International Journal of Electrical and Computer Engineering (IJECE)
Vol. 13, No. 5, October 2023, pp. 5035∼5046
ISSN: 2088-8708, DOI: 10.11591/ijece.v13i5.pp5035-5046

Unknown input observer for Takagi-Sugeno implicit models with unmeasurable premise variables

Mohamed Essabre[1], Ilham Hmaiddouch[2,3], Abdellatif El Assoudi[2,3], El Hassane El Yaagoubi[2,3]
[1]Laboratory of Materials, Energy and Control Systems, Faculty of Sciences and Technologies Mohammedia, Hassan II University of Casablanca, Morocco
[2]Laboratory of High Energy Physics and Condensed Matter, Faculty of Science, Hassan II University of Casablanca, Casablanca, Morocco
[3]ECPI, Department of Electrical Engineering, Higher National School of Electricity and Mechanics, Hassan II University of Casablanca, Casablanca, Morocco

Article Info

Article history:

Received Dec 8, 2022
Revised Jan 20, 2023
Accepted Feb 4, 2023

Keywords:

Implicit system
Linear matrix inequalitie technique
Lyapunov method
Observer design
Takagi-Sugeno model
Unknown input

ABSTRACT

Recent years have seen a great deal of interest in implicit nonlinear systems, which are used in many different engineering applications. This study is dedicated to presenting a new method of fuzzy unknown inputs observer design to estimate simultaneously both non-measurable states and unknown inputs of continuous-time nonlinear implicit systems defined by Takagi-Sugeno (T-S) models with unmeasurable premise variables. The suggested observer is based on the singular value decomposition approach and rewritten the continuous-time T-S implicit models into an augmented fuzzy system, which gathers the unknown inputs and the state vector. The exponential convergence condition of the observer is established by using the Lyapunov theory and linear matrix inequalities are solved to determine the gains of the observer. Finally, the effectiveness of the suggested method is then assessed using a numerical application. It demonstrates that the estimated variables and the unknown input converge to the real variables accurately and quickly (less than 0.5 s).

Corresponding Author:

Mohamed Essabre
Laboratory of Materials, Energy and Control Systems, Faculty of Sciences and Technologies Mohammedia, Hassan II University of Casablanca
Casablanca, Morocco
Email: mohamed.essabre@fstm.ac.ma

1. INTRODUCTION

It is well-known that equations in descriptor form offer significant flexibility in modelling nonlinear systems. Indeed, mobile robots, electrical systems, biological processes and several industrial applications are described by a system of differential and algebraic equations called implicit, singular or descriptor models, see for example [1], [2] and references therein for some applications. Furthermore, many physical phenomena, like impulses and hysteresis which are important in circuit theory, cannot be handled properly in ordinary models. Descriptor representation provides an appropriate way to deal with such problems.

This work focuses on the development of algorithms to estimate the state and unknown inputs (UIs) for implicit nonlinear models through the application of Takagi-Sugeno (T-S) formalism. In fact, many processes such as nonlinear auto regressive moving average model with eXogenous inputs (NARMAX), Hammerstein, Wiener, Hammerstein-Wiener are described by nonlinear systems [3]–[7]. Consequently, the resulting models are very complex and the analysis and the controller/observer synthesis based on the model becomes more dificult to achieve. The standard T-S approach [8], [9] is a powerful and practical modeling tool for these complex systems. It allows to obtain a model taking into account the non-linearities of the system and offering a simple and easily exploitable structure from a mathematical point of view which facilitates observer synthesis and the control design of nonlinear systems, see for example [10]–[14]. Additionally, we distinguish between a T-S fuzzy model with measurable premise variables (MPVs) and a T-S fuzzy model with unmeasurable premise variables (UPVs) depending on the nature of the variables involved in the activation functions called premise variables. Note that in [15], [16] fuzzy implicit model is defined by expanding the standard T-S model [8].

It is well known that industrial processes are often subject to disturbances. They are called UI when they affect the process input. They can spring from measurement uncertainties, sensor, actuator faults, or noises due to the environment of the process. Given their harmful effects on the normal operation of the process, their real-time estimation can be used for the synthesis of a control algorithm which is capable of minimizing these effects. Due to the increasing demand for reliability and maintainability of automatic control systems, the synthesis of observers for dynamic models subjected to UIs (called unknown inputs observers (UIO)) is one of the most important areas of research during the last two decades. Based on different approaches, this theme has been the subject of numerous theoretical developments. This is especially due to its essential role in the fault diagnosis and fault tolerant control strategy development. Several research works using different approaches have been published in [17]–[20]. Many publications have been interested in the design of UIs for the case of T-S explicit models, we may refer to [21]–[26]. In the case of T-S implicit systems, several UIOs have been developed. In [27], a design of UIO is proposed for a class of nonlinear implicit systems described by T-S structure with MPV. The basic idea of the proposed approach is based on the separation between dynamic and static relations in the T-S implicit model to estimate both the system state and the UIs concurrently. Developing an UIO for T-S fuzzy systems satisfying Lipschitz conditions is the aim of the work presented in [28]. In [29] the authors used the result obtained in [27] to develop a new approach making it possible to estimate simultaneously both non-measurable states and unknown faults in the actuators and sensors for T-S implicit model with MPV.

Most of the UIOs are synthesized to estimate the state and UI in the case of T-S implicit models with MPVs. The main contribution of this research is to introduce a novel method of fuzzy unknown inputs observer (FUIO) design for a class of continuous-time Takagi-Sugeno implicit models (CTSIMs) with UPVs permitting simultaneous estimation of the unmeasurable states and UIs. Furthermore, as compared to T-S systems with MPVs, the T-S structure with UPVs can describe a broader variety of non-linear systems. Indeed, one of the most well-known strategies for converting a nonlinear model to a T-S model is the sector nonlinearity approach [30]. This transformation usually allows for the creation of a T-S model with UPVs. It should also be noted that if the output of the system is chosen as a premise variable and this output is affected by disturbances, the resulting T-S system does not exactly describe the system. T-S models with UPVs are more harder to treat than those with UPVs. For this reason, few studies are dedicated to this class of models despite of their interest. The design method in this work is based on the singular value decomposition (SVD) approach and the use of an augmented system structure. The Lyapunov theory is used to investigate the global exponential stability of the state estimation error, and the stability conditions are expressed in terms of linear matrix inequalities (LMIs). Besides, in most works, the authors proposed observers for CTSIMs in descriptor form instead of the standard TS observer. It is important to keep in mind that designing a descriptor observer is not easy because the slow dynamic (resulting from the algebraic equations) is directly estimated, which can depend on impulsive behavior and influence the observer stability. In addition to this, the main drawback of these observers lies in their implementations. However, the solution we offer in this paper, which is presented in an explicit form, allows us to avoid this problem.

The paper is organized as: in section 2, the mathematical of the considered CTSIMs subject to UI to be studied is described. Section 3 presents the suggested method and the main result of this work. More precisely, the structure of the FUIO is proposed. Besides, the stability analysis, and the convergence conditions are demonstrated. Finally, a numerical example of this theoretical result and simulation results are given to show the interests of the proposed design framework.

2. MATHEMATICAL FORMULATION OF THE CONSIDERED MODEL

Let consider nonlinear implicit systems subject to UIs given by (1):

$$\begin{cases} M\dot{\zeta} = A(\zeta)\zeta + B(\zeta)u + \Lambda(\zeta)d \\ y = C\zeta + Dd \end{cases} \quad (1)$$

where $\zeta \in \mathbb{R}^n$, $u \in \mathbb{R}^m$ and $y \in \mathbb{R}^p$ represent, respectively, the state, the input and the output vectors of the sysytem. The unknown inputs are modeled by $d \in \mathbb{R}^\sigma$. $A(\zeta) \in \mathbb{R}^{n\times n}$, $B(\zeta) \in \mathbb{R}^{n\times m}$, $\Lambda \in \mathbb{R}^{n\times\sigma}$ are nonlinear matrices functions. $C \in \mathbb{R}^{p\times n}$, $M \in \mathbb{R}^{n\times n}$, $D \in \mathbb{R}^{p\times\sigma}$ are real known constant matrices, with $rank(M) = r < n$.

By using the nonlinear sector transformation [30], the system (1) can be represented by the CTSIM:

$$\begin{cases} M\dot{\zeta} = \sum_{i=1}^{q} \rho_i(\beta)(A_i\zeta + B_iu + \Lambda_i d) \\ y = C\zeta + Dd \end{cases} \quad (2)$$

where $A_i \in \mathbb{R}^{n\times n}$, $B_i \in \mathbb{R}^{n\times m}$, $\Lambda_i \in \mathbb{R}^{n\times\sigma}$ are real known constant matrices. The activating functions $\rho_i(\beta)$ collect the contribution of all sub-systems.

$$\begin{cases} M\dot{\zeta} = A_i\zeta + B_iu + \Lambda_i d \\ y = C\zeta + Dd \end{cases} \quad (3)$$

Along this paper, we assume that the weighing functions $\rho_i(\beta)$ depend on unmeasurable premise variables (state of the system), and satisfy the following constraints:

$$0 \le \rho_i(\beta) \le 1, \quad \sum_{i=1}^{q} \rho_i(\beta) = 1 \quad (4)$$

Assume that d is a constant unknown control input every time interval, as (5):

$$\dot{d} \;=\; 0 \qquad t \in [T_1,\ T_2], \qquad \forall\, T_1, T_2 \in \mathbf{R}^+ \quad (5)$$

Let's start by defining the augmented state vector:

$$z = [\zeta^T \quad d^T]^T$$

As a result, the system (2) can be extended in the augmented form as (6):

$$\begin{cases} \Gamma\dot{z} = \sum_{i=1}^{q} \rho_i(\beta)(\widetilde{A}z + \widetilde{B}_iu) \\ y = \widetilde{C}z \end{cases} \quad (6)$$

where

$$\Gamma = \begin{pmatrix} M & 0 \\ 0 & I \end{pmatrix}; \widetilde{A}_i = \begin{pmatrix} A_i & \Lambda_i \\ 0 & 0 \end{pmatrix}; \widetilde{B}_i = \begin{pmatrix} B_i \\ 0 \end{pmatrix}; \widetilde{C} = \begin{pmatrix} C & D \end{pmatrix} \quad (7)$$

Let us make the following assumption before giving the main result [1], [31].

Assume that:

H_1) $(\Gamma, \widetilde{A}_i)$ is regular, $det(s\Gamma - \widetilde{A}_i) \neq 0\ \forall s \in \mathbb{C}$

H_2) All sub-models (3) are impulse observable and detectable.

H_3) $rank \begin{pmatrix} \Gamma \\ \widetilde{C} \end{pmatrix} = n_1 = n + \sigma$

According to hypothesis H_3), there is a non-singular matrix $\begin{pmatrix} \Psi_1 & \Psi_2 \\ \Psi_3 & \Psi_4 \end{pmatrix}$ such that:

$$\left\{ \begin{array}{ccccc} \Psi_1\Gamma & + & \Psi_2\widetilde{C} & = & I \\ \Psi_3\Gamma & + & \Psi_4\widetilde{C} & = & 0 \end{array} \right. \tag{8}$$

where $\Psi_1 \in \mathbb{R}^{n_1 \times n_1}$, $\Psi_2 \in \mathbb{R}^{n_1 \times p}$, $\Psi_3 \in \mathbb{R}^{p \times n_1}$, $\Psi_4 \in \mathbb{R}^{p \times p}$ are constant matrices that may be obtained via the singular value decomposition of $\begin{pmatrix} \Gamma \\ \widetilde{C} \end{pmatrix}$.

3. THE PROPOSED METHOD AND MAIN RESULT

According to the transformation of CTSIM (2) into the form (6), the suggested FUIO which is not in implicit form allowing simultaneous estimation of UIs and unmeasurable states of system (2) takes the following structure:

$$\begin{cases} \dot{x} = \displaystyle\sum_{i=1}^{q} \rho_i(\hat{\beta})(N_i x + L_{1i} y + L_{2i} y + G_i u) \\ \hat{z} = x + \Psi_2 y + K\Psi_4 y \end{cases} \tag{9}$$

where x represents the estimated vector, the activation functions $\rho_i(\hat{\beta})$ are dependent on the unmeasurable premise variables and $\hat{z}$ is the estimated augmented state vector. The unknown matrices N_i, L_{1i}, L_{2i}, G_i and K must be obtained in the aim of a exponential convergence of $\hat{z}$ converges to z. The matrices Ψ_2 and Ψ_4 satisfy (8).

The augmented state estimate error is defined as (10):

$$e = z - \hat{z} \tag{10}$$

by inserting (8) and (9) into (10) we obtain:

$$e = (\Psi_1 + K\Psi_3)\Gamma z - x \tag{11}$$

so, error dynamics will be (12):

$$\dot{e} = (\Psi_1 + K\Psi_3)\Gamma \dot{z} - \dot{x} \tag{12}$$

from (6) and (9), we obtain:

$$\dot{e} = \sum_{i=1}^{q} \rho_i(\beta)(\Psi_1 + K\Psi_3)(\widetilde{A}_i \xi + \widetilde{B}_i u) - \sum_{i=1}^{q} \rho_i(\hat{\beta})(N_i x + L_{1i} y + L_{2i} y + G_i u) \tag{13}$$

by substituting (11), (13) becomes:

$$\dot{e} = \sum_{i=1}^{q} \rho_i(\beta)(\Psi_1 + K\Psi_3)(\widetilde{A}_i z + \widetilde{B}_i u) + \sum_{i=1}^{q} \rho_i(\hat{\beta})(N_i e - \Phi_i z - G_i u) \tag{14}$$

where

$$\Phi_i = N_i(\Psi_1 + K\Psi_3)\Gamma + L_{1i}\widetilde{C} + L_{2i}\widetilde{C} \tag{15}$$

provided the matrices N_i, L_{1i}, L_{2i}, G_i and K satisfy:

$$\Phi_i = (\Psi_1 + K\Psi_3)\widetilde{A}_i \tag{16}$$

and

$$G_i = (\Psi_1 + K\Psi_3)\widetilde{B}_i \tag{17}$$

as a result, the system (14) becomes:

$$\dot{e} = \sum_{i=1}^{q} (\rho_i(\beta) - \rho_i(\hat{\beta}))(\Psi_1 + K\Psi_3)(\widetilde{A}_i z + \widetilde{B}_i u) + \sum_{i=1}^{q} \rho_i(\hat{\beta}) N_i e \tag{18}$$

then, from (8), (16) and (17), we have (19)

$$N_i = (\Psi_1 + K\Psi_3)\widetilde{A}_i - L_{2i}\widetilde{C} + (N_i(\Psi_2 + K\Psi_4) - L_{1i})\widetilde{C} \tag{19}$$

take

$$L_{1i} = N_i(\Psi_2 + K\Psi_4) \tag{20}$$

then

$$N_i = (\Psi_1 + K\Psi_3)\widetilde{A}_i - L_{2i}\widetilde{C} \tag{21}$$

using the fact that

$$\begin{cases} \sum_{i=1}^{q} (\rho_i(\beta) - \rho_i(\hat{\beta}))\widetilde{A}_i = \sum_{i,j=1}^{q} \rho_i(\beta)\rho_j(\hat{\beta})\Delta\widetilde{A}_{ij} \\ \sum_{i=1}^{q} (\rho_i(\beta) - \rho_i(\hat{\beta}))\widetilde{B}_i = \sum_{i,j=1}^{q} \rho_i(\beta)\rho_j(\hat{\beta})\Delta\widetilde{B}_{ij} \end{cases} \tag{22}$$

where $\Delta\widetilde{A}_{ij} = \widetilde{A}_i - \widetilde{A}_j$ and $\Delta\widetilde{B}_{ij} = \widetilde{B}_i - \widetilde{B}_j$.

Then, the (18) becomes:

$$\dot{e} = \sum_{i,j=1}^{q} \rho_i(\beta)\rho_j(\hat{\beta})((\Psi_1 + K\Psi_3)(\Delta\widetilde{A}_{ij} z + \Delta\widetilde{B}_{ij} u) + \sum_{i=1}^{q} \rho_i(\hat{\beta}) N_i e \tag{23}$$

multiplying by $\sum_{i=1}^{q} \rho_i(\beta)$, (23) can be simplified as (24):

$$\dot{e} = \sum_{i,j=1}^{q} \rho_i(\beta)\rho_j(\hat{\beta})(N_j e + \Omega_{ij} z + \Upsilon_{ij} u) \tag{24}$$

where

$$\begin{cases} \Omega_{ij} = (\Psi_1 + K\Psi_3)\Delta\widetilde{A}_{ij} \\ \Upsilon_{ij} = (\Psi_1 + K\Psi_3)\Delta\widetilde{B}_{ij} \\ i, j \in \{1, ..., q\} \end{cases} \tag{25}$$

Let $\widetilde{e} = [e^T \quad z^T]^T$, we have (26)

$$\begin{cases} \widetilde{\Gamma}\dot{\widetilde{e}} = \sum_{i,j=1}^{q} \rho_i(\beta)\rho_j(\hat{\beta})[\Theta_{ij}\widetilde{e} + \Pi_{ij} u] \\ z = R\,\widetilde{e} \end{cases} \tag{26}$$

where

$$\widetilde{\Gamma} = \begin{pmatrix} I & 0 \\ 0 & \Gamma \end{pmatrix}; \Theta_{ij} = \begin{pmatrix} N_j & \Omega_{ij} \\ 0 & \widetilde{A}_i \end{pmatrix}; \Pi_{ij} = \begin{pmatrix} \Upsilon_{ij} \\ \widetilde{B}_i \end{pmatrix}; R = \begin{pmatrix} I & 0 \end{pmatrix} \tag{27}$$

Thus, the goal is to find the observer gains N_i, L_{1i}, L_{2i}, $G_i (i = 1, \ldots, q)$ and K that will ensure the stability of (26) while attenuating the effect of the input u on z. Therefore, the following theorem can be used to express the convergence condition of (9).

Theorem (Under assumptions 1 and 2): Given $\lambda > 0$, the state estimation error between the CTSIM (2) and its FUIO (9) converges towards zero, if there are matrices $P = \begin{pmatrix} P_1 & 0 \\ 0 & P_2 \end{pmatrix}$, $Q, W_j, j = 1, \ldots, q$ and a positive scalar α, such that the following LMIs hold:

$$\widetilde{\Gamma}^T P = P\widetilde{\Gamma} \geq 0 \tag{28}$$

$$\Delta_{ij} = \begin{pmatrix} \sigma_{11} & * & * \\ \sigma_{21} & \sigma_{22} & * \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{pmatrix} < 0 \quad \forall i, j \in 1, \ldots.., q \tag{29}$$

where

$$\begin{cases} \sigma_{11} = J_j + J_j^T + I + 2\lambda P_1 \\ \sigma_{22} = \widetilde{A}_i^T P_2 + P_2 \widetilde{A}_i + I + 2\lambda \Gamma P_2 \\ \sigma_{33} = -\alpha I \\ \sigma_{21} = \Delta \widetilde{A}_{ij}^T (P_1 \Psi_1 + Q\Psi_3)^T \\ \sigma_{31} = \Delta \widetilde{B}_{ij}^T (P_1 \Psi_1 + Q\Psi_3)^T \\ \sigma_{32} = \widetilde{B}_i^T P_2 \end{cases} \tag{30}$$

with

$$J_j = P_1 \Psi_1 \widetilde{A}_j + Q\Psi_3 \widetilde{A}_j - W_j \widetilde{C} \tag{31}$$

The observer gains N_j , L_{1j} , L_{2j} , G_j and K are given by (32)

$$\begin{cases} K = P_1^{-1} Q \\ G_j = (\Psi_1 + K\Psi_3)\widetilde{B}_j \\ L_{2j} = P_1^{-1} W_j \\ N_j = (\Psi_1 + K\Psi_3)\widetilde{A}_j - L_{2j}\widetilde{C} \\ L_{1j} = N_j(\Psi_2 + K\Psi_4) \end{cases} \tag{32}$$

and the attenuation level is

$$\delta = \sqrt{\alpha} \tag{33}$$

where Ψ_1, Ψ_2, Ψ_3 and Ψ_4 satisfy (8)
proof: Consider the quadratic Lyapunov function:

$$V = \widetilde{e}^T \widetilde{\Gamma} P \widetilde{e}\,, \qquad P = P^T > 0 \tag{34}$$

with

$$\widetilde{\Gamma}^T P = P\widetilde{\Gamma} \geq 0 \tag{35}$$

and

$$P = \begin{pmatrix} P_1 & 0 \\ 0 & P_2 \end{pmatrix} \tag{36}$$

By using (26), the time derivative of V can be written as (37):

$$\dot{V} = \dot{\widetilde{e}}^T \widetilde{\Gamma}^T P \widetilde{e} + \widetilde{e}^T \widetilde{\Gamma}^T P \dot{\widetilde{e}} \tag{37}$$

By using (26) and (35), the (37) becomes:

$$\dot{V} = \sum_{i,j=1}^{q} \rho_i(\beta)\rho_j(\hat{\beta})[\Theta_{ij}\widetilde{e} + \Pi_{ij}u]^T P\widetilde{e} + \sum_{i,j=1}^{q} \rho_i(\beta)\rho_j(\hat{\beta})\widetilde{e}^T P[\Theta_{ij}\widetilde{e} + \Pi_{ij}u] \tag{38}$$

Then

$$\dot{V} = \sum_{i,j=1}^{q} \rho_i(\beta)\rho_j(\hat{\beta})[\widetilde{e}^T(\Theta_{ij}^T P + P\Theta_{ij})\widetilde{e} + u^T\Pi_{ij}^T P\widetilde{e} + \widetilde{e}^T P\Pi_{ij}u] \tag{39}$$

The stability of (26) and the bounded transfer from u to z by α is assured as:

$$\frac{\|z\|_2}{\|u\|_2} < \delta, \quad \|u\|_2 \neq 0, \quad \delta > 0 \tag{40}$$

which leads to:

$$\dot{V} + z^T z - \delta^2 u^T u < 0 \tag{41}$$

The state estimation error converges exponentially if the following condition holds:

$$\dot{V} + z^T z - \delta^2 u^T u < -2\lambda V, \ \ \lambda > 0 \tag{42}$$

From (34) and (39), inequality (42) becomes:

$$\sum_{i,j=1}^{q} \rho_i(\beta)\rho_j(\hat{\beta})(\widetilde{e}^T\Upsilon_{ij}\widetilde{e} + u^T\Pi_{ij}^T P\widetilde{e} + \widetilde{e}^T P\Pi_{ij}u - u^T\delta^2 u) < 0 \tag{43}$$

where

$$\Upsilon_{ij} = \Theta_{ij}^T P + P\Theta_{ij} + R^T R + 2\lambda\Gamma P. \tag{44}$$

This implicates

$$\sum_{i,j=1}^{q} \rho_i(\beta)\rho_j(\hat{\beta}) \begin{pmatrix} \widetilde{e} \\ u \end{pmatrix}^T J_{ij} \begin{pmatrix} \widetilde{e} \\ u \end{pmatrix} < 0 \tag{45}$$

where

$$\Delta_{ij} = \begin{pmatrix} \Upsilon_{ij} & P\Pi_{ij} \\ Pi_{ij}^T P & \delta^2 I \end{pmatrix} \tag{46}$$

Then, from (21), (25), (27), (36) and the use of the following change of variables:

$$\begin{cases} Q = P_1 K \\ W_j = P_1 L_{2j} \\ \alpha = \delta^2 \end{cases} \tag{47}$$

it follows that (46) is equivalent to (29). Finally, according to Lyapunov stability theory, if the LMI conditions (29) are satisfied, the (24) is exponentially stable.

4. SIMULATION RESULTS

In this section, an academic example is used to show the validness of the proposed method. Consider a CTSIM defined by (48):

$$\begin{cases} M\dot{\zeta} = \sum_{i=1}^{2} \rho_i(\beta)(A_i\zeta + Bu + \Lambda d) \\ y(t) = Cx \end{cases} \tag{48}$$

where $\zeta = (\zeta_1, \zeta_2, \zeta_3, \zeta_4)^T \in \mathbb{R}^4$, $u \in \mathbb{R}$, $d \in \mathbb{R}$ and $y \in \mathbb{R}^2$. The matrices numerical values are:

$$A_1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -2.50 & -0.75 & 0 & 0.03 \\ 0 & 1 & -0.40 & 0 \\ -2.50 & -0.75 & 0 & 0.08 \end{pmatrix}; A_2 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -2.70 & -0.75 & 0 & 0.03 \\ 0 & 1.00 & -0.40 & 0 \\ -2.70 & -0.75 & 0 & 0.08 \end{pmatrix}$$

$$B = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -0.13 \end{pmatrix}; \Lambda = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}; C = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}; M = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

The weighting functions are given by:

$$\begin{cases} \rho_1(\zeta) = 1 - 12.76\zeta_1^2 \\ \rho_2(\zeta) = 12.76\zeta_1^2 \end{cases}$$

Under hypothesis H_3), the following matrices Ψ_1, Ψ_2, Ψ_3, Ψ_4 satisfying (8) were obtained

$$\Psi_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \Psi_2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}; \Psi_3 = \begin{pmatrix} 0 & 0 & 0.71 & -0.71 & 0 \\ 0 & 0 & -0.71 & -0.71 & 0 \end{pmatrix}, \quad \Psi_4 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Using the MATLAB LMI toolbox, The following observer gains result from the resolution of LMIs (29) of Theorem 1 with $\lambda = 5$:

$$N_1 = \begin{pmatrix} -6.084 & 6.563 & -1.252 & -1.977 & 0 \\ -130.3 & -132 & 37.51 & 30.39 & 1 \\ -3.172 & -5.628 & -15.55 & 2.585 & 0 \\ 131.1 & 123.6 & -39.87 & -48.75 & -1 \\ -1280 & -1196 & 398.2 & 320 & 0 \end{pmatrix}; N_2 = \begin{pmatrix} -6.734 & 6.540 & -1.193 & -2 & 0 \\ -138.8 & -132 & 38.33 & 29.61 & 1 \\ -1.968 & -5.628 & -15.73 & 2.723 & 0 \\ 139.4 & 123.6 & -40.79 & -47.97 & -1 \\ -1367 & -1196 & 398.2 & 312 & 0 \end{pmatrix}$$

$$L_{11} = \begin{pmatrix} -1.253 & -1.977 \\ 37.51 & 30.4 \\ -15.55 & 2.586 \\ -39.87 & -48.75 \\ 389.3 & 320 \end{pmatrix}; L_{12} = \begin{pmatrix} -1.193 & -2 \\ 38.33 & 29.61 \\ -15.73 & 2.723 \\ -40.79 & -47.97 \\ 398.2 & 312 \end{pmatrix}; L_{21} = \begin{pmatrix} -2.957 & 2.249 \\ 11.46 & -26.88 \\ 20.82 & -3.115 \\ -13.20 & 45.27 \\ 51.69 & -283.1 \end{pmatrix}$$

$$L_{22} = \begin{pmatrix} -3.017 & 2.272 \\ 10.65 & -26.09 \\ 21 & -3.252 \\ -12.28 & 44.48 \\ 42.82 & -275.1 \end{pmatrix}; G_1 = \begin{pmatrix} -0.452 \\ -5.817 \\ 0.883 \\ 5.767 \\ -61.41 \end{pmatrix}; G_2 = \begin{pmatrix} -0.452 \\ -5.817 \\ 0.883 \\ 5.767 \\ -61.41 \end{pmatrix}; K = \begin{pmatrix} 5 & -10 \\ -120 & 54 \\ -4 & 14 \\ 127 & -610 \\ -1127 & 432 \end{pmatrix}$$

The unknown input d is described as shown in Figure 1, and the following is the definition of the input signal u:

$$u = \begin{cases} t-2 & 0 \le t \le 2 \\ 0 & otherwise \end{cases}$$

The initial conditions of the state of the system and FUIO are: $\zeta_0 = [0.1 \;\; 0.3 \;\; 0.75 \;\; 3.03]^T$, $\hat{\zeta}_0 = [0 \;\; 0 \;\; 0 \;\; 0]^T$. The simulation results are given in Figure 1 that shows the unknown input and its estimate and Figures 2(a) to 2(d) which represnte the state variables $x_1(t)$, $x_2(t)$, $x_3(t)$, $x_4(t)$ and their estimates. They demonstrate that the observer provides a good estimation of unknown states and unknown input. It shows that the estimated variables catch up rapidly (around 0.5s), and accurately the real variables during the application time of the UI.

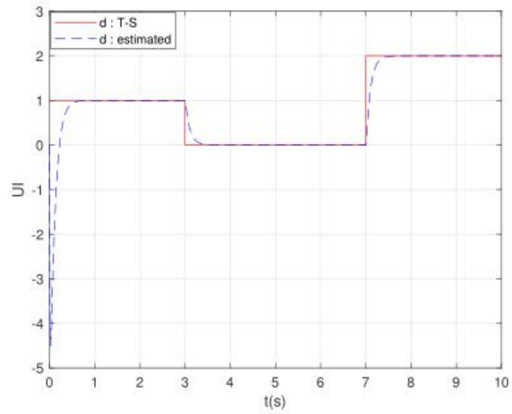
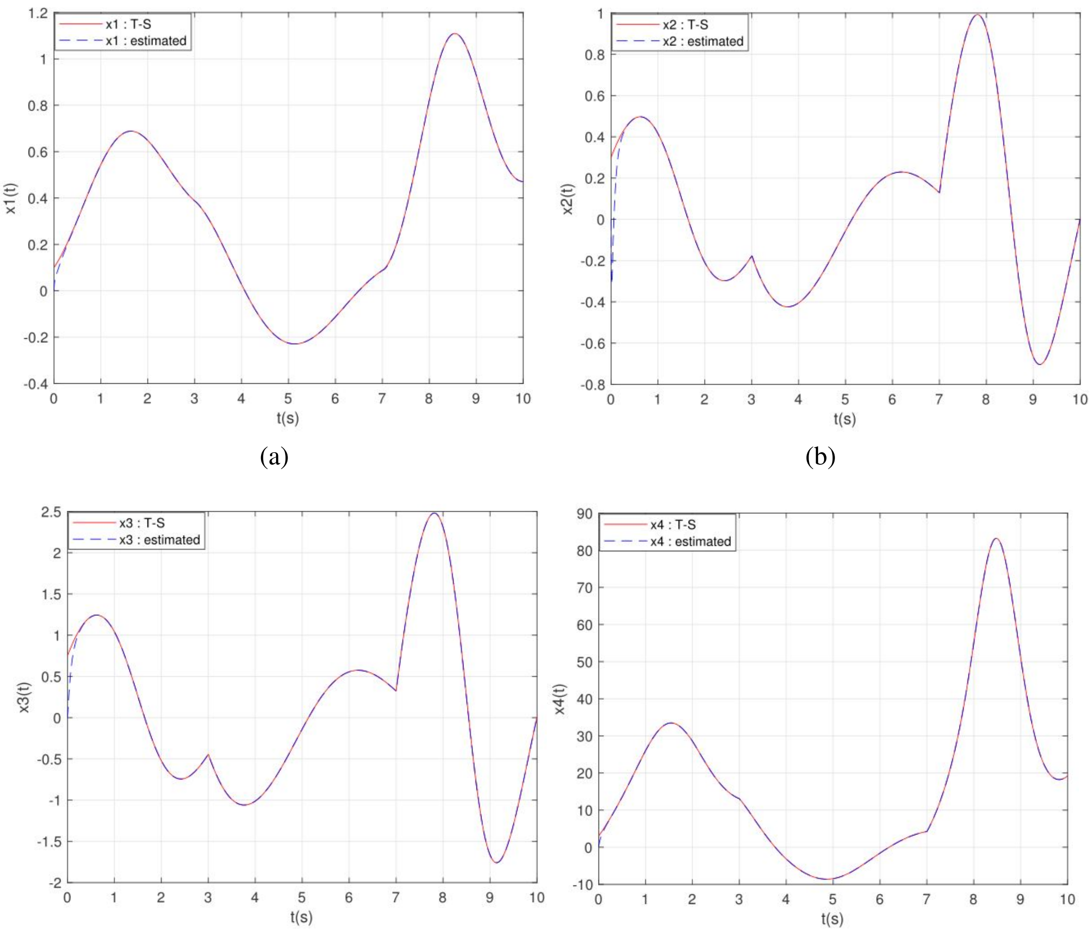


Figure 1. Unknown input and its estimate



(a) (b) (c) (d)

Figure 2. States with their estimates (a) state variable $x_1(t)$ and its estimate, (b) state variable $x_2(t)$ and its estimate, (c) state variable $x_4(t)$ and its estimate, and (d) state variable $x_4(t)$ and its estimate

It is obvious from this example that the proposed strategy presents better results than the approaches given by [27] and [28]. Indeed, the method developed is based on the separation between static and dynamic relations in the CTSIM. Only systems that ensure the criterion of rank described in the study allow for this separation. It is also pointed out that, to express the nonlinear system as a CTSIM, the authors took the weighing functions that are dependent on the measured variables (the output of the system). Due to the use of the same premise variables by the observer and the model, it is obvious that this class with measurable premise variables will allow a factorization of the weighting functions $\rho_i(\beta)$) while analyzing the convergence of the estimate error. Noises affect the output in all practical situations. As a result, the accuracy of the model describing the system, as well as the results obtained, will be affected. In our situation, we used a CTSIM with an unmeasurable variable to simulate the rolling disc process. The activation functions used in our proposed observer are not the same as those employed in CTSIM. Furthermore, the elimination of the Lipschitz assumption of the weighting functions is another advantage of this method compared to those that use CTSIM with premise variables satisfying Lipschitz conditions (see [28]) which allowed us to to relax the LMI conditions.

5. CONCLUSION

This article proposes a new method to design FUIO in explicit structure for a class of CTSIMs with UPVs. Using an augmented system structure, the suggested FUIO enables to estimate, exponentially , the UIs and the system states. The Lyapunov theory is used to investigate the convergence of the observer and the the conditions that ensure the convergence of estimation errors are obtained in the LMIs formulation. A numerical example is used to verify and confirm the efficiency of the proposed method. As future works, it would be interesting to extend the present result to the design of a FUIO for a class of discrete-time T-S implicit models also for a class of time-delay T-S implicit systems and try to relax the LMI conditions and reduce its conservatism, by using different Lyapunov functions such as poly-quadratic ones.

REFERENCES

[1] L. Dai, *Singular control systems*, vol. 118. Berlin/Heidelberg: Springer-Verlag, 1989.

[2] G.-R. Duan, *Analysis and design of descriptor linear systems*, vol. 23. New York, NY: Springer New York, 2010.

[3] C. McHugh, S. Coleman, and D. Kerr, "Hourly electricity price forecasting with NARMAX," *Machine Learning with Applications*, vol. 9, Sep. 2022, doi: 10.1016/j.mlwa.2022.100383.

[4] J. J. Jui and M. A. Ahmad, "A hybrid metaheuristic algorithm for identification of continuous-time Hammerstein systems," *Applied Mathematical Modelling*, vol. 95, pp. 339–360, Jul. 2021, doi: 10.1016/j.apm.2021.01.023.

[5] G. Savaia, G. Panzani, M. Corno, J. Cecconi, and S. M. Savaresi, "Hammerstein–Wiener modelling of a magneto-rheological dampers considering the magnetization dynamics," *Control Engineering Practice*, vol. 112, Jul. 2021, doi: 10.1016/j.conengprac.2021.104829.

[6] I. Chihi, L. Sidhom, and E. N. Kamavuako, "Hammerstein–wiener multimodel approach for fast and efficient muscle force estimation from EMG signals," *Biosensors*, vol. 12, no. 2, Feb. 2022, doi: 10.3390/bios12020117.

[7] M. Roohi, K. Erazo, D. Rosowsky, and E. M. Hernandez, "An extended model-based observer for state estimation in nonlinear hysteretic structural systems," *Mechanical Systems and Signal Processing*, vol. 146, Jan. 2021, doi: 10.1016/j.ymssp.2020.107015.

[8] T. Takagi and M. Sugeno, "Fuzzy identification of systems and its applications to modeling and control," *IEEE Transactions on Systems, Man, and Cybernetics*, vol. SMC-15, no. 1, pp. 116–132, Jan. 1985, doi: 10.1109/TSMC.1985.6313399.

[9] T. Taniguchi, K. Tanaka, H. Ohtake, and H. O. Wang, "Model construction, rule reduction, and robust compensation for generalized form of Takagi-Sugeno fuzzy systems," *IEEE Transactions on Fuzzy Systems*, vol. 9, no. 4, pp. 525–538, 2001, doi: 10.1109/91.940966.

[10] Z. Lahlou, A. EL-Amrani, and I. Boumhidi, "Finite frequency H∞ control design for nonlinear systems," *International Journal of Power Electronics and Drive Systems (IJPEDS)*, vol. 12, no. 1, pp. 567–575, Mar. 2021, doi: 10.11591/ijpeds.v12.i1.pp567-575.

[11] M. A. Ghany and M. A. Shamseldin, "Parallel distribution compensation PID based on Takagi-Sugeno fuzzy model applied on egyptian load frequency control," *International Journal of Electrical and Computer Engineering (IJECE)*, vol. 10, no. 5, pp. 5274–5287, Oct. 2020, doi: 10.11591/ijece.v10i5.pp5274-5287.

[12] S. Aboulem, A. EL-Amrani, and I. Boumhidi, "Finite frequency H∞ control for wind turbine systems in T-S form," *International Journal of Power Electronics and Drive Systems (IJPEDS)*, vol. 11, no. 3, pp. 1313–1322, Sep. 2020, doi: 10.11591/ijpeds.v11.i3.pp1313-1322.

[13] W. S. Pambudi, R. A. Firmansyah, R. R. Issany, R. Y. Adhitya, and M. Syai'in, "The application of Sugeno fuzzy to control active power load and remaining battery usage time modelling," *Indonesian Journal of Electrical Engineering and Computer Science (IJEECS)*, vol. 26, no. 3, pp. 1266–1273, Jun. 2022, doi: 10.11591/ijeecs.v26.i3.pp1266-1273.

[14] J. Soulami, A. El Assoudi, M. Essabre, E. H. El Yaagoubi, and M. Habibi, "Observer design for a class of Takagi-Sugeno descriptor systems: application to an aerobic culture of a recombinant yeast," *Applied Mathematical Sciences*, vol. 9, pp. 2999–3017, 2015, doi: 10.12988/ams.2015.5167.

[15] T. Taniguchi, K. Tanaka, and H. O. Wang, "Fuzzy descriptor systems and nonlinear model following control," *IEEE Transactions on Fuzzy Systems*, vol. 8, no. 4, pp. 442–452, 2000, doi: 10.1109/91.868950.

[16] T. Taniguchi, K. Tanaka, K. Yamafuji, and H. O. Wang, "Fuzzy descriptor systems: Stability analysis and design via LMIs," in *Proceedings of the American Control Conference*, 1999, vol. 3, pp. 1827–1831, doi: 10.1109/acc.1999.786165.

[17] Z. Mejri, L. Sidhom, and A. Abdelkrim, "Three-dimensional structure from motion recovery of a moving object with noisy measurement," *International Journal of Electrical and Computer Engineering (IJECE)*, vol. 10, no. 1, pp. 117–128, Feb. 2020, doi: 10.11591/ijece.v10i1.pp117-128.

[18] V.-P. Vu, M.-T. Nguyen, A.-V. Nguyen, V.-D. Tran, and T. M. N. Nguyen, "Disturbance observer-based controller for inverted pendulum with uncertainties: linear matrix inequality approach," *International Journal of Electrical and Computer Engineering (IJECE)*, vol. 11, no. 6, pp. 4907–4921, Dec. 2021, doi: 10.11591/ijece.v11i6.pp4907-4921.

[19] K. O. Omali, M. N. Kabbaj, and M. Benbrahim, "Fault-tolerant control with high-order sliding mode for manipulator robot," *International Journal of Power Electronics and Drive Systems (IJPEDS)*, vol. 13, no. 3, pp. 1854–1869, Sep. 2022, doi: 10.11591/ijpeds.v13.i3.pp1854-1869.

[20] N. A. Sulaiman, M. P. Abdullah, H. Abdullah, M. N. S. Zainudin, and A. Md Yusop, "Fault detection for air conditioning system using machine learning," *IAES International Journal of Artificial Intelligence (IJ-AI)*, vol. 9, no. 1, pp. 109–116, Mar. 2020, doi: 10.11591/ijai.v9.i1.pp109-116.

[21] M. Lungu, "Reduced-order multiple observer for Takagi–Sugeno systems with unknown inputs," *ISA Transactions*, vol. 85, pp. 1–12, Feb. 2019, doi: 10.1016/j.isatra.2018.10.026.

[22] V. H. Tellez-Izquierdo, J. Anzurez-Marin, and E. Espinosa-Juarez, "LMI approach: design of unknown inputs observers to Takagi-Sugeno fuzzy model," in *2018 International Conference on Computational Science and Computational Intelligence (CSCI)*, Dec. 2018, pp. 174–178, doi: 10.1109/CSCI46756.2018.00041.

[23] D. Zhao, H. K. Lam, Y. Li, S. X. Ding, and S. Liu, "A novel approach to state and unknown input estimation for Takagi–Sugeno fuzzy models with applications to fault detection," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 67, no. 6, pp. 2053–2063, Jun. 2020, doi: 10.1109/TCSI.2020.2968732.

[24] J. Pan, S. Zhang, L. Bai, and W. Zhang, "Fuzzy unknown input observer design for autonomous ground vehicles," in *2021 IEEE 16th Conference on Industrial Electronics and Applications (ICIEA)*, Aug. 2021, pp. 1972–1977, doi: 10.1109/ICIEA51954.2021.9516283.

[25] J. Yoneyama, "An improved unknown input observer design for fuzzy bilinear systems," in *2022 13th Asian Control Conference (ASCC)*, May 2022, pp. 252–257, doi: 10.23919/ASCC56756.2022.9828167.

[26] A.-T. Nguyen, T. Q. Dinh, T.-M. Guerra, and J. Pan, "Takagi–Sugeno fuzzy unknown input observers to estimate nonlinear dynamics of autonomous ground vehicles: theory and real-time verification," *IEEE/ASME Transactions on Mechatronics*, vol. 26, no. 3, pp. 1328–1338, Jun. 2021, doi: 10.1109/TMECH.2020.3049070.

[27] K. Bouassem, J. Soulami, A. El Assoudi, and E. H. El Yaagoubi, "Fuzzy observer design for a class of Takagi-Sugeno descriptor systems subject to unknown inputs," *Nonlinear Analysis and Differential Equations*, vol. 5, pp. 117–134, 2017, doi: 10.12988/nade.2017.61191.

[28] A. Louzimi, A. El Assoudi, J. Soulami, and E. H. El Yaagoubi, "Unknown input observer design for a class of nonlinear descriptor systems: a Takagi-Sugeno approach with Lipschitz constraints," *Nonlinear Analysis and Differential Equations*, vol. 5, pp. 99–116, 2017, doi: 10.12988/nade.2017.61190.

[29] K. Ouarid, M. Essabre, A. El Assoudi, and E. H. El Yaagoubi, "State and fault estimation based on fuzzy observer for a class of Takagi-Sugeno singular models," *Indonesian Journal of Electrical Engineering and Computer Science (IJEECS)*, vol. 25, no. 1, pp. 172–182, Jan. 2022, doi: 10.11591/ijeecs.v25.i1.pp172-182.

[30] K. Tanaka and H. O. Wang, *Fuzzy control systems design and analysis*. Wiley, 2001.

[31] C. Mechmeche, H. Habib, M. Rodrigues, and N. B. Braiek, "State and unknown inputs estimations for multi-models descriptor systems," *American Journal of Computational and Applied Mathematics*, vol. 2, no. 3, pp. 86–93, Aug. 2012, doi: 10.5923/j.ajcam.20120203.04.

BIOGRAPHIES OF AUTHORS

Mohamed Essabre is a professor at the department of Electrical Engineering in Faculty of Science and Technology Mohammedia (FSTM), in Hassan II University of Casablanca (Morocco). His research interests focus in nonlinear implicit model, nonlinear observer design, unknown input observer design, fault detection, Takagi-Sugeno fuzzy control. He can be contacted at email: mohamed.essabre@fstm.ac.ma.

Ilham Hmaiddouch received the Ph.D. in Electrical Engineering from the National Higher School of Electricity and Mechanics (ENSEM), Hassan II University of Casablanca, Morocco. His research interests focus in nonlinear implicit model, nonlinear observer design, unknown input observer design, fault detection, Takagi-Sugeno fuzzy control. She can be contacted at email: ilham.hmaiddouch@yahoo.fr.

Abdellatif El Assoudi is a professor at the department of Electrical Engineering in National Higher School of Electricity and Mechanics (ENSEM), in Hassan II University of Casablanca (Morocco). His research interests focus in nonlinear implicit model, nonlinear observer design, unknown input observer design, fault detection, Takagi-Sugeno fuzzy control. He can be contacted at email: a.elassoudi@ensem.ac.ma.

El Hassane El Yaagoubi is a professor at the department of Electrical Engineering in National Higher School of Electricity and Mechanics (ENSEM), in Hassan II University of Casablanca (Morocco). His areas of interest include nonlinear implicit model, nonlinear observer design, unknown input observer design, fault detection, Takagi-Sugeno fuzzy control. He can be contacted at email: h.elyaagoubi@ensem.ac.ma.