A leading cause of accidents during the landing phase of a flight lies in a considerable altitude loss by an aircraft as a result of the impact of a microburst of wind. One of the significant factors focuses primarily on the need to simultaneously satisfy various requirements regarding conditions of environmental disturbances and a wide range of systemic changes. The paper presents an algorithm for synthesizing an optimal controller that solves the mixed H_2/H_∞ control problem for the stabilization of aircraft in glide-path landing mode in the presence of uncertainty. Firstly, the principles of multi-criteria optimization are presented, and the mixed H_2/H_∞ problem is interpreted as the synthesis of a system with optimal quadratic performance, subject to its readiness to operate with the worst disturbance. Then, the ensuing section expounds upon the mathematical depiction of the vertical trajectory of aircraft, duly considering the perturbations imposed by wind phenomena. Subsequently, the effectiveness of mixed H_2/H_∞ control is confirmed compared to autonomous H_2 or H_∞ regulators through simulation outcomes acquired from the created system. Optimization based on a hybrid (mixed) criterion allowed combining the strengths of locally optimal systems based only on H_2 or H_∞ theory.

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1. INTRODUCTION

A high accuracy in determining motion parameters and controlling the aircraft is an essential requirement for modern control system design [1]–[8]. This emergence necessitates considering various uncertainty factors during the development phase of appropriate control algorithms. Particular importance is attached to random uncertainties affecting aircraft flight include the disturbances in the atmosphere, such as density deviation from the standard value and wind shear, as well as processing errors in control actions, deviations in the aerodynamic, geometric, and several other factors [9]–[13]. It is important to note that the vast majority of flight accidents occur due to adverse meteorological conditions. The meteorological phenomenon of a local disturbance of atmospheric state, known as the vortex ring microburst, poses a significant threat to aircraft flights, particularly during take-off and landing phases [14]–[18]. In the context of the examined control algorithms within this domain, the comprehensive review of existing literature uncovers a multitude of diverse strategies employed for the purpose of aircraft control [19]–[23].

In a comprehensive review of intelligent transforming aircraft, Chu *et al.* [19] discuss both general and specific challenges in their development. Ghazali *et al.* [20] proposes a multinodal hormone regulation of neuroendocrine proportional-integral-derivative (PID) controller of multiple-input-multiple-output (MIMO) systems grounded on adaptive safe experimentation dynamics (ASED). Similarly, Ghazali *et al.* [21] investigate the incorporation of controlled sigmoid-based secretion rate neuroendocrine PID in a twin-rotor MIMO system using ASED algorithm. In reference to the findings presented by Kiselev *et al.* [22], the research delves into the examination of flight dynamics exhibited by a hypothetical maneuverable aircraft. Additionally, it investigates the application of algorithms aimed at augmenting stability and controllability, thereby compensating for inherent limitations in these characteristics. Notably, a sophisticated boundary delineating the permissible angle of attack is introduced, contingent upon the specific flight mode under consideration. Idrissi *et al.* [23] explores vertical take-off and landing arrangements, presents applicable modeling tools and control strategies, and applies them to a quadrotor.

The problem of ensuring high-quality landing control is highly relevant, especially in the presence of atmospheric disturbance. Robust controllers based on H_∞ control method is extensively applied extensively in order to address this problem. The H_∞ theory provides a powerful framework for the synthesis of multivariable robust control systems. The standard (unstructured) and structured H_∞ control development techniques have been effectively used to ensure the establishment of robust controllers. The investigation in [15] revolves around the examination and formulation of a robust glide-path approach controller of the H_∞ structure. The controller is an integral component of automated landing system formulated in response to the aircraft landing challenge proposed by Airbus. In [16], an integrated control method is considered for the Autoland system of a civil aircraft, which combined stable inversion swarm intelligence (SI) algorithm and H_∞ synthesis to simultaneously solve the problem of tracking the trajectory and deflection disturbances.

In the realm of linear parameter-varying (LPV) systems, wherein faults in actuators and sensors occur concurrently, the issue of robust active fault-tolerant control is the focal point of investigation within Tayari *et al.* [24]. The assurance of stability for the systems operating in closed-loop configuration is ensured through the application of H_∞ performance measures. Within in [25], an integrated sliding-mode controller incorporating self-adaptation is devised, aiming to attain finite-time convergence in system control, regardless of the underlying parameters. The study focuses on the LPV model, which experiences significant alterations in sweep angle and expansion, encompassing a broad range of parameters. The state-feedback linear fractional representation (LFR)-H_∞ controller is derived through the utilization of constraints based on linear matrix inequalities. Subsequently, the necessary prerequisites for the existence of sliding mode characterized by integral action are derived by means of pole assignment.

Yue *et al.* [26] describes the development of a morphing aircraft engine multi-loop controller, which ensures the steadiness of the process of wing transition. The offered controller employs a collection of inner loop gains in order to guarantee stability, leveraging basic methodologies as the foundation for its design. A self-tuning H_∞ controller is formulated for the outer loop gain to attain a satisfactory degree of robust stability and operational effectiveness, particularly in the presence of non-stationary dynamics. A comprehensive research in [27] focus on the determination of robust controller parameters for the lateral control of aircraft, wherein the utilization of auxiliary damping automatic devices (ADAD) plays a pivotal role. The synthesis of the suggested controller is founded upon the utilization of both H_∞ and μ techniques, serving as the fundamental framework for it is development.

The structured H_∞ paradigm has emerged as a versatile approach for implementation of multi-requirement and multi-variable control systems. In research [14], a structured H_∞ method based on a standard H_∞ control structure is examined for a vertical speed controller. Biannic *et al.* [17] concentrates on the demanding flare phase in the conditions of high wind and parametric uncertainties based on a structured principle of H_∞ control. The results of the research provide important insights into the problem of aircraft vertical speed control before landing phase of a flight, minimizing the impact of variations in airspeed, wind gradient, and ground proximity. Marcos *et al.* [28] provides an extensive comparative study, centered around the assessment of two distinct control schemes utilized to actively suppress flutter in a flexible unmanned aerial vehicle, with thorough analysis and evaluation. The H_∞ approach is applied in the development of both controllers, however, the first is based on a standard (i.e., unstructured) synthesis, and the second is based on a structured technique. Beisenbi and Basheyeva [29] describes the application of the Lyapunov function to construct robustly stable aircraft control systems. Karimtaevna and Asylbekkyzy [30] outlines a design methodology and implementation of robust control using H_∞ synthesis tools, which allows to cope more effectively with parameters and load perturbation. The research conducted in Karimtaevna *et al.* [31] delves into a meticulous investigation of the H_2 and H_∞ synthesis methods, specifically exploring their potential in the realization systems responsible for controlling the flight of an aircraft during the crucial landing phase, while effectively mitigating the impact of external disturbances.

A promising approach consists of system optimizing using several criteria, each of which applies under certain circumstances; consequently, there arises a necessity of considering the problem of robust controller synthesis in terms of simultaneously satisfying two optimization H_2/H_∞ robust controller criteria [32]–[34]. An analysis of scientific publications dedicated to the field of the mixed H_2/H_∞ robust controller synthesis indicates that the issue of using the mixed H_2/H_∞ controller for solving the problem of aircraft control under conditions of uncertainty has not received sufficient attention. The investigation of the H_2/H_∞ controller is carried out only from the perspective of robust stability, and the issue of improving the technical characteristics therefore remains relevant. The problem of developing mixed H_2/H_∞ robust controllers for aircraft flight control under conditions of uncertainty is of relevance to both academic research and industrial applications.

This paper describes the synthesis of the mixed H_2/H_∞ robust controller for regulating aircraft motion in the vertical plane throughout the critical landing phase, even in the presence of uncertain disturbances. This solution effectively enhances the robustness of the system, effectively mitigating the adverse effects of uncertainties induced by disturbances caused by wind conditions. Section 2, entitled "research method," offers an exhaustive assessment of the fundamental principles underlying multi-objective optimization, interprets the mixed H_2/H_∞ control approach as the problem of optimal quadratic quality under the condition of robust stability, and constructs a mathematical model capturing the intricate dynamics of airplane in the vertical dimension, accounting for the influence of uncertain disturbances. Section 3, entitled "results and analysis," presents the findings of the application of the mixed H_2/H_∞ optimal controller to aircraft's flight control mechanisms, specifically addressing the challenges encountered during the critical landing phase in the face of turbulent wind interferences. The simulation outcomes provide evidence supporting the effectiveness of the blended H_2/H_∞ control strategy in terms of its efficiency. The simulation results provide evidence supporting the effectiveness of the mixed H_2/H_∞ control strategy in terms of its efficiency. Finally, section 4 presents the primary findings and imparts recommendations for forthcoming investigations, thus culminating the study.

2. RESEARCH METHOD

Controller synthesis based on various criteria (i.e., norms) that are related to either to one or different system outputs is a common aspect of multi-objective optimization. To accurately represent the output, a quadratic or uniform-frequency index is typically employed. The development of a controller that optimally represents the first or second indicator is achieved using well-known algorithms described in literature [35], [36]. Recently, the optimization of the system output based on both frequency-uniform and quadratic criteria simultaneously, known as mixed H_2/H_∞-control, has gained significant attention.

Contemplate a stationary linear system depicted in Figure 1, which possesses finite dimensions. Assume the closed-loop control system exhibits internal stability. The plant $G(s)$ and controller $K(s)$ are described by the state-space equations in (1) and (2) [35], [36].

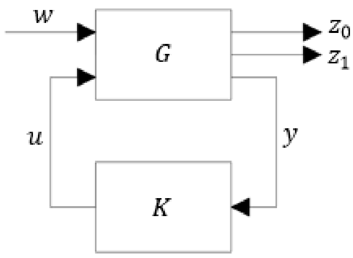

Figure 1. Scheme of a linear finite-dimensional stationary system

$$\begin{aligned} \dot{x} &= Ax + B_1 w + B_2 u; \\ z_0 &= C_0 x + D_0 u; \\ z_1 &= C_1 x + D_1 u; \\ y &= C_2 x + D_2 w. \end{aligned} \tag{1}$$

$$\begin{aligned} \dot{x}_c &= A_c x_c + B_c y; \\ u &= C_c x_c. \end{aligned} \tag{2}$$

By substituting expression (2) into (1), the expression (3) is obtained,

$$\begin{aligned} \dot{\tilde{x}} &= \tilde{A}\tilde{x} + \tilde{B}w; \\ z_0 &= \tilde{C}_0 \tilde{x}; \\ z_1 &= \tilde{C}_1 \tilde{x}, \end{aligned} \tag{3}$$

where

$$\tilde{A} = \begin{bmatrix} A & B_2C_c \\ B_cC_2 & A_c \end{bmatrix}, \quad \tilde{B} = \begin{bmatrix} B_1 \\ B_cD_2 \end{bmatrix}, \quad \tilde{C}_0 = [C_0 \quad D_0C_c], \quad \tilde{C}_1 = [C_1 \quad D_1C_c].$$

Let T_{zw} be the transfer function matrix of a closed-loop control system from input w to z:

$$T_{zw} = \begin{bmatrix} T_{z_0w} \\ T_{z_1w} \end{bmatrix}. \tag{4}$$

The synthesized controller must meet the following conditions [36], [37]:

a) A closed-loop system exhibits stability properties, i.e., $\tilde{A}$ is a stable matrix.
b) The transfer function $T_{z_1w}(s) = \tilde{C}_1\left(sI - \tilde{A}\right)^{-1}\tilde{B}$ satisfies the constraint $\left\|T_{z_1w}\right\|_\infty < \gamma$.
c) The quality functional is minimized: $J(T_{z0w}) = \lim_{t\to\infty}\int_0^t\{Z_0^T(t)Z_0(t)\}\,dt = \lim_{t\to\infty}\int_0^t\{\tilde{x}^T(t)\tilde{R}\tilde{x}(t)\}\,dt = \lim_{t\to\infty}\int_0^t\{x^T(t)R_1x(t) + u^T(t)R_2u(t)\}\,dt,\ R_1 = C_0^TC_0,\ R_2 = D_0^TD_0,\ \tilde{R} = \tilde{C}_0^T\tilde{C}_0 = \begin{bmatrix} C_0^T \\ C_c^TD_0^T \end{bmatrix}[C_0 \quad D_0C_c] = \begin{bmatrix} C_0^TC_0 & 0 \\ 0 & C_c^TD_0^TD_0C_c \end{bmatrix} = \begin{bmatrix} R_1 & 0 \\ 0 & C_c^TR_2C_c \end{bmatrix}$; where $J\left(T_{z_0w}\right)$ is a special case of the functional of stochastic linear optimal control task $\lim_{t\to\infty}\frac{1}{t}E\left\{\int_0^t Z_0^T(t)Z_0(t)\,dt\right\}$ for systems with constant parameters [35]. Minimization of the functional $J\left(T_{z_0w}\right)$ is equivalent to the minimization of H_2 norm of the transfer matrix T_{z_0w}, which is regular, and consequently $\left\|T_{z_0w}\right\|_2$ is finite [35].

As the problem formulation includes both H_2 and H_∞ quality components, similar to the R_1 and R_2 matrices of the H_2, corresponding matrices for the H_∞ are introduced. Let $R_{1\infty} = C_1^TC_1$, $R_{2\infty} = D_1^TD_1$, $\tilde{R}_\infty = \tilde{C}_1^T\tilde{C}_1$. Similarly, $C_1^TD_1 = 0$, and let $R_{2\infty} = \beta^2R_2$, where the non-negative scalar β is a design variable. Let L_c denote the controllability Gramian for an $\left(\tilde{A},\tilde{B}\right)$ pair. It satisfies the (5),

$$\tilde{A}L_c + L_c\tilde{A}^T + \tilde{B}\tilde{B}^T = 0 \tag{5}$$

then [35]:

$$J\left(T_{z_0w}\right) = \left\|T_{z_0w}\right\|_2^2 = trace\left(\tilde{C}_0L_c\tilde{C}_0^T\right) = trace\left(\tilde{R}L_c\right)$$

Therefore, solving Riccati equations Y:

$$R(Y) = \tilde{A}Y + Y\tilde{A}^T + Y\tilde{R}_\infty Y\gamma^{-2} + \tilde{V} = 0 \tag{6}$$

where $\tilde{V} = \tilde{B}\tilde{B}^T = \begin{bmatrix} B_1B_1^T & 0 \\ 0 & B_cD_2D_2^TB_c^T \end{bmatrix} = \begin{bmatrix} V_1 & 0 \\ 0 & B_cV_2B_c^T \end{bmatrix}$ by analogy with (5), the following quality measure is established:

$$J(T_{zw}, Y) = trace\left(\tilde{C}_0Y\tilde{C}_0^T\right) = trace\left(Y\tilde{R}\right) \tag{7}$$

which is a measure consisting of the mixed H_2/H_∞ norm, according to the aforementioned property of Y (6). As a result, the solution of the Riccati (6) provides the upper bound for the H_2 norm criterion subject to the H_∞ norm constraints. According to [35], [36] (A_c, B_c, C_c, Y) solve an additional minimization problem. Therefore, there are non-negative definite matrices Q, P, $\hat{Q}$ such that the (8) equalities hold:

$$\begin{aligned} A_c &= A - Q\bar{\Sigma} - \Sigma PS + \gamma^{-2}QR_{1\infty}; \\ B_c &= QC_2^TV_2^{-1}; \\ C_c &= -R_2^{-1}B_2^TPS, \end{aligned} \tag{8}$$

while

$$Y = \begin{bmatrix} Q + \hat{Q} & \hat{Q} \\ \hat{Q} & \hat{Q} \end{bmatrix} \tag{9}$$

$$0 = AQ + QA^T + V_1 + \gamma^{-2}QR_{1\infty}Q - Q\bar{\Sigma}Q \quad (10)$$

$$0 = \left(A + \gamma^{-2}[Q + \hat{Q}]R_{1\infty}\right)^T P + P\left(A + \gamma^{-2}[Q + \hat{Q}]R_{1\infty}\right) + R_1 - S^T P\Sigma PS \quad (11)$$

$$0 = (A - \Sigma PS + \gamma^{-2}QR_{1\infty})\hat{Q} + \hat{Q}(A - \Sigma PS + \gamma^{-2}QR_{1\infty})^T + \gamma^{-2}\hat{Q}(R_{1\infty} + \beta^2 S^T P\Sigma PS)\hat{Q} + Q\bar{\Sigma}Q \quad (12)$$

where $\Sigma = B_2R_2^{-1}B_2^T$, $\bar{\Sigma} = C_2^TV_2^{-1}C_2$, $S = \left(I_n + \beta^2\gamma^{-2}\hat{Q}P\right)^{-1}$, $\beta > 0$, and $R_{2\infty} = \beta^2R_2$. In addition, the auxiliary cost for the system can be represented by the subsequent (13),

$$J(T_{zw}, Y) = trace\left([Q + \hat{Q}]R_1 + \hat{Q}S^TP\Sigma PS\right) \quad (13)$$

where Q, P, and $\hat{Q}$ are solutions of modified Riccati (10)-(12). Consequently, the mixed H_2/H_∞ control problem can be construed as referring to optimal quadratic quality, provided robust stability. In the instant case, the upper bound for $\|T_{z_0w}\|_2$ is minimized under the condition $\|T_{z_1w}\|_\infty < \gamma$, and the boundary is commonly called the mixed H_2/H_∞ norm. The mixed H_2/H_∞ optimization algorithm is presented in the flowchart as shown in Figure 2. The concept of the algorithm assumes that the problem is approximated by the H_2 control theory for sufficiently large γ, what allows to obtain a reliable initial value of the solution. The parameter γ is successively reduced until the required value is reached, or further reduction becomes impossible. The convergence of the algorithm is determined by the number ε.

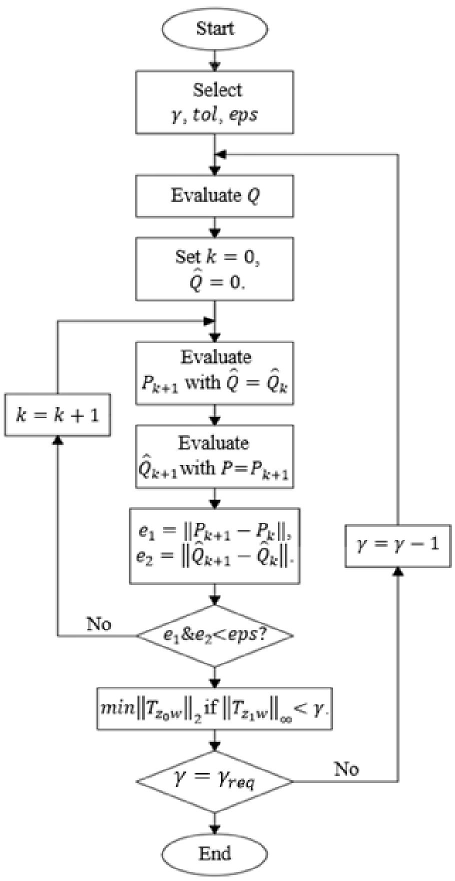

Figure 2. Flowchart of the mixed H_2/H_∞ optimization algorithm

The synthesis of the mixed H_2/H_∞ controller investigated in this paper is applicable to the problem of aircraft control. Two crucial control variables of an aircraft, namely engine thrust force T and angle of attack α, are contingent upon the deflection of throttle and elevator, respectively. The equations of flight dynamics for an aircraft in the vertical dimension, influenced by wind disruption in projection on the coordinate axes, are defined by a system of nonlinear differential equations [31], [38]:

$$\begin{cases} m\dot{V} = Tcos\alpha - D - mgsin\theta - m(\dot{w}_X cos\theta + \dot{w}_Y sin\theta), \\ mV\dot{\theta} = Tsin\alpha + L - mgcos\theta + m(\dot{w}_X sin\theta - \dot{w}_Y cos\theta), \\ J_z\dot{\omega}_z = M_z, \\ \dot{\vartheta} = \omega_z. \end{cases} \quad (14)$$

M is aircraft weight, J_z is aircraft moment of inertia about the transverse axis z, T is engine thrust force, M_z is moment of forces about the z axis, $\vartheta = \theta_{\text{в}} + \alpha$ is pitch angle, ω_z is angular velocity about the z axis, $\dot{w}_X$, $\dot{w}_Y$ is derivative of horizontal and vertical components of wind speed. The mentioned equations are valid in the supposition, that the direction of engine thrust force coincides with the axis of the aircraft, aircraft weight remains constant, the Earth is flat, and wind flow is stationary. The effect of the earth's rotation is neglected. The differential equation for the height of the center of mass h, and the incremental equation modeling the engine dynamics are formulated as (15) and (16),

$$\dot{h} = Vsin\theta + W_h \quad (15)$$

$$\Delta\dot{T} = \frac{1}{T_{\text{дв}}}\left(-\Delta T + K_{\text{дв}}\Delta\delta_t\right) \quad (16)$$

where δ_t throttle deflection from the target value. The elevator deflection δ_e is determined by taking into account the flight contour of the aircraft in its short-term periodic motion, can be summarized as following equation:

$$\delta_e = K_{\omega_z}\Delta\omega_z + K_\vartheta\Delta\vartheta + K_{\text{cy}}\Delta\vartheta_{\text{cy}},$$

where K_{ω_z}, K_ϑ и K_{cy} numerical coefficients, $\Delta\vartheta_{\text{cy}}$ control generated with the assistance of a robust controller.

A significant simplification of the aircraft mathematical model is its linearization. Let linearize the non-linear aircraft model for system of differential (14) determined by taking into consideration (15), (16). As a result, the non-linear aircraft model is transformed into a system of linear differential equations in increments. The matrix representation of linear system takes the form (1), where key vectors: $x = (\Delta V, \Delta\theta, \Delta\omega_z, \Delta\vartheta, \Delta h, \Delta T)^T$ represents the state, $w = (w_Y, \dot{w}_X, \dot{w}_Y)^T$-wind disturbance, $u = \left(\Delta\vartheta_{\text{cy}}, \Delta\delta_t\right)^T$-control [31], [36].

3. RESULTS AND ANALYSIS

This research is devoted to the analysis of a particular aircraft glide path trajectory, characterized by a linear trajectory with a defined flight path angle θ_{gl} ($\theta_{gl} = 2.7$ degrees) in height and range coordinates [31], [36]. The main purpose of synthesized system is to maintain a consistent airspeed $V_0 = 71.375$ m/s and a predetermined height $h = 400$ m under the influence of wind disturbances, when moving on a glide path. The model is presented in [31]. Studies have found that the output signal energy is minimized when a stochastic perturbation model in the form of white noise is served as an input in H_2 theory. On the other hand, the perturbation model is not defined, but its power is restricted in H_∞ theory. However, H_∞ theory provides robust control that is appropriate for systems with disturbances having significant power over an arbitrarily small frequency band. In contrast, H_2 theory permits obtaining control for systems with uniform spectral density of disturbances. Therefore, the H_2 controller is well applicable for noise processing, nevertheless, a potential weak point lies in providing robustness and tracking performance. The H_∞ controller offers a notable advantage in terms of achieving a high level of system robustness. However, it exhibits relative limitations when it comes to effectively handling noise interference. As a result, this paper contains a synthesis of robust controllers mainly based mainly on a mixed H_2/H_∞ approach, which provides an estimate of all the above-mentioned requirements.

A comparative analysis was conducted to evaluate the transient response characteristics of closed-loop systems employing the aforementioned H_2, H_∞ [31], and H_2/H_∞ controllers. In the process of simulation an identical input signal was fed to each closed-loop system, imitating the atmospheric disturbance w caused by wind that affected the aircraft's motion in the area characterized by microburst-type wind conditions. Figure 3 [31] illustrates the graphical representation of the vertical component w_y and horizontal component w_x of the wind field in relation to the position of the vortex center within the microburst airflow pattern.

Figures 4 and 5 illustrate the deviation graphs of altitude Δh and speed ΔV from their nominal values for H_2, H_∞ and mixed H_2/H_∞ controllers, as shown in Tables 1 and 2. An analysis of deviation graphs reveals that the mixed H_2/H_∞ controller provides less deviation of flight altitude h and speed V than the H_2 controller, but greater deviation than the H∞ controller. However, a comparison of control signals as shown in Figure 6 and Table 3 demonstrates that the H∞ controller provides a greater deviation than the H_2 controller. In summary: the H∞ controller requires heavy engine loads, whereas the H_2 controller requires less loads, but provides slightly lower quality. As a result, if heavy engine loads are not acceptable, implementing a mixed H_2/H∞ controller would be appropriate.

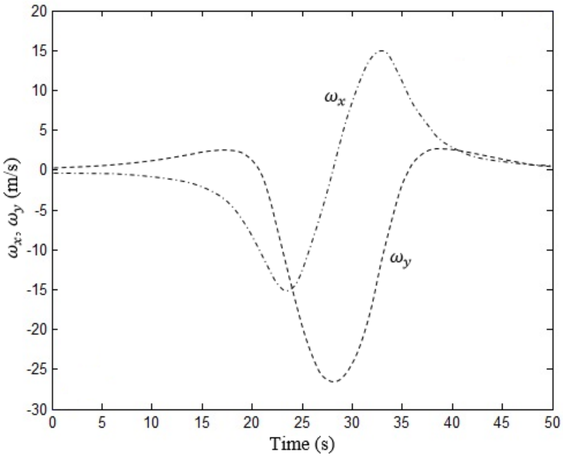

Figure 3. Vertical component ω_y and horizontal component ω_x of the wind field

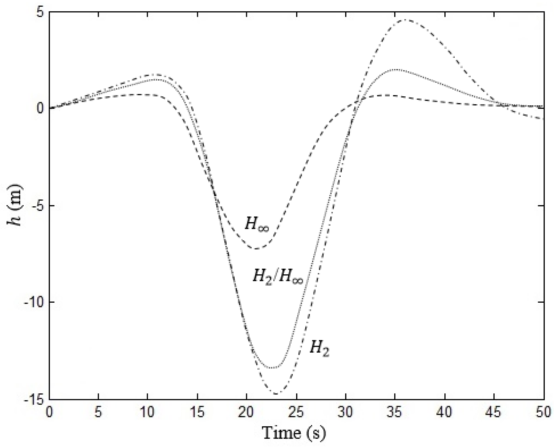

Figure 4. Flight altitude h deviation in cases of H_2, H_∞ and mixed H_2/H_∞ controllers using

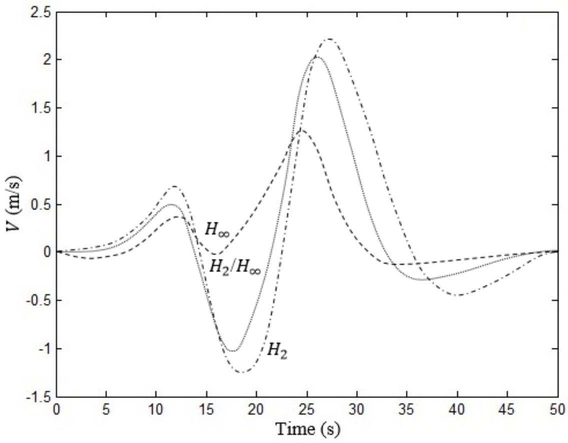

Figure 5. Speed V deviation in cases of H_2, H_∞ and mixed H_2/H_∞ controllers using

Table 1. Flight altitude deviation from the nominal value under the action of wind disturbances

Controller type	Flight altitude h deviation (m)		
	h_{min}	h_{max}	$h_{max}-h_{min}$
H_2	-14.375	4.38	18.75
H_∞	-7	0.7	7.7
H_2/H_∞	-13.125	1.875	15

Table 2. Flight speed deviation from the nominal value under the action of wind disturbances

Controller type	Flight speed V deviation (m)		
	V_{min}	V_{max}	$V_{max}-V_{min}$
H_2	-1.25	2.24	3.49
H_∞	-0.125	1.25	1.375
H_2/H_∞	-1	2	3

Consequently, a mixed H_2/H∞ controller can be obtained by manipulating the parameter γ and the weighting matrices, possessing almost equivalent qualities of H_2 or H∞ control depending on the conditions of a specific task. It is worth emphasizing that the primary cause of accidents during aircraft landings consist in a sharp loss of aircraft altitude in conditions of microburst wind action. From this perspective, the results demonstrate the technical feasibility of the proposed mixed H_2/H∞ optimal controller for solving such problems. Despite the significantly complicated algorithm of calculation, manipulating the level γ and the weighting coefficients provides an opportunity to obtain access to a wide range of transient processes, each of which is capable of exhibiting high efficiency in certain circumstances, as opposed to optimization by a single criterion. This article further advances the ongoing exploration of devising and investigating effective techniques for synthesizing robust controllers to facilitate aircraft flight control during the landing phase, specifically focusing on the glide path mode. These efforts are conducted in the face of uncertainties arising from extrinsic and intrinsic disturbances, building upon the foundation established in the previous study [31].

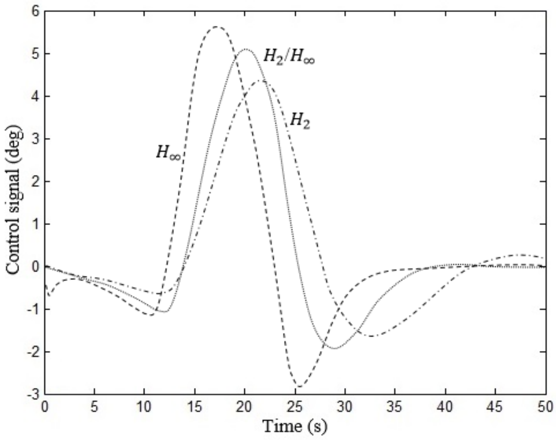

Figure 6. Control signal's reaction to the assigned wind disturbance

Table 3. Control signals deviation from the nominal value under the action of wind disturbances

Controller type	Control signal δ deviation (degree)		
	δ_{min}	δ_{max}	$\delta_{max} - \delta_{min}$
H_2	-1.7	4.25	5.95
H_∞	-2.8	5.5	8.3
H_2/H_∞	-1.95	5	6.95

4. CONCLUSION

The landing phase of aircraft flight embodies the most dangerous flight stage because of the high risk of an accident. Given the prevalence of substantial external disturbances and uncertainties during this particular phase of flight, it becomes imperative to employ robust synthesis methods such as H_2 and H_∞ techniques. These approaches offer a promising foundation for effectively addressing and resolving the challenges at hand. The H_2 controller has the capability of handling and minimizing noise but, on the other side, plays a weak role in ensuring robustness and tracking performance. The H_∞ controller contributes to the implementation of a high-quality robust system, but is not applicable in noise processing in comparison. Consequently, this research emphasizes an important aspect of robust controller synthesis by focusing on the application of a mixed H_2/H_∞ method that fully complies with the above-mentioned requirements. A mixed H_2/H_∞ controller of the required quality, functioning similarly to H_∞ or mostly H_2 depending on the conditions, can be developed by applying the technique of manipulating the parameters of γ and the weighting matrices. The proposed robust systems exhibit a broad spectrum of applications within the realm of moving object control, encompassing a wide array of technological challenges that extend beyond the confines of aircraft flight control. Further research is planned to perform directed towards the development of robust H_2, H_∞ and mixed H_2/H_∞ control in relation to other objects.

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