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Recommender systems: a novel approach based on singular value decomposition

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ABSTRACT

Due to modern information and communication technologies (ICT), it is increasingly easier to exchange data and have new services available through the internet. However, the amount of data and services available increases the difficulty of finding what one needs. In this context, recommender systems represent the most promising solutions to overcome the problem of the so-called information overload, analyzing users' needs and preferences. Recommender systems (RS) are applied in different sectors with the same goal: to help people make choices based on an analysis of their behavior or users' similar characteristics or interests. This work presents a different approach for predicting ratings within the model-based collaborative filtering, which exploits singular value factorization. In particular, rating forecasts were generated through the characteristics related to users and items without the support of available ratings. The proposed method is evaluated through the MovieLens100K dataset performing an accuracy of 0.766 and 0.951 in terms of mean absolute error and root-mean-square error.

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1. INTRODUCTION

Recommender systems (RS) are widely used to analyze and filter information supporting an individual's choice of a service or a specific object. The RS developed considerably in the nineties thanks to the beginning of the first e-commerce sites, solving the problem of managing a considerable amount of data. They then adapted to the technologies developed over the years to improve their performance. Just think, for example, of the birth of social networks. In this context, RS can benefit user groups' opinions belonging to the same community [1]. In particular, the importance of these systems significantly increased in the last decade, representing an opportunity in the modern context characterized by big data [2]. By processing large quantities of data and using increasingly sophisticated algorithms, the ultimate goal is to provide precise suggestions to users [3]. The field of use of recommender systems is very varied. Its use is known in e-commerce, streaming services, and dissemination sites. Many applications concern scenarios based on a large number of services or objects where only a part of them is interesting or relevant to the user. The main problem of RSs is the ability to find rating forecasts based on the information provided to the system itself. In content-based RSs the rating forecasts are done through the user and item profiles; instead of in collaborative filtering, the known ratings are exploited.

In this paper, a novel content-based approach defined rating through singular value decomposition (RSVD), that exploits the properties of singular value decomposition is described. The main purpose of the approach is to develop rating forecasts through operations on users and items profiles. This article is structured as: section 1 continues with the background on RS, section 2 is focused on the description of the properties of singular value decomposition (SVD) and the formal presentation of the proposed methodology through a constructive demonstration, in section 3 the numerical results are presented, section 4 presents conclusions and future works.

2. A NEW CONTENT-BASED APPROACH BASED ON SINGULAR VALUE DECOMPOSITION

In this section, after the description of the background, the SVD is quickly described through the main properties, and the proposed method is presented. The principal aim is to exploit the SVD in a content-based approach instead of a collaborative filtering one. The matrix factorization is usually used when some rating matrix elements are known.

2.1. Background

The main elements on which a RS operates are user, item and transaction [4]. The user represents the target of the recommender phase, which can be identified through its needs and characteristics. The item represents elements that recommender system [5] suggests to the user, and it can be classified according to its features and the associated complexities. The transaction represents the interaction between the system and the user. The most common information is the rating, an evaluation of a user's consideration of an item. A quantitative-numerical representation is attributed to a user's usefulness or could assign to an item through the rating. The concept of rating is formalized.

Definition 1. Let U be the set of users and I the set of items. We define the rating function or utility function as the function r, described in relation (1), which to each pair of the domain $(u, i) \in U \times I$ associates the evaluation $r_{ui} \in R$.

$$r: (u, i) \in U \times I \mapsto r_{ui} \in R \tag{1}$$

One of the main aims of a recommender system is to determine the item $i'_u = arg \max_{i \in I} r\,(u, i) \in I$ which maximize the rating function for the user $u \in U$. A common problem to all recommender systems is the low number of known assessments. Therefore, the need to provide an estimate to evaluations $\hat{r}_{ui}$, for each element of the domain $U \times I$ by which the rating function is not defined arises. The ability to provide reliable forecasts distinguishes a good recommender system from an ineffective one.

Recommender systems are faced with several practical issues. Among these, the main ones are scalability, which indicates the ability of the recommender systems to adapt to increases in the number of data to be managed, sparse rating matrix that indicates the presence of a small number of known ratings. The lack of ratings implies the need to have a recommender system capable of providing suggestions based on a limited number of available evaluations. At last, the cold start problem indicates a recommender system's difficulty in providing suggestions to a new user or about a new item.

Among the main types of RS are content-based systems, collaborative systems, and hybrid systems [6]. These are also advanced recommendation services that manage and use the entire context in which a user is located: RS based on context-aware technologies [7], [8]. As previously mentioned, one of the main characteristics of recommender systems is to predict the consideration that an individual may have about an item that has not yet been evaluated. The ways in which this forecast is made is one of the distinguishing criteria for the RS. It should also be added that the ability to predict ratings correctly distinguishes a reliable recommender system from an ineffective one. Methods that exploit some known ratings to make forecasts will be presented during the article. These methods will be compared with the novel developed approach, which generates estimates without the need-to-know initial assessments.

Content-based RSs [4], [9] recommend items that are similar to what the user has preferred in the past. These techniques require a preliminary phase for constructing and representing a user profile and the objects' features. Content-based recommender systems generate rating forecasts through the feature vectors of the items and the user profile. The construction of user profiles originates from the information retrieval to acquire information about the preferences of the user [3]. Content-based recommender systems are exploited for different scopes, in fact, Wang *et al.* [10] develop a content-based RS in order to suggest where a paper can be published; instead Casillo *et al.* [11] propose a content-based RS in the cultural heritage field.

Collaborative filtering RSs [4], [9], [12] suggest to the user items that are well valued by users with similar characteristics [13], [14]. In this way, recommender systems are entirely based on evaluating the interests expressed by users of a community. Collaborative filtering systems are based on an ancient concept for humans: sharing opinions. In fact, predictions are made through the opinions of a community [15]. Through the internet, the trivial "word of mouth" is replaced, passing from the analysis of hundreds of opinions to thousands or more. The speed of the calculation systems allows to process the vast amount of information obtained in real-time and to determine both the preferences of a large community of people and the preferences of the individual through the most reasonable opinions for the specific user or for a group of users [16]. Collaborative filtering systems are subdivided into two groups: memory-based CF and model-based CF. In memory-based CF, users are divided into groups called neighborhood. Model-based CF aims to create a model through the factorization of the matrix of known ratings. The most common methods for this are principal component analysis (PCA) [7], probabilistic matrix factorization (PMF) [17], [18], non-negative matrix factorization (NMF) [17], and SVD [19], [20]. Model-based collaborative filtering methods based on the singular value decomposition are average rating filling [21], stochastic gradient descent [22], [23] and biased stochastic gradient descent [22], [23].

Hybrid RSs [24], [25] consist of combining two or more different techniques from those listed so far. The commonly used techniques are content-based, and collaborative filtering [4], [26]. In this way, the limits of the individual methods can be overcome. Hybrid RSs combine the two recommendation techniques trying to exploit the advantages of one to correct the disadvantages of the other [27].

2.2. The singular value decomposition

The singular value decomposition is used to reduce a rectangular matrix in a diagonal form by a suitable pre- and post-multiplication by orthogonal matrices. We recall that an orthogonal matrix is a square invertible matrix whose inverse coincides with its transpose, and that a square matrix $A \in R^{n\times n}$ is similar to a matrix $B \in R^{n\times n}$ if there exists a square invertible matrix $S \in R^{n\times n}$, such that $S^{-1}AS = B$. Then, let $m, n \in N$ and let $R \in R^{m\times n}$, then $\exists U \in R^{m\times m}$ and $\exists V \in R^{n\times n}$ orthogonal matrices such that the relation (2) is valid:

$$U^T RV = D = diag(\sigma_1, \dots, \sigma_p) \in R^{m\times n} \quad p = \min\{m, n\} \tag{2}$$

where $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_k > \sigma_{k+1} = \cdots = \sigma_p = 0$. From (2) it easily follows $R = UDV^T$.

The columns of the matrix $U \in R^{m\times m}$ are defined left singular vectors, the columns of the matrix $V \in R^{n\times n}$ are defined right singular vectors and $D \in R^{m\times n}$ is the matrix of the singular values. Some properties inherent to singular value factorization are discussed.

Lemma 1 Let $R \in R^{m\times n}$ and let U, V, D be the matrices obtained through relation (2). Then the following properties apply: i) $RR^T = UDD^TU^T \in R^{m\times m}$ and ii) $R^TR = VD^TDV^T \in R^{n\times n}$. From property 1 of Lemma 1, we obtain that the real matrix $RR^T \in R^{m\times m}$ is similar to the diagonal matrix $DD^T = diag(\lambda_1, \dots, \lambda_p, 0, \dots, 0) = diag(\sigma_1^2, \dots, \sigma_p^2, 0, \dots, 0) \in R^{m\times m}$ through the orthogonal matrix U. This implies that the left singular vectors are eigenvectors of the real symmetric matrix RR^T, while the eigenvalues $\lambda_i = {\sigma_i}^2 i = 1, \dots, p$ of RR^T are the squares of the first p singular values. The property 2 of Lemma 1 returns the same considerations about matrix R^TR.

It is helpful to find low-rank approximations of a given matrix R to know the associated approximation error in the computational context. Let $R \in R^{m\times n}$ and U, V, D be given by relation (2) and let $r = r(R)$ the rank of the given matrix, with $r \leq p = min\{m, n\}$. We define the matrix R_k in the realtion (3):

$$R_k = \sum_{i=1}^{k} u_i \sigma_i v_i^T = U_k D_k V_k^T \tag{3}$$

with $k \leq r$, $U_k = (u_1, \dots, u_k) \in R^{m\times k}$ matrix of the left singular vectors without the last $m - k$ columns, $V_k = (v_1, \dots, v_k) \in R^{n\times k}$ matrix of the right singular vectors without the last $n - k$ columns and $D_k = diag(\sigma_1, \dots, \sigma_k) \in R^{k\times k}$ matrix of singular values without the last $m - k$ lines and $n - k$ columns, then the Eckart-Young theorem [20] allows to estimate the approximation error.

Moreover, Eckart-Young theorem establishes that, considering only the first $k \leq r(R)$ singular values of the matrix R we obtain an approximation of the matrix R by means of a low rank matrix R_k having rank k. In this way, it is possible to reduce the computational cost relating to singular value factorization with an estimate of the acceptable error based on the decreasing property of singular values. The low-rank approximation of matrices is used in many applications such as control theory, signal processing, machine learning, image compression, information retrieval, quantum physics, see, e.g., [28]–[31] and references therein. In many of these applications, the matrices are not constant, but they vary with time, thus requiring *dynamical* low-rank approximation.

Process-based on SVD in collaborative filtering RSs let $R = (r_{ij})_{m\times n} \in R^{m\times n}$ be the matrix of known ratings, in collaborative filtering methods, the SVD is exploited to reduce the quantity of used memory and obtain rating forecasts. According to the notation of Eckart-Young theorem, the matrices P, and Q are defined in the relation (4).

$$P = U_k\sqrt{D_k} \in R^{m\times k} \qquad Q = \sqrt{D_k}V_k^T \in R^{k\times n} \tag{4}$$

These matrices can be seen as a linear combination of the k main features of user and item or can be defined randomly and fixed through a learning role [22], [23].

2.3. The proposed approach

The purpose of this subsection is to show the theoretical prerequisites for creating rating forecasts when we assume to know some features associated with users and items. We denote by $\bar{R} \in R^{m\times n}$ the matrix of rating forecasts, which will be built from the input matrices $\hat{P} \in R^{m\times k}$, for which, in the i-th row ($i = 1, \dots, m$), k features of i-th user are stored, with $k \le m$ and $\hat{Q} \in R^{k\times n}$, for which, in the j-th column ($j = 1, \dots, n$), k features of j-th item are stored, with $k \le n$. We observe that the number k of columns of the matrix $\hat{P}$ must be equal to the number of rows of the matrix $\hat{Q}$. Intuitively, the predictions generated through the matrix $\hat{R} = \hat{P}\hat{Q}$ seem reliable. The error returned by the resulting method will then be tested that the provided results are not reliable.

Some properties inherent to the matrices defined in the theorem Eckart-Young theorem are obtained. Lemma 2 let $R \in R^{m\times n}$, let $k \le r(R)$ and let U_k, V_k, D_k be the matrices defined in (3) and let I_k be the identity matrix of dimension k. The following properties apply: i) $U_k^T U_k = I_k$; ii) $V_k^T V_k = I_k$; iii) $R_k R_k^T = U_k D_k^2 U_k^T \in R^{m\times m}$; and iii) $R_k^T R_k = V_k D_k^2 V_k^T \in R^{n\times n}$. From properties 1 and 2 of Lemma 2, it follows that the matrices U_k and V_k maintain the orthogonality of the columns but lose, concerning the matrices U and V of the relation (2), the orthogonality of the rows.

The following lemma is presented beforehand, in which some properties of the matrices defined in the relationship (4) are analyzed. Lemma 3 Let $R \in R^{m\times n}$ and let the matrix $R_k = U_k D_k V_k^T$ be obtained according to relation (3). Let P and Q the matrices defined in the relation (4). Then the following properties apply: i) $PP^T = U_k D_k U_k^T \in R^{m\times m}$; ii) $P^T P = D_k \in R^{k\times k}$; iii) $Q^T Q = V_k D_k V_k^T \in R^{n\times n}$; and iv) $QQ^T = D_k \in R^{k\times k}$. From Lemma 3 we deduce that the matrix P columns constitute a system of orthogonal vectors. The same can be said for the rows of the Q matrix. We will then modify the input matrices $\hat{P}$ and $\hat{Q}$ in order to satisfy the properties 2 and 4 of the Lemma 3.

Theorem 1. Let the input matrices $\hat{P} \in R^{m\times k}$ and $\hat{Q} \in R^{k\times n}$ have rank $r(\hat{P}) = r(\hat{Q}) = k$. Denote with μ_i the eigenvalues of the matrix $\hat{P}^T\hat{P}$ and with λ_i the eigenvalues of the matrix $\hat{Q}\hat{Q}^T$, $i = 1, \dots, k$. Then:

a. There exist orthogonal matrices $X, Y \in R^{k\times k}$ such that,

$$Y^T\left(\widehat{P^T}\hat{P}\right)Y = D_{\hat{P}} = diag(\mu_1, \dots, \mu_k) \; X^T\left(\hat{Q}\widehat{Q^T}\right)X = D_{\hat{Q}} = diag(\lambda_1, \dots, \lambda_k) \tag{5}$$

   with $D_{\hat{P}}, D_{\hat{Q}}$ invertible matrices and $\mu_1 \ge \dots \ge \mu_k$ and $\lambda_1 \ge \dots \ge \lambda_k$.

b. The matrices $\widetilde{U_k} = \hat{P}Y\sqrt{D_{\hat{P}}^{-1}}$ and $\widetilde{V_k} = \widehat{Q^T}X\sqrt{D_{\hat{Q}}^{-1}}$ satisfy properties 1 and 2 of Lemma 2, i.e., they have orthogonal columns;
c. The matrices $\widetilde{U}_k, \widetilde{V}_k, \widetilde{D}_k = \sqrt{D_{\hat{P}}}\sqrt{D_{\hat{Q}}}$ and $\tilde{P} = \widetilde{U}_k\sqrt{\widetilde{D}_k}, \tilde{Q} = \sqrt{\widetilde{D}_k}\widetilde{V}_k^T$ satisfy properties 1-4 of Lemma 3;
d. The matrix $\bar{R} = \tilde{P}\tilde{Q}$ coincides with its reduced factorization matrix, defined in (3), i.e. $\bar{R}_k = \bar{R}$;
e. The matrix $\bar{R}$ has singular values $\sigma_1, \dots, \sigma_k$ satisfying $\sigma_i = \sqrt{\mu_i}\sqrt{\lambda_i}, i = 1, \dots, k$.

Proof in order to prove (a), we observe that the matrix $\hat{Q}\hat{Q}^T \in R^{k\times k}$ is a real symmetric matrix, then for the spectral theorem it is diagonalizable. In particular, it is similar to the diagonal matrix $D_{\hat{Q}} \in R^{k\times k}$ containting its eigenvalues on the diagonal, through the orthogonal matrix $X \in R^{k\times k}$ containing the corresponding eigenvectors. Analogously the matrix $\hat{P}^T\hat{P} \in R^{k\times k}$ is similar to the diagonal matrix $D_{\hat{P}} \in R^{k\times k}$ containing its eigenvalues on the diagonal, through the orthogonal matrix $Y \in R^{k\times k}$ containing the corresponding eigenvectors. Furthermore, from the hypothesis that matrices $\hat{P}^T\hat{P}$ e $\hat{Q}\hat{Q}^T$ have maximal rank k, we deduce that the respective eigenvalues λ_j $j = 1, \dots, k$ and μ_i $i = 1, \dots, k$ are all non-zero. In particular, the matrices $D_{\hat{Q}}$ e $D_{\hat{P}}$ are invertible.

In order to prove (b), by exploiting (5), we obtain $\widetilde{U}_k^T\widetilde{U}_k = \sqrt{D_{\hat{P}}^{-1}}Y^T\hat{P}\hat{P}Y\sqrt{D_{\hat{P}}^{-1}} = I_k$ and $\widetilde{U}_k^T\widetilde{U}_k = \sqrt{D_{\hat{P}}^{-1}}Y^T\hat{P}\hat{P}Y\sqrt{D_{\hat{P}}^{-1}} = I_k$, where I_k is the identity matrix of dimension k. As regards (c), the properties 1-4 of Lemma 3 immediately follow from $\tilde{P} = \widetilde{U}_k\sqrt{\widetilde{D}_k}$ and $\tilde{Q} = \sqrt{\widetilde{D}_k}\tilde{V}^T$k, by using (b).

We finally prove properties (d) and (e). The matrix $\bar{R}$ can be obtained through the relation $\bar{R} = \tilde{P}\tilde{Q} = \tilde{U}_k\sqrt{\tilde{D}_k}\sqrt{\tilde{D}_k}\tilde{V}_k^T = \tilde{U}_k\tilde{D}_k\tilde{V}_k^T$. Then, by construction, the obtained matrix $\bar{R}$ coincides with the matrix $\bar{R}_k$ associated with relation (3). Furthermore, again by construction, the singular values $\sigma_i = (\tilde{D}_k)_{ii} = \sqrt{\mu_i}\sqrt{\lambda_i}$ $i = 1, \dots, k$ of $\bar{R}$ coincide with the product of the square roots of the eigenvalues of the matrices $\hat{P}^T\hat{P}$ and $\hat{Q}\hat{Q}^T$.

Figure 1 summarizes how the proposed approach works. The users and items profiles memorized in $\hat{P}$ and $\hat{Q}$ are elaborated through the users profiles elaboration module and the items profiles elaboration module, respectively. These modules calculate the matrices $\tilde{P}$ and $\tilde{Q}$ described in Theorem 1. The rating forecasts module calculates the matrix $\bar{R}$ that contains the rating forecasts.

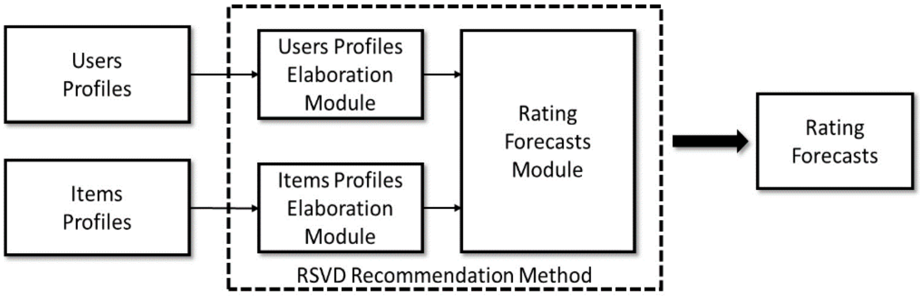

Figure 1. Summary of the RSVD method developed through Theorem 1

3. RESULTS AND DISCUSSION

Some numerical tests of the RSVD procedure generated by Theorem 1 compared with the average rating filling [21], stochastic gradient descent [22], [23] and biased stochastic gradient descent [22], [23] methods are presented. The values used for the numerical experiments related to latent factors is $k = 18$ because of the number of features related to the used dataset, the learning rate of stochastic gradient descent (SGD) is $\alpha = 0.01$, the parameters of biased stochastic gradient descent (BSGD) are $\alpha = 0.04$, $\lambda = 0.005$. Finally, the SGD and BSGD methods are tested for different iterations. The values of parameters are obtained through numerical experiments on the accuracy, and these experiments are reported in Figure 2. In particular, Figure 2(a) describes the parameter estimation for SGD algorithm, Figure 2(b) describes parameter estimation for BSGD algorithm.

The considered database is the MovieLens100k database made available by the GroupLens Research Project of the University of Minnesota [32]. This database provides a matrix $\hat{Q}$ where the element (c, j) is 1 if the movie j-th is part of the category c-th otherwise, it is zero. The matrix $\hat{P}$ was constructed through the means of the known ratings by category associated with each user. It is emphasized that this approach constitutes a Theorem 1 method adaptation for the test on the MovieLens100k database. In fact, the proposed approach is a content-based RS that needs users' and items' profiles. The errors evaluated are the RMSE and the mean absolute error (MAE). Let N be the set of pairs (i, j) for which the numerical test is carried out and $\hat{r}_{ij}$ the prediction made for the pair (i, j). The above errors are defined in relation (6).

$$RMSE = \sqrt{\sum_{(i,j)\in N} \frac{(\hat{r}_{ij} - r_{ij})^2}{|N|}} \qquad MAE = \sum_{(i,j)\in N} \frac{|\hat{r}_{ij} - r_{ij}|}{|N|} \tag{6}$$

3.1. Results

Due to oscillating results on the value of MAE for the methods SGD and BSGD, the reported results are the median of the value obtained in 11 experiments. The RSVD method calculates the rating forecasts through the formula $\bar{R} = \tilde{P}\tilde{Q}$ presented in Theorem 1. Instead, the PQ represents the procedure that generates rating forecasts $\bar{R}_{PQ} = \hat{P}\hat{Q}$ through the product of the matrices $\hat{P}$ e $\hat{Q}$ defined without the changes suggested by Theorem 1. Furthermore, these predictions are regularized on the interval [1,5]. The evaluation of the PQ method aims to give consistency to the practical application of Theorem 1.

Table 1 and Figure 3 show that the RSVD method has a worse mean absolute error value than the MAE of the average rating filling (ARF), SGD, and BSGD methods. The root mean squared error value is also worse than the above cases, except for applying the stochastic gradient descent procedure associated with $iter = 10$. However, these results should not be read in a negative key. Indeed, they testify to the goodness of the evaluation forecasts provided. The MAE and the RMSE values resulting from RSVD, although worse, are not far from those of competing methods. This implies that it is possible to build reliable ratings without starting evaluations instead of necessary for the comparison procedures. Moreover, the use of matrix factorization gives RSVD better scalability than content-based methods.

From a semantic point of view, the value associated with k in RSVD represents the number of categories of interest to the user. Therefore, it is possible to intuitively build user and item profiles needed to RSVD, without adding too much cost to the method. RSVD method presents a temporal mean of execution equal to 0.9975 s, comparable to that of other methods. Finally, focus on the differences between the RSVD and PQ procedures. The poor results found for the second method justify the increase in complexity associated with the first, which, as mentioned, is comparable with methods already tested and recognized.

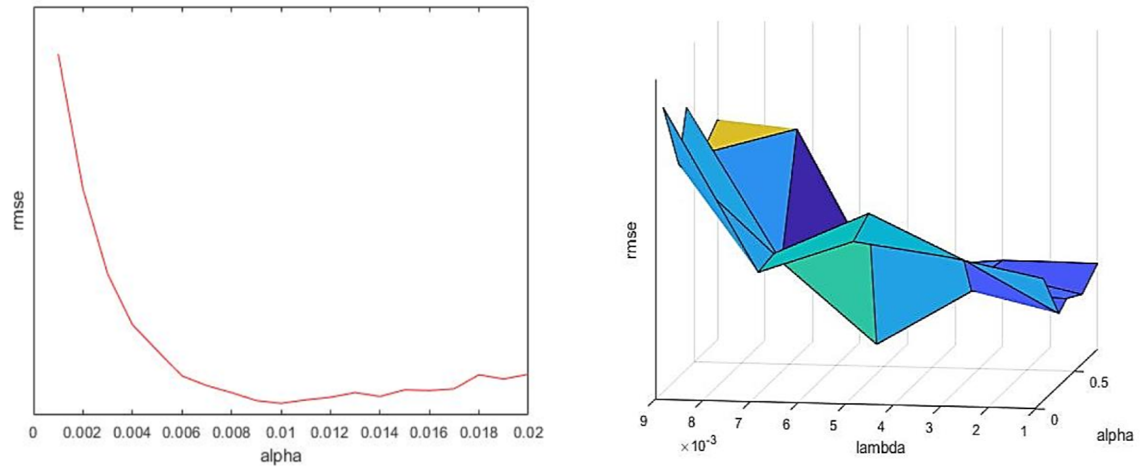

Figure 2. Graphical representation of root mean squared error (RMSE) error of method (a) SGD when α changes and (b) BSGD when α and λ change

Table 1. Results of the methods ARF, SGD, BSGD, RSVD and PQ

	k	*iter* (if any)	MAE	RMSE
ARF	18		0.6882	0.8741
SGD	18	10	0.7227	0.7813
	18	20	0.7210	0.6973
	18	40	0.6506	0.6586
BSGD	18	10	0.4829	1.0721
	18	20	0.4821	0.9198
	18	40	0.5824	0.8043
RSVD	18		0.7664	0.9512
PQ	18		1.6458	1.8986

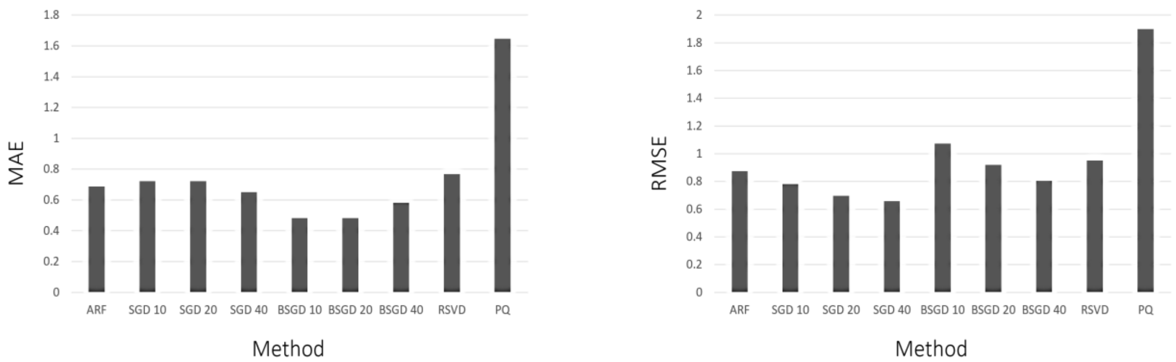

Figure 3. Results of methods ARF, SGD, BSGD, RSVD, and PQ

4. CONCLUSION

After the introduction and the description of the RS background, this paper describes a novel Content-Based approach, namely RSVD. The method's accuracy on the MovieLens100k dataset is 0.7664 for MAE and 0.9512 for RMSE. These results are compared to Collaborative Filtering methods returning promising results. This method exploited the properties of SVD in a new way. In fact, the SVD is usually used in collaborative filtering methods; instead, the proposed approach exploits SVD in a Content-Based recommendation method and allows providing an appropriate starting point for a system that cannot take advantage of standard procedures. Due to the encouraging results regarding the RSVD method, future works include introducing the context variables to complete and finalize the research work. Finally, the proposed approach will be tested in a real case of study.

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