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Optimal trajectory tracking control for a wheeled mobile robot using backstepping technique

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ABSTRACT

This work studies an optimal trajectory tracking of a wheeled mobile robot with the objective of minimizing energy consumption. First, the mathematical model, which takes into account the kinematic model of the mobile robot and the dynamic model of the actuators is presented. Then, a backstepping controller is designed and its parameters are tuned to satisfy several strict criteria such as rapid convergence, matching desired trajectory, and minimizing energy. For that, two cost functions were investigated and the best one has been selected. The significant reduction in energy losses achieved for all the proposed motion scenarios proves the effectiveness of our approach.

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1. INTRODUCTION

Designing mobile robots to meet the needs of mankind in various sectors of society is of huge interest today. Indeed, by their capability to move into wider fields, mobile robots demonstrate great adaptability [1], [2]. Unfortunately, this ability is hampered by the fact that the robot carries a limited amount of energy [3]. Therefore, to resolve these problems, many works have emerged and focused on improving the mobile robot energy efficiency. The first works in this perspective were done by Kim *et al.* [4], [5]. In this work, they succeed to save 10% more energy for a differential drive robot by optimizing velocity profile. Ho [6], added the case study of simple motion trajectories, such as S-curve and trapezoidal. Wahab [7] investigated the various energy loss components and presented a well-defined and complete energy model. The above methods could effectively propose models for mobile robots that describe globally the flow of energy. However, they did not take into account the contribution of the motion control to reduce the power consumption, which can improve efficiency if addressed.

Quite recently, a large variety of publications have been undertaken for trajectory tracking with various control methods used, such as the adaptive output feedback control [8], the input-output feedback linearization method [9], the two-step feedback linearization control [10], the backstepping-based control [11], the proportional integral derivative (PID) control [12], the Lyapunov function-based control [13], [14], the adaptive and sliding mode control [15]–[17], the neural-network-based control [18], and the robust adaptive-based control [19]. Stefek *et al.* [20] found during their review of these studies, most of them refer only to path morphology, whereas energy consumption is not considered. They, therefore, undertook a work in which they compared the energy of mobile robots based on the most common controllers. The main result observed is that the smooth control leads to low energy consumption with a lack of accuracy, and inversely an accurate control motion needs a higher consumption.

A considering the works mentioned, it appears necessary to build an energy model of a tow wheeled differential mobile robot (WDMR) that includes the influence of motion control. The originality of our approach resides in using the nonlinear least square method to tune the gain parameters of the Backstepping controller for a WDMR to reach minimum energy and achieve a good performance tracking trajectory criterion. The model is implemented in MATLAB/Simulink to generate optimal trajectories and to demonstrate the effectiveness of our approach.

The rest of the paper is organized as: in section 2, the kinematic model of the mobile robot is presented, and the dynamic of DC-motors is considered. Then, the control law is developed using the Backstepping technique, and its stability property is proved using the Lyapunov theory. Later, the model of energy and the cost function are introduced. In section 3, simulations results are discussed. Finally, section 4 concludes this paper.

2. METHOD

As mentioned above, this paper presents an energy model that considers the influence of motion control. The first step to achieve this goal is to describe the states and outputs of the model influencing energy consumption. The kinematic model of the mobile robot is described in the next section where the relationship between the inputs (coordinates and orientation) and outputs (linear and angular velocities) is determined.

2.1. Kinematic and dynamic model

To understand the mechanical behavior of the robot we need a Kinematic model. This helps to design appropriate mobile robots for tasks and understand how to create control software for an instance of mobile robot hardware [21]. For that, A two WDMR is modeled as a rigid body having mass m and rear castor added for balance. As depicted in Figure 1, we denote with OXY and $P_0X_0Y_0$ the world inertial frame and the body frame attached to the WDMR, respectively. Where r is drive wheel radius, b is the axle length, x and y denote the position of P_0 point, θ is the angle between the robot axle and X- axis. So, the pose of the robot is noted $q = [x\ y\ \theta]^T$.

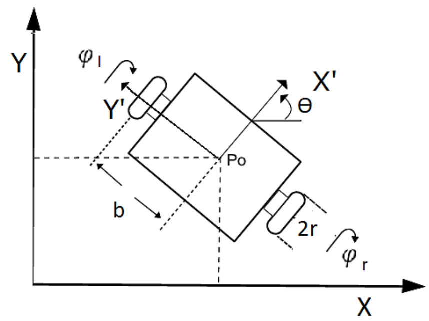

Figure 1. Schematic diagram of the mobile robot

Assuming that: i) the wheels are rolling without slipping, ii) the center of gravity G coincides with the point P_0, and iii) the guidance axis is perpendicular to the robot plane. The robot changes its direction and speed by varying the angular velocities of wheels, $\dot{\varphi}_L$ and $\dot{\varphi}_R$, which are related to the linear velocitie v and the angular velocitie ω by (1), (2):

$$\mathrm{v} = \frac{r(\dot{\varphi}_R+\dot{\varphi}_L)}{2} \tag{1}$$

$$\omega = \frac{r(\dot{\varphi}_R-\dot{\varphi}_L)}{2b} \tag{2}$$

Then the kinematics model can be described as [22]:

$$\begin{cases} \dot{x} = v\ cos\theta \\ \dot{y} = v\ sin\theta \\ \dot{\theta} = \omega \end{cases} \tag{3}$$

and formulated by (4).

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} cos\theta & 0 \\ sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \quad (4)$$

2.2. Controller design

Motion control is an important function for WDMR as it ensures that the desired task is well performed. Among all motion control methods, Backstepping remains to this day the most suitable for tracking trajectories problems. Indeed, backstepping control has the most advantages in terms of simplicity, robustness, and integration. All the more so since a recent study shows that these benefits have been increased, by improving its main defect, namely the velocity jumps and overshoots [23]. The objective of the controller is to synthesize v the linear velocity and ω the angular velocity such that position and orientation follow the desired trajectories.

Let be $q_d = [x_d \; y_d \; \theta_d]^T$ the desired pose and orientation of the mobile robot, then as (4) we deduce as (5) [24],

$$\begin{bmatrix} \dot{x}_d \\ \dot{y}_d \\ \dot{\theta}_d \end{bmatrix} = \begin{bmatrix} cos\theta & 0 \\ sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_d \\ \omega_d \end{bmatrix} \quad (5)$$

where v_d and ω_d are the desired linear and the angular velocities, respectively. The desired linear velocity and the desired angular velocity for the reference trajectory are calculated by (6) and (7), respectively [25]:

$$v_d = \sqrt{\dot{x}_d^{\,2} + \dot{y}_d^{\,2}} \quad (6)$$

$$\omega_d = \frac{\ddot{y}_d \dot{x}_d - \ddot{x}_d \dot{y}_d}{\dot{x}_d^{\,2} + \dot{y}_d^{\,2}} \quad (7)$$

The controller receives the references linear and angular velocities, and generates another pair of linear and angular velocities to be delivered to the robot DC motors. The configuration error can be presented by [26]:

$$S_e = \begin{bmatrix} x_e \\ y_e \\ \theta_e \end{bmatrix} \begin{bmatrix} cos\theta & sin\theta & 0 \\ -sin\theta & cos\theta & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_d - x \\ y_d - y \\ \theta_d - \theta \end{bmatrix} \quad (8)$$

Differential (8) and rearranging with (4) and (7). Now the configuration error becomes:

$$\dot{S}_e = \begin{bmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{\theta}_e \end{bmatrix} = \begin{bmatrix} v_d cos\theta_e + y_e\omega - v \\ v_d sin\theta_e + x_e\omega \\ \omega_d - \omega \end{bmatrix} \quad (9)$$

with $v_d > 0$ and $\omega_d > 0$, for all the time, determine a velocity control law v= f(S_e, v_d, ω_d, K) such that $\lim_{t\to+\infty} S_e = 0$ is asymptotically stable. Where $S_e, v_d, \omega_d, K = [K_x \; K_y \; K_\theta]$ are the tracking configuration error, the desired linear and angular velocity and control gain parameters vector, respectively. Finally, the proposed nonlinear kinematic trajectory tracking control law based on backstepping technique can be described by (10).

$$\boldsymbol{v} = \begin{bmatrix} v \\ \omega \end{bmatrix} = \begin{bmatrix} v_d cos\theta_e + K_x x_e \\ \omega_d + K_y v_d y_e + K_\theta v_d sin\theta_e \end{bmatrix} \quad (10)$$

Assuming that K_x , K_y and K_θ are positive control gains. To prove that the control law is asymptotically stable the Lyapunov candidate function is considered as (11) [27].

$$V = \frac{1}{2}(x_e^{\,2} + y_e^{\,2}) + \frac{1}{K_y}(1 - cos\theta_e) \quad (11)$$

The time derivative of (11) leads to:

$$\dot{V} = x_e\dot{x}_e + y_e\dot{y}_e + \frac{1}{K_y}\dot{\theta}_e sin\theta_e \quad (12)$$

and using (12) we can rearrange (9) as (13):

$$\dot{S}_e = \begin{bmatrix} (\omega_d + v_d(K_x y_e + K_\theta sin\theta_e))y_e - K_x x_e \\ -\left(\omega_d + v_d\left(K_y y_e + K_\theta sin\theta_e\right)\right)y_e + v_d sin\theta_e \\ -v_d\left(K_y y_e + K_\theta sin\theta_e\right) \end{bmatrix} \quad (13)$$

One can deduce that:

$$\dot{S}_e = \begin{bmatrix} (\omega_d + v_d(K_x y_e + K_\theta sin\theta_e))y_e - K_x x_e \\ -\left(\omega_d + v_d\left(K_y y_e + K_\theta sin\theta_e\right)\right)y_e + v_d sin\theta_e \\ -v_d\left(K_y y_e + K_\theta sin\theta_e\right) \end{bmatrix} \quad (14)$$

which lead to:

$$\dot{V} = -K_x {x_e}^2 - v_d\frac{K_\theta}{K_y}sin^2\theta_e \quad (15)$$

which is clearly negative. So, the stability property is proved using the Lyapunov theory.

2.3. Modeling DC motor

DC motor is typically, the most used actuator for mobile robots. Adjusting the voltage and current permit to control the torque and the angular velocity. As this actuator is controlled by voltage input, we need to determine the transfer function that allows using of (1) and (2) to drive the mobile robot along the desired path. This angular velocity is characterized using an equivalent circuit model. Figure 2 shows the DC motor circuit with torque and rotor angle consideration.

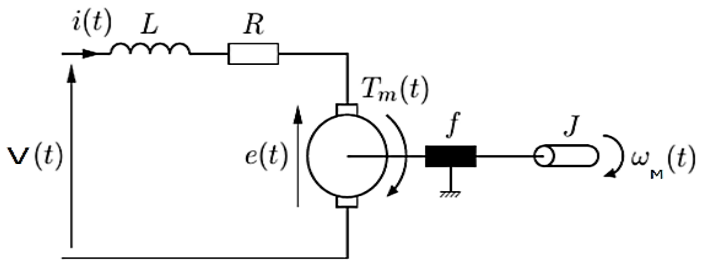

Figure 2. Schematic diagram of the DC motor

The differential equations that describe the dynamic model are:

$$\begin{cases} L\frac{di}{dt} + Ri + K_\omega\dot{\varphi} = V \\ J\ddot{\varphi} + K_t i + f\dot{\varphi} = T_m \end{cases} \quad (16)$$

where V and i are the armature voltage and current, R and L are the armature resistance and inductance, f is the viscous friction coefficient, T_m is the dynamic load applied to the motor, K_t is the motor torque constant, K_ω is the voltage constant, J is the motor shaft inertia, φ is the angular position of the shaft. Let $\omega_M(s) = \dot{\varphi}$, we can deduce from (6) the transfer function of the angular velocity $\omega_M(s)$ to the input voltage $V(s)$ as (17).

$$F(s) = \frac{\omega_M(s)}{V(s)} = \frac{K_t}{(R+Ls)(Js+f)+K_tK_\omega} \quad (17)$$

2.4. Energy modeling

To build an energy model, we focused on the kinetic energy losses of the robot. The other energy losses have been studied in per [27]. Then the kinetic energy equation, can be expressed using the control provided velocities same as in (18) [28]:

$$E_k = \frac{1}{2}(m\, v^2(t) + I\, \omega^2(t) \quad (18)$$

where m is the mass, and I is the moment of inertia of the robot. The total energy can be estimated by (19).

$$E = \int_0^{t_f} E_k dt \quad (19)$$

2.5. Cost functions

To evaluate the performance of the controller two cost functions were adopted. The first one is:

$$J_1 = \int_0^{t_f} q^T Q q\, dt \quad (20)$$

as the trajectory of the mobile robot and desired velocities are known we can estimate a minimum energy using (14) as (21):

$$E_m(t) = \frac{1}{2}(m\, {v_d}^2(t) + I\, {\omega_d}^2(t) \quad (21)$$

where Q is 3×3 identity matrix. and compare it with the kinetic of the mobile robot. Thus, we define the second cost function as (22):

$$J_2 = \int_0^{t_f} q^T Q q\, dt + \int_0^{t_f} \left(E_k(t) - E_m(t)\right)^2 dt \quad (22)$$

3. RESULTS AND DISCUSSION

In this section, the simulation results evaluating the performance of the proposed backstepping controller for solving the trajectory tracking problem of a wheeled mobile robot will be presented and thoroughly discussed. The models of the WDMR, the Backstepping controller, and DC-motors are implemented in the MATLAB/Simulink environment using the parameters listed in Table 1. And numerically simulated with sampling time and Dormand-prince method based. To test the tracking performance, three motion scenarios, each addressing two case studies are considered. The control gain parameters were set to fit the states of the model to reference stats by a non-linear least squares method using the Marquardt-Levenburg algorithm with 0.01 as tolerance. The MATLAB optimization toolbox was used for this purpose.

Table 1. Parameters of the mobile robot

Parameters	Values	Parameters	Values
r	0.095 m	R	0.5 Ω
b	0.165 m	L	0.01 H
m	6.5 kg	J	0.01 kgm^2/s^2
I	0.01 kgm^2	$K_t = K_\omega$	0.01 Nm/A
		f	0.1 Nm/(rad/s)

3.1. Circular trajectory

In this scenario, the desired trajectory is chosen to be a circle, and the coordinates are:

$$x_d = cos(\omega_d t), y_d = sin(\omega_d t), \theta_d = \frac{\pi}{2} + \omega_d t \quad (23)$$

The robot starts from the initial state $q_0 = [2\ 0\ 0]^T$ and ends its trajectory at the same point within the final time t_f =126 s. It is seen from Table 2 that by using the cost function J_2 instead of J_1, the best set of gain parameters is found. Indeed, with the second set, the pose path and orientation errors converge rapidly, and fewer energy losses are observed. Consequently, these performances can be illustrated in Figure 3. Figure 3(a) shows the actual trajectory of the mobile robot and desired circular trajectory. The linear and angular velocities as described by (10) are shown in Figure 3(b). And Figures 3(c)-(e) show the actual and desired circular trajectory pose and orientation of the mobile robot. Finally, the desired total energy and actual total energy are compared in Figure 3(f).

Table 2. Gain parameters, mean tracking errors, and minimum total energy based on cost functions

	K_x	K_y	K_θ	Mean x_e [m]	Mean y_e [m]	Mean θ_e [rad]	Minimum total energy [J]	Mean minimum total energy error [J]
Case study with J_1	5	4.38	21.38	0.0018	0.0200	0.1030	42.26	-
Case study with J_2	0.1	20	21.12	0.0349	0.3814	0.1038	32.81	0.4153

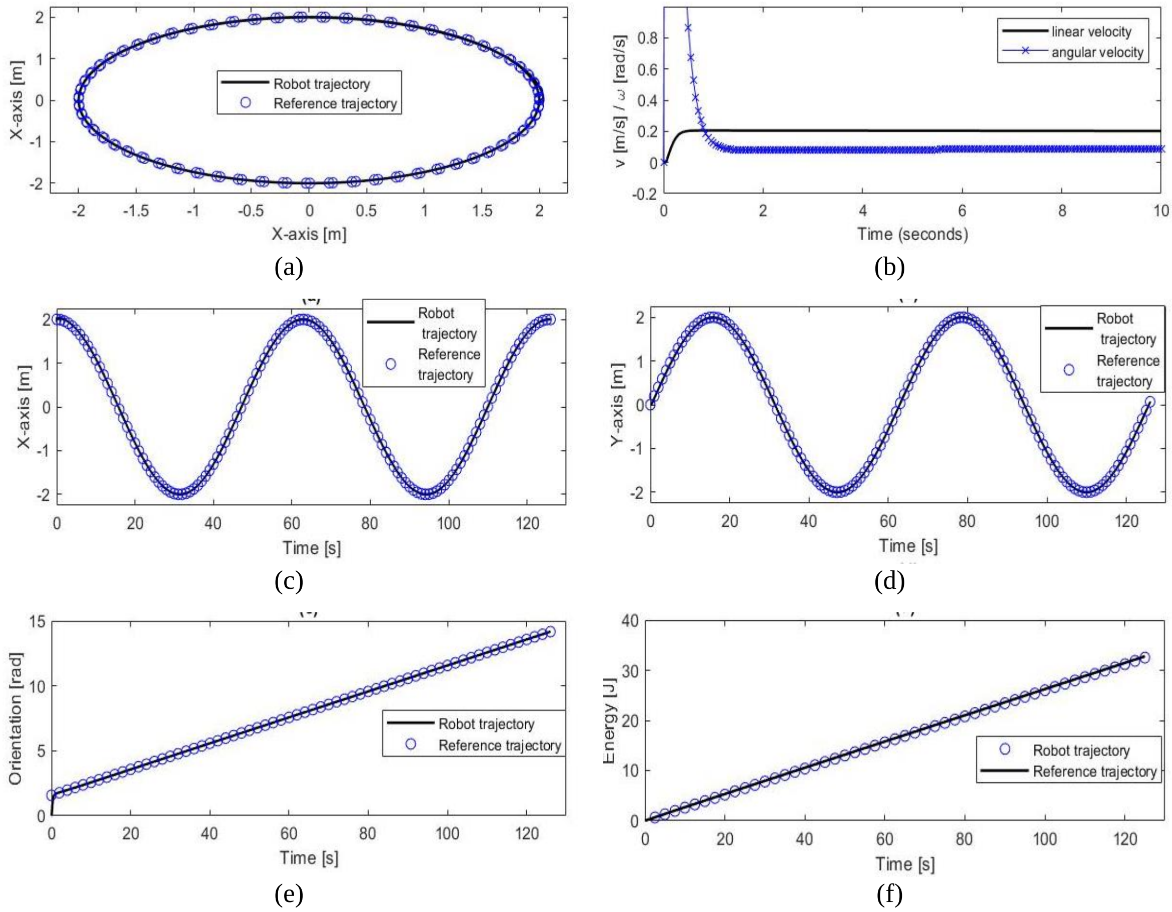

Figure 3. Comparison of obtained optimal circular trajectory (red dotted line) and reference trajectory (blue solid line) (a) X-Y coordinates, (b) linear and angular velocities, (c) X coordinates, (d) Y coordinates, (e) orientation, and (f) total minimum energy

3.2. Lemniscates trajectory

In this scenario, the desired trajectory is chosen to be a circle, and the coordinates are:

$$x_d = 0.75 + 0.75 * sin\left(\frac{2\pi}{50}t\right), y_d = sin\left(\frac{4\pi}{50}t\right)$$
$$\theta_d = \begin{cases} g(t), & 0 \le t < 12.5\ sec \\ -\pi - g(t) & 12.5 \le t < 37.5\ sec \\ g(t) & 37.5 < t \end{cases} \quad (24)$$

where: $g(t) = tan^{-1}\left(\frac{8cos\left(2sin^{-1}\left(\frac{4\pi x_d}{3}-1\right)\right)}{\sqrt[3]{1-\left(\frac{4\pi x_d}{3}-1\right)^2}}\right)$

The robot starts from the initial state $q_0 = [0.75\ 0\ 0]^T$ and ends its trajectory at the same point within the final time t_f =100 s. It is seen from Table 3 that with the cost functions J_2 we can save more energy than with using J_1. Consequently, these observations can be illustrated by Figure 4. Figure 4(a) shows actual trajectory of the mobile robot and desired lemniscates trajectory. The linear and angular velocities are shown in Figure 4(b). While Figures 4(c)-(e), show actual and desired lemniscates trajectory pose and orientation of the mobile robot. Finally, the desired total energy and actual total energy are compared in Figure 4(f).

Table 3. Gain parameters, mean tracking errors, and minimum total energy based on cost functions

	K_x	K_y	K_θ	mean x_e [m]	mean y_e [m]	mean θ_e [rad]	Minimum total energy [J]	Mean minimum total energy error [J]
Case study with J_1	1.516	723.5	92.83	0.0092	0.0177	0.0227	28.40	-
Case study with J_2	0.2609	418.5	42.28	0.0420	0.2745	0.2781	24.32	0.4455

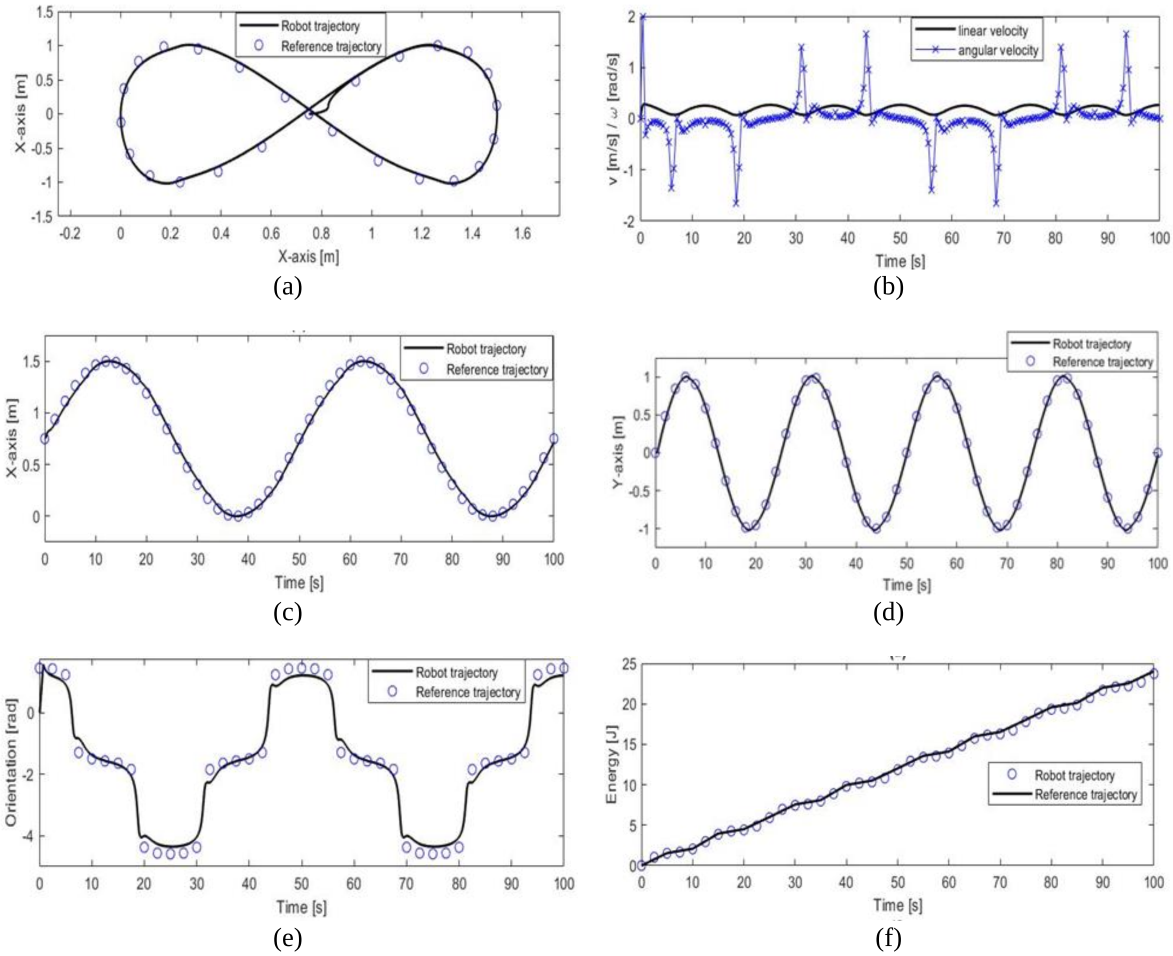

Figure 4. Comparison of obtained optimal lemniscate trajectory (red dotted line) and reference trajectory (blue solid line) (a) X-Y coordinates, (b) linear and angular velocities, (c) X coordinates, (d) Y coordinates, (e) orientation, and (f) total minimum energy

4. CONCLUSION

This work has developed an optimal trajectory tracking of a wheeled mobile robot with the objective of minimizing energy consumption. First, the mathematical model, which takes into account the kinematic model of the mobile robot and the dynamic model of the actuators was presented. Then, a backstepping controller is designed to satisfy several strict criteria such as rapid convergence, matching desired trajectory, and minimizing energy. For both cost functions, the gain parameters were tuned and the best set has been selected. Our simulation results showed that the proposed control method allowed a good control performance and a significant reduction of energy losses.

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