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Diffusion recursive least squares algorithm based on triangular decomposition

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ABSTRACT

In this paper, diffusion strategies used by QR-decomposition based on recursive least squares algorithm (DQR-RLS) and the sign version of DQR-RLS algorithm (DQR-sRLS) are introduced for distributed networks. In terms of the QR-decomposition method and Cholesky factorization, a modified Kalman vector is given adaptively with the help of unitary rotation that can decrease the complexity from inverse autocorrelation matrix to vector. According to the diffusion strategies, combine-then-adapt (CTA) and adapt-then-combine (ATC) based on DQR-RLS and DQR-sRLS algorithms are proposed with the combination and adaptation steps. To minimize the cost function, diffused versions of CTA-DQR-RLS, ATC-DQR-RLS, CTA-DQR-sRLS and ATC-DiQR-sRLS algorithms are compared. Simulation results depict that the proposed DQR-RLS-based and DQR-sRLS-based algorithms can clearly achieve the better performance than the standard combine-then-adapt-diffusion RLS (CTA-DRLS) and ATC-DRLS mechanisms.

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1. INTRODUCTION

Recently, the distributed network with the multiple agents has interested over the wireless sensor networks [1], spectrum estimation [2], clustering [3] and intrusion detection system [4]. Hence, the diffusion method draws much attention in terms of stable and fast convergence [5]. Recursive least squares (RLS) algorithm has been widely fascinated in many fields of engineering, e.g., adaptive control and signal processing for communications. In [6], an antenna arrays calibration has been designed with RLS for modulated signal using the quadrature amplitude modulation. In practice, RLS has been applied in the power communication technology for the current measurement in the islanding detection in microgrid system [7]. For the wireless communications technique for frequency domain equalizer, an affine projection algorithm (APA) with adaptive step-size algorithm [8] has been proposed.

According to the diffusion strategies, there are many methods of diffusion adaptation presented in the area of multi-agent networks. The sign version of proportionate affine projection algorithm has been demonstrated in [9]. Experimental results for distributed sparse estimation have shown that the impulsive interference could reduce significantly. In [10], the distributed adaptation over graph signals has been developed to process the effective strategies for the streaming data. RLS-based algorithm using inverse square-root mechanism over distributed network has been presented in [11]. Over the multi-agent networks, the authors have proposed the coupled multi-agent based on stochastic optimization over distributed network [12] and sensor network [13]. Rastegarnia [14] have derived the reduced-communication based on diffusion RLS algorithm for sparse system in conjunction with the bias-compensated scenario compared to the unbiased noise. For the multi-task diffusion strategies, diffusion APA in [15] has been applied. Simulation results show that the magnitude of proportional regularization is able to compensate the degradational signal.

The According to nonlinear cooperative adaptation, the diffusion spline adaptive filterings in [16] have shown that they can perform well in comparison with the non-cooperative network. Sitjongsataporn [17] a class of nonlinear diffusion adaptive filtering has been furnished in the framework of orthogonal gradient-based algorithm. While, the combined diffusion affine algorithm has been presented over the distributed networks [18], Merched [19], [20] has derived with the diffusion adaptation to fuse data based on least squares mechanism against the colored inputs in the single-task and multi-task scenes. Gao *et al.* [21] transient behavior of a multi-task diffusion on RLS has been investigated over distributed network. Futhermore, the convergence of diffusion RLS (DRLS) has been examined against the cyclostationary colored inputs in [22]. You *et al.* [23], the co-operative DRLS has been estimated in terms of Kalman filter in the mobile sensor network.

The contribution of this paper is to simplify the computation of inverse autocorrelation matrix using the QR-decomposition and Cholesky factorization and then to orchestrate a sign RLS algorithm based on the Kalman vector with the concept of diffusion solution for adaptive network. Methodology in this work is to employ the Cholesky factorization and QR-decomposition with a unitary rotation to calculate the Kalman vector using in the diffusion strategies of RLS algorithm. The diffusion strategy is described in section 2. The modified Kalman vector based on QR-decomposition is derived in section 3. The combination with adaptation steps of diffusion method is considered in the form of combine-then-adapt (CTA) scheme, while adaptation with combination steps are related with adapt-then-combine (ATC) strategy. The QR-decomposition-based RLS (QR-RLS) algorithm and modified Kalman vector are described shortly in section 4. The concept of QR-RLS algorithm using Cholesky factorization has been presented in forms of stability of conventional RLS algorithm. The diffusion adaptation with the CTA concept is proposed in section 4.1, followed by ATC idea in section 4.2. Experimental results is demonstrated in section 5 and conclusion is described in section 6, respectively.

2. DIFFUSION ADAPTATION STRATEGY

Chen and Sayed [24] the strategy on diffusion adaptation is how the multi-node can diffuse information over the distributed network. It is found that the stochastic gradient noise can be diminished through the learning process. According to the model of standard network in [6], [7], we assume that $\mathbb{L}$ nodes in a network connected with $\Lambda \in \mathbb{R}^{\mathbb{L}\times\mathbb{L}}$. $\Lambda_{kl} \geq 0$ if nodes k and l are linked within a network, otherwise it is zero. To define a network consists of N nodes, the symbol N_k denotes the number of neighbors around node k. Additionally, the coefficient $\{\Lambda_{kl}\}$ is the non-negative scalar shown as the following conditions (1),

$$\sum_{k=1}^{\mathbb{L}} \Lambda_{kl} = 1 \text{ , } \Lambda_{kl} = 0 \text{ , if } l \notin N_k \text{ and for } k = 1,2,\dots,\mathbb{L} \tag{1}$$

where matrix Λ is composed of arrival $\{\Lambda_{kl}\}$ that the l^{th} row of Λ is in term of $\{\Lambda_{kl}, k = 1,2,\dots,\mathbb{L}\}$. Thus, the desired signal $\tilde{d}_{k,n}$ at node k and symbol n is given as (2),

$$d_{k,n} = x_{k,n}{}^T \widetilde{w}_{op} + \varsigma_{k,n} \tag{2}$$

where $x_{k,n}$ is an input vector, $\widetilde{w}_{op}$ denotes an optimum linear weight. A noise $\varsigma_{k,n} \sim N(0,\sigma^2)$ is in form of Gaussian distribution, where σ^2 is a variance of the Gaussian distribution.

3. MODIFIED KALMAN VECTOR BASED ON QR-DECOMPOSITION

According to the conventional RLS algorithm can be expressed as (3),

$$w_{k,n} = w_{k,n-1} + \mathcal{R}^{-1}{}_{k,n} x_{k,n} e_{k,n} = w_{k,n-1} + \kappa_{k,n} e_{k,n} \tag{3}$$

where $\kappa_{k,n}$ is Kalman vector, $x_{k,n}$ is the input vector and $e_{k,n}$ is *a priori* estimated error. $\mathcal{R}^{-1}{}_{k,n}$ denotes an inverse autocorrelation matrix as (4),

$$\mathcal{R}^{-1}{}_{k,n} = \lambda^{-1}\mathcal{R}^{-1}{}_{k,n-1} - \lambda^{-2}\mathcal{R}^{-1}{}_{k,n-1} \cdot \boldsymbol{x}_{k,n} \cdot g_n{}^{-1} \boldsymbol{x}_{k,n}{}^T \mathcal{R}^{-1}{}_{k,n-1} \tag{4}$$

where λ is a forgetting-factor, where $0 \ll \lambda \leq 1$ [11]and $g_{k,n}$ is a scalar that can be satisfied as (5),

$$g_{k,n} = 1 + \lambda^{-1} \boldsymbol{x}_{k,n}{}^T \mathcal{R}^{-1}{}_{k,n} \boldsymbol{y}_{k,n} \tag{5}$$

According to the Cholesky factorization, the matrix $\mathcal{C}_n$ is performed by 4-matrix terms from the right-side of (4) as (6) [25], [26].

$$\mathcal{C}_{k,n} = \mathcal{M}_{k,n} \cdot \mathcal{M}_{k,n}{}^T = \begin{bmatrix} 1 & \lambda^{-\frac{1}{2}} \boldsymbol{x}_{k,n}{}^T \mathcal{R}^{-\frac{1}{2}}{}_{k,n-1} \\ 0 & \lambda^{-\frac{1}{2}} \mathcal{R}^{-\frac{1}{2}}{}_{k,n-1} \end{bmatrix} \cdot \begin{bmatrix} 1 & \lambda^{-\frac{1}{2}} \boldsymbol{x}_{k,n}{}^T \mathcal{R}^{-\frac{1}{2}}{}_{k,n-1} \\ 0 & \lambda^{-\frac{1}{2}} \mathcal{R}^{-\frac{1}{2}}{}_{k,n-1} \end{bmatrix}^T \quad (6)$$

where $\mathcal{M}_{k,n}$ is a block matrix. By using the QR-decomposition, the pre-array $\mathcal{M}_{k,n}$ to post-array is given adaptively with a unitary rotation as (7),

$$\mathcal{M}_{k,n}\theta = \begin{bmatrix} 1 & \lambda^{-\frac{1}{2}} \boldsymbol{x}_{k,n}{}^T \mathcal{R}^{-\frac{1}{2}}{}_{k,n-1} \\ 0 & \lambda^{-\frac{1}{2}} \mathcal{R}^{-\frac{1}{2}}{}_{k,n-1} \end{bmatrix} \theta = \begin{bmatrix} g^{\frac{1}{2}}{}_{k,n} & 0^T \\ g^{\frac{1}{2}}{}_{k,n} \mathcal{R}^{-1}{}_{k,n} \boldsymbol{x}_{k,n} & \mathcal{R}^{-\frac{1}{2}}{}_{k,n} \end{bmatrix}^T \quad (7)$$

where θ denotes a unitary rotation. Consequently, a modified Kalman vector $\kappa_{k,n}$ is derived from the post-array in the right-side of (7) as (8).

$$\kappa_{k,n} = \left(g^{\frac{1}{2}}{}_{k,n} \mathcal{R}^{-1}{}_{k,n} x_{k,n} \right) \cdot g^{-\frac{1}{2}}{}_{k,n} \quad (8)$$

Noticed that $\kappa_{k,n}$ as a product of inverse autocorrelation matrix $\mathcal{R}^{-1}{}_{k,n}$ and input vector $x_{k,n}$ shown in (8) can simplify the computation from inverse matrix to vector. As can be seen, it can apply directly to RLS algorithm in (3).

4. THE PROPOSED DIFFUSION MODIFIED QR-BASED SIGN RLS ALGORITHM

Based on QR-decomposition, a modified Kalman vector with the diffusion strategies is described. The proposed diffusion QR-based sign RLS (DQR-sRLS) algorithm is furnished by Kalman vector [7]. Diffusion strategies consisting of CTA and ATC are adapted in the proposed DQR-sRLS mechanisms.

4.1. The proposed CTA-DQR-sRLS algorithm

We define the CTA diffusion strategy on sign QR-RLS as CTA-DQR-sRLS algorithm. According to a network N, each node k has the connected nodes, where $k = 1, 2, \dots, N$ and deploy as an adaptive filter. To minimize the error vector that lies on the estimated weight subject to the weighted-norm as (9),

$$\mathbb{J}(\psi_{k,n}) = \sum_{j \in N_k} \alpha_{j,k} \|\psi_{j,n}\| \text{ subject to } \|\widehat{\boldsymbol{w}}_{k,n-1} - \psi_{k,n}\|^2 \le \delta \quad (9)$$

where δ is a very small constant and $\alpha_{j,k}$ is a weight connected other node. $\psi_{k,n}$ is a combination of weight vector given as (10),

$$\psi_{j,n} = \sum_{j \in N_k} \alpha_{j,k} \widehat{w}_{j,n} \quad (10)$$

Hence, a tap-weight vector $\widehat{w}_{k,n}$ of CTA-DQR-RLS algorithm can be decribed by (11) and (12),

$$\widehat{w}_{k,n} = \sum_{j \in N_k} \alpha_{j,k} \widehat{w}_{j,n} + \kappa_{k,n} \hat{e}_{k,n} \quad (11)$$

$$\hat{e}_{k,n} = d_{k,n} - \widehat{w}_{k,n}{}^T - x_{k,n} \quad (12)$$

where $\kappa_{k,n}$ is defined in (8) and $\alpha_{j,k}$ is the combination of coefficient, where $\sum_{j=1}^{N} \alpha_{j,k} = 1$ and $\alpha_{j,k} \ge 0$ at symbol n and node k.

Accordingly, a tap-weight vector $\grave{w}_{k,n}$ of CTA-DQR-sRLS algorithm is calculated by (13),

$$\grave{w}_{k,n} = \sum_{j \in N_k} \alpha_{j,k} \grave{w}_{j,n} + \kappa_{k,n} sgn(\grave{e}_{k,n}) \quad (13)$$

$$\grave{e}_{k,n} = d_{k,n} - \grave{w}^T{}_{k,n-1} x_{k,n} \quad (14)$$

where $sgn(\cdot)$ is a sign operator. CTA process composes of combination and adaptation steps as shown in (13). The vector $\sum_{j \in N_k} \alpha_{j,k} \grave{w}_{j,n}$ is modified to combine the weight vector that the information can diffuse over the network. Hence, the result of proposed $\grave{w}_{k,n}$ is given in the adaptation step. The schematic diagram of proposed CTA-DQR-sRLS algorithm is shown in Figure 1(a).

4.2. The proposed ATC-DQR-sRLS algorithm

In this section, we introduce ATC diffusion strategy on sign QR-RLS called ATC-DQR-sRLS. Following the minimized error on an estimated tap-weight subject to the constraint of weighted-norm, it arrives at (15) and (16),

$$\mathcal{J}(\varpi_{k,n}) = \sum_{j\in N_k} \alpha_{j,k}\|\varpi_{j,n}\| \text{ subject to } \|\breve{w}_{k,n-1} - \varpi_{k,n}\|^2 \leq \delta \quad (15)$$

$$\xi_{k,n} = d_{k,n} - \breve{w}^T{}_{k,n-1}x_{k,n} \quad (16)$$

where $\alpha_{j,k}$ denotes a weighting parameter around neighbored node and $\varpi_{j,n}$ is an adaptation weight vector. Considering the diffusion framework [3], the estimated error $\xi_{k,n}$ for ATC strategy is calculated at each node.

Hence, the estimated tap-weight update of ATC-DQR-RLS algorithm at symbol n and node k can be arranged by (17) and (18).

$$\varpi_{k,n} = \varpi_{k,n-1} + \kappa_{k,n}\xi_{k,n} \quad (17)$$

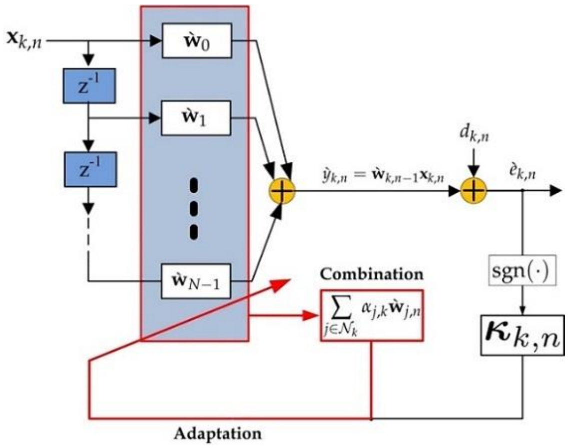
$$\breve{w}_{k,n} = \sum_{j\in N_k} \alpha_{j,k}\varpi_{j,n} \quad (18)$$

Therefore, the estimate update of ATC-DQR-sRLS algorithm can be expressed as (19) and (20).

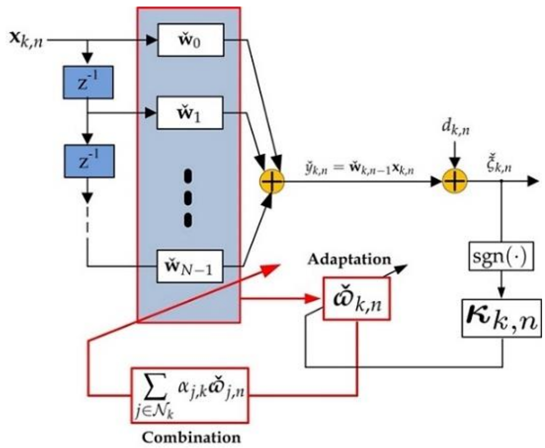
$$\breve{\varpi}_{k,n} = \breve{w}_{k,n-1} + \kappa_{k,n} sgn(\xi_{k,n}) \quad (19)$$

$$\breve{w}_{k,n} = \sum_{j\in N_k} \alpha_{j,k}\breve{\varpi}_{j,n} \quad (20)$$

ATC process composes of an adaptation step in (17) and combination step in (18). It is noticed that the tap-weighted vector can be performed in the combination step. The schematic diagram of proposed ATC- DQR-sRLS algorithm presents in Figure 1(b).



Proposed CTA-DQR-sRLS algorithm
(a)



Proposed ATC-DQR-sRLS algorithm
(b)

Figure 1. Sample of tap-weight vector $w_{k,n}$ of proposed DQR-sRLS algorithm updated (a) before preprocessing, $w_{k,n}$ is combined, then adapted and (b) after preprocessing, $w_{k,n}$ is adapted, then combined

5. EXPERIMENTAL RESULTS

Simulations for the proposed methods were conducted in the random process on the random channel over 200 Monte Carlo trials with AR(2,2) and an unit variance of white Gaussian noise random sequence is given through AR(3) as (21) and (22),

$$H_1(z) = \frac{1-0.5z^{-1}+0.81z^{-2}}{1-0.59z^{-1}+0.4z^{-2}}, \tag{21}$$

$$H_2(z) = \frac{1}{1-1.5z^{-1}+z^{-2}-0.25z^{-3}}, \tag{22}$$

and the input signals are the colored signal generated as [16]

$$x_{k,n} = \varepsilon x_{k,n-1} + \sqrt{1-\varepsilon^2 \zeta_{k,n}}, \tag{23}$$

where $\varepsilon = [0, 0.8]$ and $\zeta_{k,n}$ is a white Gaussian noise with variance and zero mean.

Illustration of network topology presents in Figure 2 with $N_k = 9$ connected nodes. Signal to noise ratio (SNR) involving with the noise variance at k node is $SNR_k = \{30, 40, 20, 35, 40, 50, 25, 20, 35\}$ dB. There are initial parameters for proposed ATC-DQR-RLS, ATC-DQR-sRLS, CTA-DQR-RLS and CTA-DQR-sRLS algorithm as: $\varepsilon = 0.2, \widehat{w}(0) = \grave{w}(0) = \breve{w}_{k,n} = \tilde{w}_{k,n} = [1\ 0 \cdots 0]^T, \widehat{\mathbb{R}}^{-1}(0) = \varrho\mathbb{I}, \varrho = 1.25 \times 10^{-3}, \mathbb{M} = 9, \lambda = 0.9995$ and for proposed ATC-DRLS and CTA-DRLS [7] as: $\mathbb{M} = 9, \lambda = 0.95, \varrho = 1.75 \times 10^{-3}$.

Figures 3 and 4 show the mean square error (MSE) trends from the proposed ATC-DQR-RLS and ATC-DQR-sRLS algorithms compared with the ATC-DRLS [7] using zero-mean uniform white Gaussian noise processes through AR(2,2) and AR(3) in (21) and (22), respectively. It is found that the MSE curve of the proposed ATC-DQR-RLS closes to ATC-DQR-sRLS algorithm, while the MSE curve of ATC-DRLS method is higher than the proposed algorithms. Figures 5 and 6 depict the comparison of MSE trajectories, those belong to the proposed CTA-DQR-RLS, CTA-DQR-sRLS and the standard CTA-DRLS algorithm [7]. It clearly shows that the proposed DQR-RLS-based and DQR-sRLS-based algorithm achieve better performance by using a modified Kalman vector based on QR-decomposition compared with the standard DRLS-based algorithm.

The complexity of CTA-DRLS, CTA-DQR-RLS and CTA-DQR-sRLS algorithms take into account the choice of diffusion strategy with the combination and adaptation steps. Table 1 shows the computational complexity of the proposed CTA-based algorithms, which are not significantly different. However in Table 2, the average MSEs of the proposed ATC-DQR-sRLS and CTA-DQR-sRLS algorithms are lower than the conventional ATC-DRLS and CTA-DRLS algorithms.

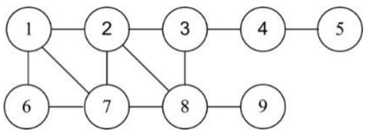


Figure 2. Illustration of network topology with $N_k = 9$

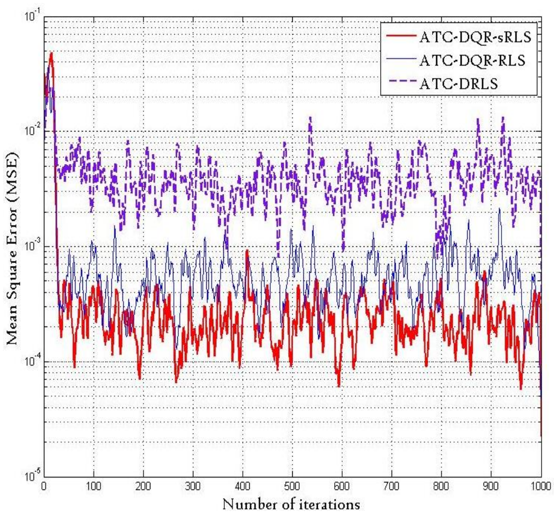


Figure 3. MSE trends of the proposed ATC-DQR-RLS, ATC-DQR-sRLS algorithms added Gaussian noise through AR(2,2) compared with ATC-DRLS

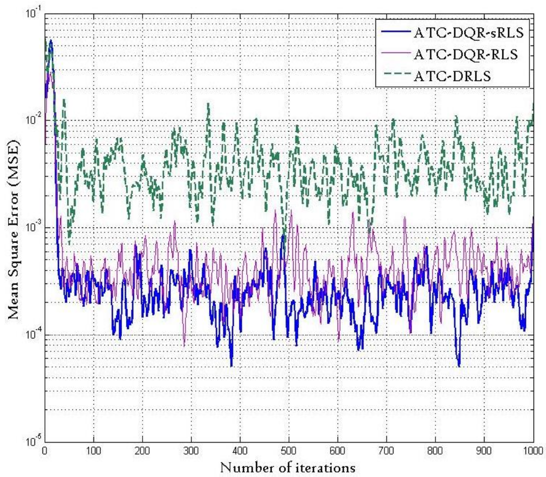


Figure 4. MSE curves of the proposed ATC-DQR-RLS, ATC-DQR-sRLS algorithms in comparison with ATC-DRLS added Gaussian noise through AR(3)

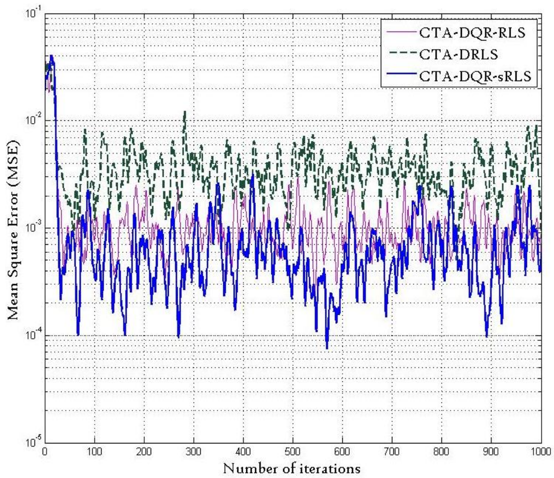


Figure 5. MSE curves of the proposed ATC-DQR-RLS, ATC-DQR-sRLS algorithms compared with ATC-DRLS using Gaussian noise through AR(2,2)

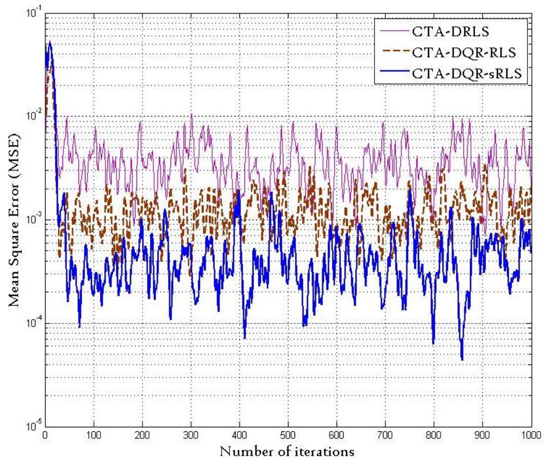


Figure 6. MSE curves of the proposed ATC-DQR-RLS, ATC-DQR-sRLS algorithms compared with ATC-DRLS added white Gaussian noise through AR(3)

Table 1. Computational complexity of the proposed DQR-RLS and DQR-sRLS-based algorithms, where $\mathbb{M}$ is number of tap-weight coefficient

Algorithm	Number of multiplication	Number of addition
CTA-DRLS [7]	$\mathbb{M}^2 + 6\mathbb{M} + 1$	$\mathbb{M}^2 + 4\mathbb{M}$
CTA-DQR-RLS	$2\mathbb{M}^2 + \mathbb{M} - 1$	$\mathbb{M}^2 + 2\mathbb{M}$
CTA-DQR-sRLS	$2\mathbb{M}^2 + \mathbb{M} + 2$	$\mathbb{M}^2 + 2\mathbb{M} + 1$
QR-RLS	$2\mathbb{M}^2 - \mathbb{M} + 3$	$\mathbb{M}^2 + 2\mathbb{M}$
RLS [7]	$2\mathbb{M}^2 + 5\mathbb{M} + 1$	$\mathbb{M}^2 + 3\mathbb{M}$

Table 2. Summary of average MSE of the proposed DQR-sRLS-based algorithms compared with the standard DRLS-based algorithm, where $\varepsilon = 0.2$

Algorithm	Average MSE	
	AR(2,2)	AR(3)
ATC-DQR-sRLS	2.229×10^{-4}	2.232×10^{-4}
ATC-DQR-RLS	6.656×10^{-4}	4.487×10^{-4}
ATC-DRLS [7]	4.503×10^{-3}	4.248×10^{-3}
CTA-DQR-sRLS	4.640×10^{-4}	4.545×10^{-4}
CTA-DQR-RLS	9.865×10^{-4}	1.292×10^{-4}
CTA-DRLS [7]	3.871×10^{-3}	4.172×10^{-3}

6. CONCLUSION

In this paper, a set of CTA and ATC strategies on DQR-RLS and DQR-sRLS has been introduced with a modified Kalman vector based on Cholesky factorization and QR-decomposition to simplify the calculation. Computational complexity of proposed algorithms is presented and compared with the conventional ones. Simulation results demonstrate that these proposed DQR-sRLS-based algorithms are able to provide better performance, in terms of achieving lower MSE with in the similar convergence rate, compared to the conventional DRLS algorithm in the system identification.

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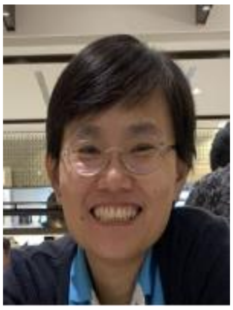

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